

Optimal money-creation in ‘pure-currency’ economies: a conjecture*

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Abstract

In a *pure-currency* economy, money is the only durable object and people have private histories. In such economies, taxation is not feasible and in some of them trade is enhanced through the use of money. For economies of that kind in which a nondegenerate distribution of money, part of the state of the economy, affects trades and real outcomes, and in which trades affect the state at the next date, the conjecture is that there are transfer schemes financed by money creation that improve ex ante representative-agent welfare relative to what can be achieved holding the stock of money fixed.

Key words: outside-money, inflation, optima

JEL classification: E52, E58

1 Introduction

Fifty years ago and earlier, the challenge for monetary theory was to integrate money into the theory of value, the general-equilibrium competitive model (see, for example, Debreu [8], page 36, note 3). More recently, as a result of work by Ostroy [23], Townsend [28], and Kocherlakota [15], the challenge has come to be posed more generally: find settings in which the presence of money allows good outcomes to be

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achieved; or, in Hahn’s [11] terminology, find settings in which money is *essential*. This new way of posing the challenge calls for a mechanism-design approach because any essentiality result requires an examination of all ways of achieving allocations. The general conclusion that has emerged from that approach is that *imperfect monitoring*, some privacy of the history of individual actions, is *necessary* for essentiality of money (see Wallace [30]). Stated very loosely, money is potentially useful in trade between strangers; it is not needed when everyone knows what everyone else has done in the past—in an idealized version of an isolated Amish community, in an Israeli kibbutz, or on Crusoe’s island even after he meets Friday.

There are, however, no general necessary and sufficient conditions for essentiality of money. In particular, imperfect monitoring is not sufficient to give a role for money.¹ Therefore, it is not surprising that many models contain extreme sufficient conditions in order to assure that money is essential. Roughly speaking, the following conditions are sufficient for essentiality: no monitoring, discounting (that is not taken to the limit of no discounting), a large number of agents, some background absence-of-double-coincidence, and no durable objects other than money.² Borrowing a term from Lucas [21], I label such economies ‘pure-currency’ economies.

Indeed, while not taking a mechanism-design point of view, there is a substantial literature that adopts many of those assumptions. For example, Lucas [21] and Bewley [4] make all those assumptions except no-monitoring. Instead, they directly rule out borrowing and lending. Although no-monitoring implies no-borrowing-and-lending, the two assumptions are very different. No-monitoring is about the environment, while no-borrowing-and-lending is not; and no-monitoring may have other implications—in particular, for the kinds of taxes that can be levied.³

As I have defined pure-currency economies, they should be thought of as extreme underground economies in which no taxation of any sort is feasible.⁴ That feature differentiates this work from almost all other work on economies that resemble pure-currency economies. (Exceptions are Deviatov [7], Hu *et. al.* [13], and Andolfatto [1] and [2], but they study very particular models.) In the absence of taxation, the feasible interventions are a rich class of schemes that involve positive transfers in the form of money.

¹Any setting in which the *folk theorem* holds is a setting in which money has no role and many such settings have imperfect monitoring (see, for example, Sagaya [24]).

²See Araujo [3] for one model that establishes essentiality using these conditions.

³Obviously, if the set of allowable taxes is too large, then money and monetary policies are irrelevant (see, for example, Correia *et. al.* [6]).

⁴Presumably, Lucas [21] called the money in his model currency, because other objects sometimes labeled money require an informational network that violates no borrowing-and-lending. And, of course, currency has always been associated with tax evasion.

There are several pure-currency models in which money creation accomplished by way of lump-sum transfers is optimal (see Levine [18], Kehoe *et. al.* [14], Molico [22], Green and Zhou [10]—even though lump-sum taxes are allowed. In those models, the transfers affect the distribution of money holdings in a beneficial way even though they lower the return on money. I suspect, however, that even economists familiar with this work view models in which money creation is beneficial as quite special. My conjecture, on the contrary, is that money creation, although not necessarily produced by way of lump-sum transfers, is almost always optimal in *pure-currency* economies when the no-taxation restriction is taken into account. The exceptions, as we will see, are models in which the distribution of money holdings is *not* a state variable of the model.

Pure-currency economies are very extreme kinds of economies. At the end, I discuss aspects of the conjecture that are likely to carry over to other economies—monetary economies with some monitoring or with other durable assets. For pure-currency economies, an implication of the conjecture is that optimal intervention, even its direction in a sense to be described, depends on all the details of the economy. That conclusion ought to carry over to almost all monetary economies.

2 The conjecture

I describe the conjecture against the background of a class of discrete-time pure-currency economies that lend themselves to an *ex ante*, representative-agent notion of welfare. The economies have infinitely-lived agents who maximize expected discounted utility. The state of the economy entering date- t is a joint distribution over types and money holdings, denoted λ_t . Then shocks, denoted ε_t , are realized, including, possibly, aggregate shocks. For example, in Kehoe *et. al.* [14], types are endowments at the previous date and ε_t determines whether a “switch” in the cross-section pattern of endowments occurs. In Molico [22] or Zhu [31], divisible-money versions of Shi [25] and Trejos and Wright [29], λ_t is simply a distribution of money holdings and ε_t determines who meets whom in pairwise meetings.

There are two stages of actions. The first stage has trade and activities, production and consumption, that determine realized period utilities. In some models, trade is centralized; in others, it occurs in pairwise meetings between agents. The second stage is solely for transfers from the planner. If a person has money holdings $x \in \mathbb{R}_+$ after trade at date t , then the post-transfer holding is $x + \tau_t(x)$, where $\tau_t : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and is weakly increasing. As spelled out below, the restriction that $\tau(x) \geq 0$ is implied by no monitoring; the restriction that τ is weakly increasing assures that people do not want to hide money at the transfer stage. I also assume

that money is a uniform divisible object.

For my purposes, it is enough to focus on the following class of two-parameter transfer functions:

$$\tau_t(x) = \max\{0, a_t + b_t x\} \text{ with } b_t \geq 0. \quad (1)$$

Notice that if $a_t = 0$, then the transfers are proportional to holdings. Such transfers, which amount to a change in the units in which money is measured, are obviously neutral. Therefore, they are equivalent to no-intervention, a fixed stock of money. It follows immediately that a scheme with $a_t > 0$ is equivalent to lump-sum transfers because it is a combination of a lump-sum transfer equal to a_t and a proportional scheme. In what follows, schemes with $a_t > 0$ are called progressive schemes because they shift wealth away from the wealthy, while those with $a_t < 0$ are called regressive schemes because they shift wealth toward the wealthy. Schemes with $a_t < 0$ give positive transfers only to those with holdings larger than $-a_t/b_t$.⁵ As we will see, regressive schemes bear some resemblance to lump-sum tax schemes in terms of their consequences.

Consistent with a mechanism-design point of view, I view the planner as choosing both the transfer scheme and the trades that occur subject to some conditions. I denote the trade that a person makes as $g_t(s_t; \lambda_t, \varepsilon_t)$, where s_t is the person's individual situation (type and money holdings just prior to trade). In general, g_t determines the person's consumption (and production) at date t and the person's end-of trade money holdings, denoted x_t . We will say that an allocation, a sequence $\{g_t(s_t; \lambda_t, \varepsilon_t), \tau_t(x_t), \lambda_{t+1}\}_{t=0}^{\infty}$, is incentive-feasible or implementable given the initial state λ_0 , if it satisfies the law of motion implied by the model, denoted

$$\lambda_{t+1} = H(\lambda_t, \varepsilon_t, g_t, \tau_t), \quad (2)$$

and if (i) the trades are both individually rational (*IR*) and subject to static group defection (as in Hammond [12]) and (ii) the transfers are subject to *IR*.⁶

The *IR* parts of (i) and (ii) are not controversial. Static group defection in (i) is a static core requirement. In a model with centralized trade and a large number of agents, core convergence implies price-taking trade. (Such trade cannot be taxed if groups can defect by conducting their trade behind the back of the planner as in Hammond [12].) In a model with pairwise meetings, static group defection pertains to the pair in each meeting at each date and says that their trade should exhaust

⁵I arrived at regressive schemes by thinking about a class of nonnegative transfers for which no intervention ($a_t = 0$) is interior. I subsequently learned that Andolfatto [1] uses a regressive scheme and shows that it can be beneficial in a particular model.

⁶Throughout, the allocations I describe are weakly implementable. Typically, there is always another equilibrium in which money is not valued.

the gains from trade.⁷ The strong assumption is that there is that groups cannot defect at the transfer stage. If there is static group defection at the transfer stage, then regressive schemes are not feasible. In the concluding remarks, I hint at ways to accomplish what regressive schemes accomplish while allowing for such static group defection.

Given an incentive-feasible allocation, the ex ante welfare associated with it is date-0 expected discounted utility, where the expectation is taken with respect to the distribution λ_0 . It is also helpful to have a definition of *first-best*. Given an environment, I mean by first-best what is best if no-monitoring is replaced by perfect monitoring. (Throughout, I assume that agents, in contrast to the planner, cannot commit to future actions and, therefore, that first-best is subject to that inability to commit.)

Now I can state two assumptions and the conjecture.

A1. (Significant two-way interaction between the state and trades): λ_t (including the distribution of money) affects the date- t payoff in the welfare criterion by way of the trades, g_t , and g_t affects λ_{t+1} .

A2. (Interiority) The best no-intervention outcome ($\tau_t \equiv 0$) has valuable money and is not first best.

Now I can state the conjecture.

Conjecture. *If a pure-currency economy satisfies A1 and A2, then generically the best outcome has nonneutral intervention ($a_t \neq 0$) at almost every date.*

Before I say why the conjecture is plausible, some comments are in order about assumptions A1 and A2. There are well-known models that do not satisfy A1. An *OLG* model of two-period lived agents does not satisfy A1 because the old at a date have all the money and are at a corner: they offer all their money no matter how money is distributed among them. Any model in which money holdings are the same for all agents does not satisfy A1. These include Shi [26] in which any dispersion in money holdings is eliminated by sharing within a so-called *large family* and Lagos and Wright [17] in which any such dispersion is eliminated by a stage of centralized trade with transferable utility. Another model which violates A1 is the random-matching model of Shi [25] and Trejos-Wright [29] in which money is discrete and individual holdings are in $\{0, 1\}$. There, because trades necessarily have buyers and sellers switching money holdings, the distribution, although nondegenerate, is not affected by trades. Needless to say, all the exceptions are very special models for which conformity to A1 is produced by obvious generalizations. As regards A2, although there are models in which a fixed stock of money gives rise to a first-best allocation, that seems to happen only in models which also violate A1 (see, for example, Hu *et.*

⁷This requirement is far from straightforward for models in which people can hide money.

al. [13]) or seem to violate static group defection (see Kocherlakota [16]).⁸ Indeed, in many models, A1 implies A2.

Although the conjecture does not say much, I do not have a proof. The ingredients of an argument would seem to be standard and are reminiscent of so-called *second-best* theory (see Lipsey and Lancaster [20]). Consider a policy $\{a_t, b_t\}_{t=0}^{\infty}$ with $a_t \equiv 0$ and $b_t > 0$ for all t . By neutrality, this is the same as no-intervention. Now, for any date t and with $b_t > 0$ fixed, consider a δ -neighborhood around 0 for a_t . This is an open set around no-intervention at date- t . By A1 and A2, there ought to be a policy with $a_t \neq 0$ that increases ex ante welfare except in special cases. In particular, absent intervention, the trades at t are doing double duty: they must be a compromise between those which are best for production and consumption at t and those which are geared solely toward producing a desirable distribution of money holdings entering the next date. Nonneutral transfers represent additional instruments. Generically, they ought to be helpful.

3 Examples

I discuss two kinds of examples, one with centralized trade and one with pairwise meetings. The centralized-trade example is one for which a steady-state version of the conjecture can be verified for a particular specification. The second example is the kind of model within which I first thought about the conjecture. It does not lend itself to any simple demonstration of its validity. For it, even verifying A1 and A2 is challenging.

3.1 Kehoe, Levine, and Woodford (KLW)

KLW [14] generalizes Levine [18] and can be regarded as a special case of Bewley [4]. The economy is an exchange economy with one good per discrete date and a unit measure of people. Each person maximizes expected discounted utility of consumption with discount factor $\beta \in (0, 1)$ and period utility function $u : \mathbb{R}_{++} \rightarrow \mathbb{R}$, where u is twice differentiable over the relevant part of the domain and $u'' < 0 < u'$. There are two possible endowment patterns at each date: in one of them, each of one-half of the people has endowment y^h and each of the remaining half has endowment y^l ; in the other the reverse is the case. Here, $y^h > y^l > 0$. Let $\pi \in [0, 1]$ be the probability that the pattern switches. The initial condition is that each pattern is

⁸As demonstrated in Hu et. al. [13] for the Lagos-Wright model [17], in the absence of A1, it is easier to attain first-best outcomes.

equally likely. Let $\bar{y} = (y^h + y^l)/2$. Also, trade is price-taking spot trade of the good for money subject to the sequence of flow budget constraints,

$$c_t + p_t x_t \leq y_t + p_t [x_{t-1} + \tau_{t-1}(x_{t-1})] \text{ for } t \geq 0, \quad (3)$$

where c_t is date- t consumption, p_t is the date- t price of money, and $x_t \geq 0$ is the end-of-trade amount of money at date- t .

KLW analyze special cases for which there are equilibria in which those who realized the high endowment at $t - 1$ hold all the money at the end of the trade stage at $t - 1$. They call such equilibria two-state Markov equilibrium. They also assume that money is distributed initially in that way. Their policies are equal per capita lump-sum transfers or taxes. For some versions of their model, they show that the best policy in their class is a lump-sum transfer. One such example is $u(c) = \min\{\ln c, \bar{u}\}$ (where $\bar{u} > \ln y^h$ and assures that transversality conditions hold, but plays no other role); $\beta = \pi = 1/2$; and $y^h/y^l = 10$ (see [14], page 520).

I use their model for two purposes. First, for the special case $\pi = 1$ (see Townsend [27]), I construct the best constant allocation and show that it can be supported by a constant-inflation regressive scheme from the class above. That result illustrates in a very simple way the beneficial role of regressive schemes. (It is well-known (see Townsend [27]) and easily shown that progressive schemes reduce welfare when $\pi = 1$.) Then, I study constant-inflation intervention schemes for the above particular example (but without setting π). In that example, a two-state valued-money Markov equilibrium with no intervention exists for all $\pi \in (1/9, 1]$ (see condition (6.4) on page 519 of [14]).

3.1.1 The special case $\pi = 1$.

Let s be the unique solution to

$$\frac{u'(y^h - s)}{\beta u'(y^l + s)} = 1, \quad (4)$$

where, in accord with assumption A2, I assume that s is positive. (That is, the best constant no-intervention outcome has valued money.) Notice that s is the constant goods value of money acquired by each y^h person when money has rate-of-return zero.

The first-best constant allocation is constructed as follows. Let s^* be the unique positive solution to

$$u(y^h - s^*) + \beta u(y^l + s^*) = u(y^h) + \beta u(y^l) \quad (5)$$

and let $\hat{s} = \min\{s^*, y^h - \bar{y}\}$. The first-best constant allocation is $(\hat{c}^h, \hat{c}^l) = (y^h - \hat{s}, y^l + \hat{s})$, where \hat{c}^h (\hat{c}^l) is consumption of those with endowment y^h (y^l). Clearly, (\hat{c}^h, \hat{c}^l) gives higher ex ante utility than that implied by (4). Also, (\hat{c}^h, \hat{c}^l) is the best allocation in the set of stationary allocations. If $(\hat{c}^h, \hat{c}^l) = (\bar{y}, \bar{y})$, then it is best. If not, then any better constant allocation must satisfy $c^h + c^l \leq 2\bar{y}$ and $c^h < \hat{c}^h$. But, this cannot be an equilibrium because, in contrast to schemes which permit taxation, consuming $(c_t, c_{t+1}) = (y^h, y^l)$ for all t is in the choice set for people with $y_t = y^h$ and is strictly preferred by them to any (c^h, c^l) that satisfies $c^h + c^l \leq 2\bar{y}$ and $c^h < \hat{c}^h$. Our claim is that (\hat{c}^h, \hat{c}^l) is attained as an equilibrium under some regressive scheme.

Claim 1 *If the solution for s to (4) is positive, then there is a regressive scheme that supports (\hat{c}^h, \hat{c}^l) as an equilibrium.*

The proof (available in an on-line appendix) is a guess-and-verify argument. The idea is to construct a transfer scheme so that the two-date consumption opportunity set for people with $y_t = y^h$ given that they enter date t and date $t + 2$ with no money makes (\hat{c}^h, \hat{c}^l) an optimal choice. Here is one way to do that.

Let

$$\gamma_0 = \frac{u'(y^h)}{\beta u'(y^l)} \text{ and } \gamma_1 = \frac{u'(\hat{c}^h)}{\beta u'(\hat{c}^l)}. \quad (6)$$

Notice that $\gamma_0 < 1 < \gamma_1$. Then, our candidate supporting transfer scheme is $\tau_t(x_t) = \max\{0, a_t + b_t x_t\}$, where

$$b_t = \frac{\gamma_1}{\gamma_0} - 1 \text{ and } \frac{\tau_t(2m_t)}{2m_t} = \frac{1}{\gamma_0} - 1 \quad (7)$$

and m_t is the pre-transfer per capita stock of money at date t . Moreover, we guess that prices satisfy $p_{t+1}/p_t = \gamma_0$ and $2p_t m_t = y^h - \hat{c}^h$, and that choices satisfy

$$p_t x_t = \begin{cases} 0 & \text{if } y_t = y^l \\ 2p_t m_t & \text{if } y_t = y^h \end{cases}. \quad (8)$$

Notice, also, that the two equations in (7) and (8) imply a sequence for a_t . In particular, we have

$$\frac{a_t}{2m_t} + b_t = \frac{\tau_t(2m_t)}{2m_t} = \frac{1}{\gamma_0} - 1.$$

Therefore,

$$\frac{p_t a_t}{2p_t m_t} + b_t = \frac{1}{\gamma_0} - 1.$$

Then, using, $2p_t m_t = y^h - \hat{c}^h$, we have

$$p_t a_t = (y^h - \hat{c}^h)(1 - \gamma_1)/\gamma_0, \quad (9)$$

which is negative, as required for a regressive scheme. The rest of the argument is straightforward.

3.1.2 The special case $u(x) = \min\{\ln x, \bar{u}\}$, $\beta = 1/2$, $y^h/y^l = 10$, and $\pi \in (1/9, 1)$.

For this example, I study a class of constant-inflation intervention schemes. The class consists of the above two-parameter policies with $b_t = b > 0$ and with a constant growth rate of the stock of money, $\mu = (m_{t+1} - m_t)/m_t$, where m_t denotes the per capita stock of money just before transfers at t .

If those with $y_t = y^l$ end the trade-stage with no money and those with $y_t = y^h$ end with $2m_t$ (as required for a two-state Markov equilibrium), then the implied law of motion for m_t is

$$m_{t+1} - m_t = \begin{cases} a_t + b(2m_t)/2 = a_t + bm_t & \text{if } a_t \geq 0 \\ [a_t + b(2m_t)]/2 = a_t/2 + bm_t & \text{if } a_t \leq 0 \end{cases}. \quad (10)$$

It follows that

$$a_t/m_t = \begin{cases} \mu - b & \text{if } \mu \geq b \\ 2(\mu - b) & \text{if } \mu \leq b \end{cases}. \quad (11)$$

Notice that $a_t/a_{t-1} = 1 + \mu$. Also, a_t/m_t , and, hence, ex ante welfare, is not differentiable at $\mu = b$. We fix $b > 0$ and study ex ante welfare as a function of μ for μ near b , where $\mu = b$ is the same as no-intervention, $\mu > b$ is a progressive scheme, and $\mu < b$ is a regressive scheme.

Claim 2 *Let $u(x) = \min\{\ln x, \bar{u}\}$ with $\bar{u} > \ln y^h$, $\beta = 1/2$, and $y^h/y^l = 10$. Let $W(\mu)$ denote ex ante welfare and let $W'_-(b)$ and $W'_+(b)$ denote the left- and right derivatives of $W(\mu)$ at $\mu = b$. These derivatives satisfy*

$$(signW'_-(b), signW'_+(b)) = \begin{cases} (-, +) & \text{if } \pi \in (1/9, .74) \\ (-, -) & \text{if } \pi \in (.74, 1] \end{cases}. \quad (12)$$

Because the left derivative is always negative, a regressive scheme, one with $\mu < b$, always produces a local improvement; a progressive scheme produces a local improvement if $\pi < .74$.

As for claim 1, the proof, which appears in the on-line appendix, is a guess and verify argument. Let $\sigma \in \{0, 1\}$ denote the state at a date, where 0 at date- t means that the high-endowment people at t had the high endowment at $t-1$ (no switch at t) while 1 at date- t means that the high endowment people at t had the low endowment at $t-1$ (a switch at t). The guess that there is a valued money, two-state Markov equilibrium gives rise to one pair of simultaneous equations for the two-date saving decision for high-endowment people when $\mu \geq b$ and another pair when $\mu \leq b$. The first pair allows us to derive an expression for $W'_+(b)$ and the second pair an expression for $W'_-(b)$.

The claim says that a regressive scheme is always locally improving. To see why that is true, let $c_-^l(\sigma; \mu)$ be equilibrium consumption of a low-endowment person in state σ for $\mu \leq b$. It follows that $\partial c_-^l(1; \mu)/\partial \mu|_{\mu=b} < 0$, while $\partial c_-^l(0; \mu)/\partial \mu|_{\mu=b} = 0$, where these are left derivatives. That is, $c_-^l(0; \mu) = y^l$ and does not depend on μ , while $c_-^l(1; \mu)$ is decreasing in μ at $\mu = b$. Because $c_-^l(1; b) < \bar{y}$, there is a local improvement from a regressive scheme for any π . This source of gain is the same as occurs in the $\pi = 1$ economy and is a consequence of the result that low-endowment people in state $\sigma = 0$ are at a corner in which they consume their endowment.

The result for progressive schemes is in K LW. Let $c_+^l(\sigma; \mu)$ be equilibrium consumption of a low-endowment person in state σ for $\mu \geq b$. In general, we have $\partial c_+^l(1; \mu)/\partial \mu|_{\mu=b} < 0$ and $\partial c_+^l(0; \mu)/\partial \mu|_{\mu=b} > 0$, where these are right derivatives. That is, a progressive intervention lowers consumption of low endowment people at the date of a switch (because it discourages saving of high endowment people at such a date), but increases consumption of low endowment people when there is no switch—effects which have offsetting consequences for welfare. The result says that $\pi \approx .74$ is a cut-off that determines when the net effect is a welfare gain.

Because the source of a gain for regressive and progressive schemes is very different (one affects $c_-^l(1; \mu)$ favorably and the other affects $c_+^l(0; \mu)$ favorably), it can happen that both kinds of schemes are locally improving. That happens for $\pi \in (1/9, .74)$, so that no-intervention is a local minimum for such π 's.

The above follows K LW in studying a class of constant interventions. Of course, even if there exists a no-intervention, two-state Markov equilibrium, that does not imply that the best intervention scheme is a constant scheme. Nevertheless, the study of constant schemes suffices to prove for this example that never intervening ($a_t \equiv 0$) is not best.⁹

⁹In a variant of the K LW model that does not have two-state Markov equilibria, Lippi and

3.2 Shi and Trejos-Wright with divisible money

Shi [25] and Trejos and Wright [29] is a pure-currency economy with random pairwise meetings. Time is discrete and there is a nonatomic unit measure of people, each of whom maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. Production and consumption occur in pairwise meetings that occur at random in the following way. Just prior to such meetings, each person looks forward to being a consumer who meets a random producer with probability $1/K$, looks forward to being a producer who meets a random consumer with probability $1/K$, and looks forward to no pairwise meeting with probability $1 - (2/K)$, where K , an integer, exceeds two. The period utility of someone who becomes a consumer and consumes $y \in \mathbb{R}_+$ is $u(y)$, where u is strictly increasing and strictly concave, and $u(0) = 0$. The period utility of someone who becomes a producer and produces $y \in \mathbb{R}_+$ is $-c(y)$, where c is strictly increasing, convex, and $c(0) = 0$. In addition, $y^* = \arg \max_{y \geq 0} [u(y) - c(y)]$ exists and is positive. Production is perishable; it is either consumed or lost.¹⁰

Each person's trading history is private information and each person's money holding is in $[0, \infty)$.¹¹ As described below, we can consider versions in which people in meetings can hide money and versions in which they cannot.

The state of the economy at t is a measure, denoted λ_t , over money holdings prior to meetings. Trade in a meeting is denoted $f_t(m, m') \in \mathbb{R}_+ \times [0, m']$, where m (m') is the producer's (consumer's) money prior to trade, and the first component is output (production and consumption), and the second is the money transferred from the consumer to the producer. Transfers from the planner are as above. A symmetric allocation is a sequence $\{f_t, \tau_t, \lambda_{t+1}\}_{t=0}^\infty$.

What determines f_t ? In models of centralized trade, trade is determined by the condition that it be immune to static group defection. When people meet in pairs, possible defection by the pair in the meeting does not determine trades. And, it matters greatly whether people in a meeting can hide money. If they cannot hide money, then the pairwise core is well defined for a given continuation value of money, and is the *IR* segment of the implied contract curve. Trades can be any mapping

Tracter [19] study optimal lump-sum tax policy, while permitting intervention to be a function of the proportion of money held by their analogue of low-endowment people. Their computed optima have lump-sum transfers at some dates and lump-sum taxes at other dates. A plausible conjecture is that if interventions are limited to my class of two-parameter transfer schemes, then the optimum in their example would have progressive transfers at some dates and regressive transfers at other dates.

¹⁰In Shi [25] and Trejos and Wright [29], K is the number of distinct goods and specialization types and $1 - 2/K$ is the probability of a no-coincidence meeting.

¹¹The absence of taxation is particularly easy to justify in this setting. Where would tax collectors be?

from money holdings to the *IR* segment of the contract curve. If people can hide money, then defining cooperative defection by the pair in a meeting is problematic.

The validity of A1 and A2 in such a model is not obvious. A1 almost certainly holds. As regards A2, we know that the first best cannot be achieved. Let $v(y) = [u(y) - c(y)]/K(1 - \beta)$, the ex ante discounted expected utility if $y \in \mathbb{R}_+$ is output in every single-coincidence meeting. If $c(y^*) \leq \beta v(y^*)$, output equal to y^* in every single-coincidence is first best, where the inequality assures that $y = y^*$ is consistent with no commitment. However, no constant and positive y produced in all meetings is consistent with valued money (see Wallace [30]). The other part of A2 is that monetary trade is implementable. That is not so obvious. With a bound on individual holdings, Zhu [31] shows that consumer (buyer) take-it-or-leave-it offers give rise to a steady state with valued money.¹² If the conjecture were adapted to be consistent with bounded holdings, then Zhu's result would suffice for satisfaction of A2.

Of course, confirming A1 and A2 is not a proof of the conjecture for this model. Some existing results hint that it is true. Molico [22] provides examples of steady states in which progressive schemes improve ex ante welfare under consumer take-it-or-leave-it offers. That leaves open whether they would also be optimal if trades were chosen optimally subject to static group-defection. Deviatov [7] studies a version with individual holdings confined to $\{0, 1, 2\}$, no hiding of money, and considers all trades consistent with pairwise defection. He finds some examples in which a probabilistic approximation to a progressive scheme is welfare improving and others in which it is not. He does not study regressive schemes and his upper bound on individual money holdings seems to be too low to give scope to regressive schemes.

4 Concluding remarks

One of the least palatable assumptions I made is that groups cannot exploit the transfers. If they could, then regressive schemes involving money-creation cannot be implemented. However, there may be alternatives whose effects resemble those of regressive schemes. One possibility is that the planner offers a variety of securities, different kinds of government bonds, which are in denominations that prevent them

¹²Although Zhu [31] describes his result for a setting in which people cannot hide money, it is easy to verify that it also holds if people can hide money. One simply interprets the offers by the buyer as a menu. Then, it can be shown that the seller self-selects from the menu the offer that corresponds to his true holdings. In other words, in Zhu's [31] equilibrium, a richer seller does not envy the trade offered to a poorer seller.

from being shared when purchased. If the returns are financed by money creation, then such schemes should approximate regressive schemes.¹³

The conjecture is weak in that it says only that some intervention is beneficial. Can we hope for stronger conclusions—perhaps, a characterization of when an improvement comes from a small progressive scheme and when it comes from a small regressive scheme? I think not. In order to give money a role, the environment must have enough imperfect monitoring and discounting to prevent the folk theorem from holding. Although uncertainty is not necessary for essentiality of money, any general model will have uncertainty and risk-averse people. When that is the case, the assumptions that make money essential inhibit beneficial risk-sharing. A consequence is that the optimum depends on details of the model because progressive schemes tend to improve risk-sharing while regressive schemes tend to raise the return on money in line with the Friedman-rule recommendation.

Pure-currency economies are very special economies. The obvious way to depart from those assumptions is to add durable objects and/or some monitoring so that some credit and some taxation is incentive-feasible. Given the much richer set of instruments available in economies with some monitoring, there is no reason to expect optima to involve money creation in almost all such economies. We can, however, expect that most such models, provided they are monetary, will satisfy A1 and A2—namely; that they will have two-way interaction between trades and the distribution of assets holdings and be such that the first-best (arrived at by replacing imperfect-monitoring by perfect-monitoring) is not attained absent intervention.¹⁴ When A1 and A2 hold, we should expect that optima in such models will involve intervention, and that desirable intervention, even its qualitative nature, will depend on all the details of the model.¹⁵

¹³I began by thinking about the potential beneficial role of government bonds with interest financed by money creation in the context of the second example. However, I quickly noticed that bonds are equivalent to regressive money-transfers schemes under the assumption of no group-defection at the transfer stage. That, in turn, led to the conjecture set out here.

¹⁴If we maintain no-monitoring and add assets like capital or Lucas trees, then it matters whether those assets can be traded easily. Wallace [30] describes a version of putty-clay capital that makes such trade impossible. If, instead, the assets are uniform and, therefore, easily traded, then the classic question of coexistence of money and higher return assets must be confronted in order to get a valued-money equilibrium.

¹⁵See Deviatov and Wallace, [9] for some surprising optima in a seasonal model based on Cavalcanti and Wallace [5], a model with some perfectly monitored people and some who are not monitored at all.

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