Bidder Collusion: Accounting for All Feasible Bidders*

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Abstract

The Idaho Department of Lands (IDL) sells timber from state lands by means of ascending bid auction. In our empirical analysis of all IDL scale auctions from 2004 through 2015, accounting for all auction-specific feasible bidders, we find significant evidence of bidder collusion. Given the complexity of the empirical model and the absence of analytic results, we apply the method of simulated moments to estimate the parameters and Monte Carlo simulations to produce standard deviations of the estimates. The loss to Idaho from the bidder collusion is estimated to be approximately $43 million over this time period.

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1 Introduction

It is well recognized that ascending bid auctions, often referred to as English auctions, are susceptible to collusion by bidders. The intuition for the susceptibility is rooted in the fact that the collusive bidders, often referred to as a ring, will remain active up to the maximal willingness to pay of its highest valuing member, just as that bidder would do acting non-collusively, while all other ring bidders do not meaningfully bid or refrain from bidding at all.[1] The collusive gain comes from the suppression of the bids by those who do not have the maximal willingness to pay in the ring. The absence of an incentive to cheat on the collusive agreement comes from the fact that the ring member with the maximal willingness to pay acts exactly the same as if they were bidding non-collusively.

In practice, English auctions typically have a reserve price. Suppose we observe some English auctions where only one bidder attends. For these one bidder auctions the reserve price is paid by the sole bidder and, with this information alone, there is no reasonable inference to draw about collusion. Suppose we observe some English auctions where between two and five bidders attend. Suppose that for some of these auctions the price paid is a seemingly small premium above the reserve, but, for several other auctions the premium above the reserve price seems reasonable. Alone, this information is certainly not compelling evidence of collusion.

Within the context of the same set of observed auctions, suppose that we now learn that there were between ten and fifteen feasible bidders for each auction. By a feasible bidder we mean a bidder that should have an interest in the tract being sold (a formal definition is provided in Section 4.1 below). In addition, suppose that there is a reasonably high probability that most of the feasible bidders would have

[1] See Krishna (2010, Chapter 11).

a willingness to pay greater than the reserve. Under these circumstances, assuming that submitting a bid remains a relatively low cost activity, it would be difficult to explain within a non-collusive framework scenarios where only a small subset of feasible bidders register for the auction and, relatedly, where the price paid remained close to the reserve. Obviously, there remains a non-trivial inference problem but information relative to the number of feasible bidders is definitely of value within this context.

This paper addresses the issue of bidder collusion at English auctions where there are potentially many feasible bidders not attending the auctions. We focus on data for timber auctions conducted by the Idaho Department of Lands from 2004 through 2015. Our analysis will be conducted entirely within an independent private values framework.[2]

The literature on bidder collusion has two main strands. First, there are studies of collusive bidding where the identity of the collusive bidders, and possibly the nature of the bidder collusion, is known from litigation.[3] Second, there are studies, without direct guidance from litigation, that attempt to infer bidder collusion from the auction data.[4] Our study is in the latter category.

We do not have first-hand knowledge of potential collusive mechanisms at IDL auctions, nor is our current objective that of identifying potential colluders. Nevertheless, we frequently observe prices that are close to reserves, as well as the number

[2]Following Athey, Levin, and Seira (2011) who examine timber auctions in regions that include Idaho, we assume independent private values. Athey, Levin, and Seira (2011) allow for two distributional types in their private value setting (loggers and mills), but since IDL staff has informed us that independent contractors (loggers) are almost never bidders at the IDL auctions, our assumption regarding the distributional source of values is the same as Athey, Levin, and Seira (2011). We also note that Haile, Hong, and Shum (2003) tested for common values at timber auctions and found the evidence did not support common values at scale sales (the data we used for our estimation is exclusively from scale sales).

[3]See Porter and Zona (1993) and Asker (2010).

[4]See, for example, Baldwin, Marshall, and Richard (1997), Bajari and Ye (2003), and Athey, Levin, Siera (2011).

of registered bidders (l_*) significantly lower than that of "feasible" bidders (L), with, in particular, 82 out of 708 auctions for which $l_* = 1$. In the context of our analysis, such findings can be rationalized in two alternative ways. A non-collusive interpretation would require that reserves set by IDL are aggressive to the effect that, for any given auction, a relatively large subset of all potential bidders would draw a maximum willingness to pay, WTP, lower than the reserve, hence the low l_*. An alternative interpretation would be that of a less aggressive reserve policy combined with collusion among a subset of feasible bidders with WTPs larger than the reserve. We observe l_*, the price paid, P, the reserve price, R, the identity of the two highest bidders when $l_* > 1$ as well as auction specific covariates. Disentangling the two alternative explanations for seemingly low auction prices raises a potential identification problem.

Actually, it is generally accepted that IDL reserves are non-aggressive, being based, in part, on outdated timber market information. We could have used that information through a proper prior density but decided against doing so and instead let the data identify which of the two alternatives best fit the data.

Similarly, closer analysis of the data could identify suspicious repeated bidding behavior but, for the same reason as above, we decided against exploiting this answer for research in the present paper. Moreover, we lack information on post auction secondary transactions between mills. Actually, as mills are highly specialized, such side transactions occur even in the absence of collusion. On the other hand, they provide an ideal mechanism for collusive side payments.

All in all, we decided to incorporate in our model a simple baseline collusive mechanism whereby, in any given auction, feasible bidders who draw *i.i.d.* WTPs above the reserve, *individually* decide with probability p whether to join a single ring. The probability p could depend on L and other auction specific covariates.

Furthermore, when a ring forms, it registers a single representative for the reason that repeated combinations of high l_* and price close to the reserve would raise obvious suspicion.

The empirical model in this paper is based on the theoretical framework of Graham and Marshall (1987) and the extension of that to an empirical model as described in Baldwin, Marshall, and Richard (BMR, 1997). Our model generalizes that of BMR in two important ways, one being that rings register only a single representative bidder and the other that we incorporate information of the total number of feasible bidders.[5] The latter information was irrelevant within the BMR framework since it was conditional on all bidders with WTPs above the reserve registering for the auction, whether or not members of a ring. Unlike BMR where estimation was conducted by maximum likelihood, in this paper we use the Method of Simulated Moments (MSM) for robustness consideration.

We find significant evidence of bidder collusion at the IDL sales. The loss to the State of Idaho from bidder collusion over the time 2004 through 2015, estimated by Gaussian quadrature and corroborated by simulation, is approximately \$43million with a standard deviation of \$2.4million. Thus, the estimate of the loss is also a finding of bidder collusion at these auctions.

The United States Forest Service (USFS) also sells timber tracts in Idaho. USFS uses a mixture of English auctions and first price auctions. We examined the number of feasible bidders not attending those sales, especially the first price sales, and found that the USFS sales are similar to the IDL sales. Namely, there are many feasible bidders (as defined in Section 4) not attending the US Forest Service sales. This finding suggests that bidder collusion is potentially an issue for the USFS first price

[5] In their working paper, Aradillas-Lopez et. al. (2012) also allow for feasible bidders that do not attend oil lease auctions.

sales.

The paper proceeds as follows. Legal cases of interest as well as relevant literature are discussed in Section II. In Section III we discuss some issues for modeling collusion at IDL sales. Section IV provides a discussion of the IDL auctions and basic statistics regarding our data. Our empirical model and estimates are in Section V. The methodology for calculation of the collusive gain is in Section VI. Results and robustness checks are presented in Section VII. A discussion of our findings is in Section VIII.

2 Legal Cases and Relevant Literature

2.1 Legal Cases

There have been several cases involving allegations of bidder collusion at timber auctions.[6] Perhaps the most relevant for this paper is Reid Bros. Logging Co. (RBLC) v. Ketchikan Pulp Co. (KPC) and Alaska Lumber and Pulp Company (ALP)[7]. The auctions of interest in this case were conducted by the USFS in southeastern Alaska, primarily in the Tongass National Forest. The period of interest was 1959 to 1975. The auction rules and format are not substantively different from what IDL uses today – preregistered bidders go to a central location to bid in an open outcry ascending bid auction. The court found "irrefutable evidence of a geographical market division".

[6] See United States of America, Plaintiff-appellee, v. George E. Walker, Defendant-appellant, 653 F.2d 1343 (9th Cir. 1981); Reid Bros. Logging Co. v. Ketchikan Pulp Co., 699 F.2d 1292 (1983); United States of America, Plaintiff appellee, v. Portac, Inc., Defendant appellant. United States of America, Plaintiff appellee, v. Howard L. Wolf, Defendant appellant, 869 F.2d 1288 (9th Cir. 1989); United States of America, Appellee, v. Champion International Corporation, Appellant.united States of America, Appellee, v. Young & Morgan, Inc., and Bugaboo Timber Company, et al.,appellants.united States of America, Appellee, v. Freres Lumber Company, Inc., Freres Veneer Company Androbert T. Freres, Appellants, 557 F.2d 1270 (9th Cir. 1977)

[7] 699 F.2d 1292, 1296 (9th Cir. 1983)

KPC bid in one part of the Tongass while ALP bid for timber in the complementary parts. The court found that, "From 1959 to 1975, ALP and KPC, the two giants of the southeast Alaska lumber industry, bid against each other only three times out of 143 sales by the USFS." The court continued to explain that, "These {three} sales all occurred in 1970, and this brief flurry of competition between KPC and ALP was found to be the result of a temporary disagreement between the defendants."

The Reid case motivates an empirical modeling strategy for capturing potential collusion at IDL timber auctions. Namely, only one of the colluding bidders registers for and attends the auction. All other colluding bidders do not attend the auction. In contrast, bidder collusion at used industrial metalworking machinery auctions as well as antique auctions clearly was structured so that only ring bidders who physically attended the auction would get a payoff from the collusive gain.[8] In previous work, two of us had modeled collusion at timber auction under the presumption that all colluding bidders attended the auction (BMR 1997). The Reid case implies that this assumption is not universally applicable in examining bidder collusion at timber auctions. In this paper we presume non-attendance by all but one of the colluding bidders.

2.2 Economic Literature

The economic literature is quite clear that ascending bid auctions are typically more susceptible to collusion than first price auctions.[9] Also, although the economic literature addresses the advantages of ascending bid auctions to first price auctions when bidder valuations are affiliated, these results can be reasonably viewed as slight improvements around the second order statistic. In contrast, collusion can result in

[8] United States v. Seville Indus. Machinery Corp., 696 F. Supp. 986 (D.N.J. 1988); U.S. v. Pook (Ronald), 856 F.2d 185 (3d Cir. 1988)

[9] See, for example, Marshall and Marx (2007, 2012).

dramatic effects regarding the order statistic paid at an ascending bid auction. With regard to estimation of collusive auction models, there are papers that have looked at offshore oil leases, milk procurements, timber auctions, and road construction.[10]

Some authors have assumed that first price auctions are wholly robust to collusion and then used that information to examine ascending bid auctions.[11] There is a good reason to believe that ascending bid auctions are less robust to collusion than first price auctions, but the presumption that first price auctions, with repeated auctions and repeat players, are wholly robust to collusion is a maintained hypothesis that is highly questionable. We discuss this in Section VII.

Haile and Tamer (2003) have raised issues about discrete bid increments at ascending bid auctions. The average Doug-fir tree at maturity is 228 feet tall and 20 inches in diameter. (See https://100hourboard.org/questions/60836/) It yields about 4,000 board feet of wood or 4MBF. Such a mature Doug-fir brings about $1,000 at auction (see https://100hourboard.org/questions/60836/.) IDL sets the minimum acceptable bid raise at all of its auctions to be $.10 per MBF. (See IDL Auction Sales Bidding Procedures.) So, for a mature Doug-fir, the bid increments are $.40 for something that will sell for approximately $1,000. Furthermore, we have examined a sample of fifty detailed bid sheets provided to us by IDL that contain the sequence of bids submitted at each auction. Bidders will sometimes opt for a $1 bid increment, which in the context of the aforementioned Doug-fir tree would be a bid increment of $4. It is the case that at the beginning of the auction bid increments are larger, perhaps $5. But these increments typically decline as the auction progresses. Overall, the bid increments and jump bids appear to be de minimis in terms of a material

[10]See, for example, Hendricks and Porter (1988), Aradillas-Lopez, Haile, Hendricks, and Porter (2012), Porter and Zona (1999), Athey, Levin, and Siera (2011), Schurter (2017), and Bajari and Ye (2006).

[11]See, for example, Athey, Levin, and Siera (2011).

divergence from continuous bidding. Moreover, our main objective in this paper is that of estimating "collusive gains" (i.e. loss of revenue for IDL), defined as the difference between the second order statistic (typically unobserved under collusion) and the price effectively paid (itself an order statistic that depends on the composition of the ring). Within this context, bid increments can be interpreted as (small) measurement errors on the relevant order statistics. There are no reasons to believe that these could be correlated with the model regressors. Therefore, we expect that our estimates of collusive gains will remain consistent, though a little less efficient due to the presence of these small bid increments.

3 Practical Concerns for Modeling Collusion at IDL Auctions

Since we are focusing on English auctions, the presumption is that "Vickrey logic" applies, namely, each bidder remains active at the auction up to their valuation for the item. But, bidders often have demand for more than one tract and IDL is selling many tracts each year. Intuitively, it seems perfectly natural for each bidder to have a maximal willingness to pay as they walk into each auction, and it is the distribution of this entity that will be our focus.[12] Furthermore, each bidder's WTP is assumed to be independent of the number of feasible bidders for the tract, L. If these were first price sealed bid auctions that assumption would be unreasonable. However, IDL exclusively runs ascending bid auctions. The assumption that any given bidder's WTP has nothing to do with L is important for the identification of our model as $\ln(L)$ will be introduced as a regressor in the probability of collusion but will be

[12] Other empirical studies of ascending bid auctions are implicitly doing exactly the same thing, although the authors will typically use the label of a "valuation" instead of a maximal willingness to pay.

excluded from the WTP distribution.

In addition, a given timber tract will produce some logs of certain species/quality that are of no interest to one mill while another mill is keenly interested. So it is natural for some of the logs from each tract to be sold by the winning bidder to other mills. This is fully anticipatable by the feasible bidders when considering their bidding behavior. However, winning a timber tract provides definite benefits to the winner. First, and most importantly, the winner can keep the logs that are best suited to their mill. Second, the winner has total control over the timing of the harvest and can do so as it is relevant for the timely supply of logs to their mill. Third, the winner can use the logs coming off a tract that they find less desirable as leverage in bilateral trade negotiations with other mills that are harvesting timber on different tracts. Overall, winning a timber tract provides the winner with control and external market leverage.

A freshly cut log has quite a bit of moisture in it – perhaps 1/3 of its weight is water. A sawmill does not want logs to dry out excessively before creating lumber since logs can split and twist as the moisture leaves the log, and this greatly impairs the recoverable lumber from the log. Mills can store logs for up to a few months depending on how they are stored and the time of year, but it is not sensible for a mill to have an inventory of logs that cannot be processed in a reasonable time period. Lumber is often kiln dried after emerging from the mill. This can take up to a week depending on the kiln and the species.

It is common for logs to be bought and sold by mills. Under the presumption of non-collusive conduct, if a mill wants to be a buyer it has to compete against all other mills that want to be buyers. Smaller mills must be competitive with bigger mills in pursuing logs. The cost to transport logs as well as to log a tract and to build roads on the timber site may differ somewhat between mills. Mills know one another's specialties and production capabilities. Mills know what their competitors have purchased from public sales, both IDL and USFS. Mills will have less good information about the timber their competitors have purchased from private lands, or logs that they are obtaining from private sources. Big mills and small mills need to have a timber/log plan for the short and long term. We assume the nature of these plans is symmetric across all mills. Overall, we will be using an independent private value model with bidder symmetry for each auction, but allowing for differences across auctions.

As a corollary, the reserve prices for the IDL timber auctions are calculated using a residual value method that works backward from log prices, accounting for relevant costs in getting a standing tree converted into a log at a mill. However, the best and most suitable logs for the needs of a mill are kept by the winners of the timber tracts. Log prices come from the sale of good logs that are not suitable for the winner's mill as well as lesser quality logs. In other words, log prices in the market have a natural downward bias embedded in them since auction winners do not sell the best logs.

It is also important to observe that if ring members are using reduced log prices to compensate one another for bid suppression then the gatewood log price is artificially depressed by the collusion. This is an added benefit to ring members since the gatewood log price is the starting point for the calculation of the reserve price on timber tracts, implying that reserve prices are potentially depressed, in part, by bidder collusion.

With regard to potential collusion at the IDL auctions, it is not reasonable to think of a single ring of bidders as one might think of in other auction settings.[13] Mature trees are stationary objects and tracts of timber are sold in different locations. Thus, it is reasonable to think of rings as being dependent on the tract location and

[13] Such as US v. Seville Industrial Machinery.

species mix. In this paper, we are not attempting to identify the identities of ring participants. Rather we are only interested in assessing if collusion occurred at the IDL sales and, if it did, to measure the magnitude of lost revenue for the state of Idaho.

The timber in Idaho, especially Northern Idaho, is neither exceptionally large nor exceptionally old (unlike some of the ancient timber in Region 6). Much of the timber of Northern Idaho was destroyed in the Big Burn of 1910. This eases concern about potential adverse selection at these auctions.

4 Idaho Department of Lands (IDL) Timber Sales

The timber auction data that is the focus of our analysis pertains to sales by the IDL for the time period 2004 to 2015. As part of its statehood status, Idaho was granted 3.6 million acres of land by the US Congress to fund public schools and other public enterprises.[14] The revenues derived from the IDL's sale of timber are an important

[14]IDL history. "As it was deliberating the Idaho Admissions Act in 1889, the United States Congress displayed uncommon wisdom by granting what would become the Union's 43rd member approximately 3,600,000 acres of land for the sole purpose of funding specified beneficiaries.

The Idaho Constitution was crafted to include Article IX, Section 8, which mandates that the lands will be managed "...in such manner as will secure the maximum long-term financial return to the institution to which [it is] granted."

Chief among the beneficiaries are the public schools, which received two sections of every township in the state (1/18 of the total land base). Beneficiaries of the other funds include the University of Idaho, State hospitals for the mentally ill, Lewis-Clark State College, State veterans homes, Idaho State University, the Capitol Commission, Idaho School for the Deaf and Blind, and Idaho's juvenile corrections system and prison system.

The prescribed income is generated in a number of ways: the sale of land; the sale of timber; leases for grazing, farming, conservation, commercial buildings, recreational homesites, and mining; and earnings from invested funds. The Endowment Fund Investment Board is charged with managing the invested revenues from the endowment lands.

Management activities on state endowment trust land are not intended to benefit the general public, but are directed solely to the good of the beneficiaries of the original land grants." (https://www.idl.idaho.gov/land-board/lb/documents-long-term/history-endowment-lands.pdf accessed June 25, 2018) component of that funding.[15] Over the time span of our data IDL has sold between 210 and 250 million board feet of timber per year.

IDL crafts a 10 year harvest plan. There is more certainty in the first five years about the volumes that will be offered at auction. Years six through ten are more uncertain because of variation in natural factors such as fires, insects, etc. With regard to defining the tracts, IDL looks for natural borders, prioritizing salvage areas, and then also transport issues/costs.

IDL sells stumpage at public ascending bid auctions to the qualified bidders present at the auction. A bidder must attend the auction to bid. Bidder identities are thus revealed to everyone prior to the start of the bidding. To qualify a bidder must post a performance bond equal to 10% of the reserve price, demonstrate adequate insurance coverages, and not be delinquent in any payments owed to IDL. By qualifying and attending a bidder is stating a willingness to pay at least equal to the reserve price. Interested parties can attend the auction, but only qualified bidders can bid. The reserve price for the sale is stated in terms of the average price per mbf, and bidding progresses in discrete increments in terms of the average price per mbf.[16] The amount of the winning bid and the identity of the winning bidder are publicly known immediately at the conclusion of the auction.

Each timber tract offered for sale is informally announced well in advance of the sale. IDL describes all aspects of the tract including exact location and geographic boundaries of the sale, species mix, volume of timber, estimated road costs, estimated logging costs, the amount of time IDL allocates for the logging of the tract to

[15] IDL statement of objectives – "To professionally and prudently manage Idaho's endowment assets to maximize long-term financial returns to public schools and other trust beneficiaries and to provide professional assistance to the citizens of Idaho to use, protect and sustain their natural resources." (https://www.idl.idaho.gov/land-board/about-idl/index.html accessed June 25, 2018)

[16] The minimum bid increment for a scale sale is $.10 per mbf, for a weight sale it is $.05 per ton, and for a lump sum sale it is $.10 per mbf.

be completed, and other tract features including the reserve price. From the time of the informal announcement to the time of the formal sale announcement, potential bidders will cruise the tract and offer comments to IDL regarding potential mischaracterizations by IDL. For example, a bidder may assert to IDL that IDL's road cost estimate is far too low given the terrain and other tract conditions. IDL personnel will assess the veracity of such commentary and potentially adjust the formal sale announcement. Road costs are particularly important since bidders receive one-for-one credits for road construction costs (based on IDL estimates) that can be applied against harvested timber.

Unlike USFS, there are no small business set aside sales. As a consequence, except for some salvage sales, each IDL sale typically has a substantial volume of timber. Logging firms are almost never bidders at the IDL auctions. Infrequently, IDL will sell small tracts via bilateral negotiation. These involve special circumstances, such as insect damage or a blow down. Logging firms may be involved in these negotiations.

Most IDL sales are scale sales which means that the winning bidder pays a species specific price for the timber extracted from the tract on a per MBF basis. The species specific price is set by the winning bidder after the auction by allocating its winning average price per MBF across the species on the tract. There are also a relatively small number of weight sales as well as lump sum sales. [17]

If a winning bidder does not log the tract then they forfeit their performance bond and deposit (where the latter is 15% of the sale price), and the tract is reauctioned. If there is a legitimate delay for the logging of a tract, IDL can extend the term by one year.

[17] According to IDL, "Weight sales are primarily limited to all sales south of the Salmon River or to sales with similarly sized wood and species that are in the same value range. Lump Sum sales are primarily small sales (<100 MBF and <$15K), with the exception of sales from Eastern Idaho (Idaho Falls region) which utilized lump sum for many years before going to weight within the last 5-7 years."

Timber tract harvest results are available to all bidders from IDL, although this may require a public record request. This mitigates adverse selection issues.

The USFS, Montana State, and Washington State are all selling timber tracts in and around the sales conducted by IDL. In addition, there are ongoing timber sales from private lands. IDL ignores all other sources of timber in deciding how much timber to sell and what tracts to sell.

The species and timber tract products sold from IDL lands are, in order of volume, grand fir, Doug-fir, western redcedar, ponderosa pine, lodgepole pine, pulpwood, Englemann spruce, cedar products, western white pine, and small sawlog. The IDL timber sales are exclusively softwoods.

Finally, IDL sales will sometimes have a special designation of "pole" or "pine" or "osr" or "pulp" in the sale name.[18] A designation of "pole" in the sale name means that the tract is not to be clear cut but, instead, only the poles are to be harvested (as well as other trees needed to access the poles via roads for the harvest). "Osr" designates overstory removal, which means that only the tallest trees on a tract are to be harvested in order to alleviate competition for the smaller trees and thus give them a better chance for rapid development. "Pine" means that the predominant species is pine and "pulp" means that the sale is predominantly wood that will be used for pulp.

There are only a few bidders who are viable bidders at pole sales. We have eliminated pole sales from the data.

[18] According to IDL, they include "Ton" in the name if it sold on a weight scale basis.

Also "Salvage" in the name is used to indicate recovery of volume due to wildfire and insects. Though for part of this time period, "Salvage" could also mean an intermediate sized sale no larger than 1,000 MBF and $150,000 in appraised value. This was IDL's version of small business set asides, though there were no restrictions on who bought the sale.

4.1 Basic Content of Data

The data used in our estimation are IDL scale auctions. In what follows we separate the summary statistics for one bidder auctions from those attended by two or more bidders. We include all auctions from 2004 through 2015.[19] Below (Table 1) are the summary statistics for the 626 scale auctions where there were two or more bidders.[20]

Table 1: Descriptive Statistics (2 or more bidders, 626 auctions)

	Mean	Stan Dev	Min	Max
winning bid	921,978	739,269	16,130	4,454,524
reserve price	615,741	485,690	12,873	2,626,685
total volume (Mbf)	3,583	2,552	100	16,470
l_*	3	1	2	17
L	8	3	2	18
logging cost	703,697	515,373	16,270	3,750,736
term	33	11	3	60
acres	281	189	7	2,212
winner business size	5	2	1	8
lumber price	183	23	134	232

Variables definitions are below[21]:

- Winning Bid – price paid at the auction.

[19] We do have data for "no sales". IDL has informed us that there is perhaps one no sale each year, and that tract is invariably reoffered at a later date. There are 22 reconfigured sales in our 708 sales (626 2+bidder sales and 82 one bidder sales).

[20] We dropped auctions for which we did not have location information, top 2 bidders' information or information about the size of the sale. There was one sale with an obviously miscoded size (e.g. tract was recorded as 8,800+ acres) that we excluded. There are two sales with bidder information possibly incorrectly recorded or the number of bidders attending incorrectly recorded. We excluded these three sales. We also excluded data for 2001 to 2003 from the estimation in order to calculate a temporally consistent inventory variable for the years 2004 through 2015 (the data for 2001 to 2003 was used to calculate the 2004 inventory variable).

[21] Some of these variables are rescaled in order to eliminate large inbalances between estimated coefficients. The corresponding scales are listed in parentheses when applicable.

- Reserve Price – publicly announced reserve price by IDL.
- Total Volume measured in thousands of board feet (MBF). (Scale: volume/10,000)
- l_* is the number of bidders who registered to bid and showed up at the auction. The data only contains the identities of the top two bidders.
- L is the number of feasible bidders for an auction. A bidder is defined as "feasible" if (i) the tract is within 150 miles driving distance of their mill or closest mill (for those who operate multiple mills), (ii) business size is not the smallest (e.g not 1 on the 1 to 8 scale), and (iii) the top three species in terms of volume purchased historically by the bidder are at least 60% of the species, in terms of volume, sold at the tract in question. In addition, the top two bidders are feasible regardless of these criteria. (See Section 4.2 for further discussion of the third criterion). [22]
- Logging cost is the IDL estimate of the cost of logging the tract. (Scale: logging cost/1,000,000)
- Term is the maximum amount of months that the winning bidder can take to complete the specified cut of the tract.
- Acres is the size of the tract measured in acres. (Scale: acres/100)
- Business size of a bidder is measured in terms of employment and coded from 1 to 8. A business size of 1 has 25 or fewer employees while a business size of

[22] There are other requirements when we calculate the number of feasible bidders.

(1) We assume the number of feasible bidders is larger than or equal to l_*, When the calculated L is smaller than l_*, we set $L = l_*$.

(2) The bidder (mill) must still be open.

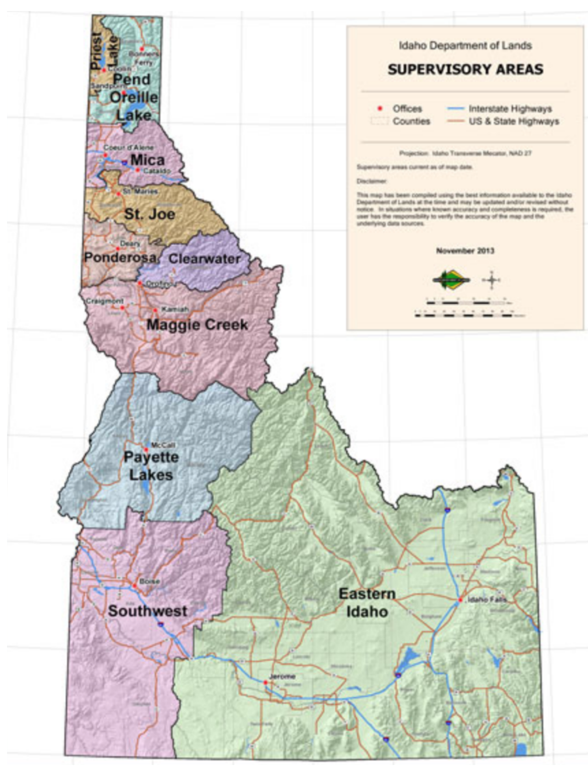
(3) Pole bidders (McFarland, Bell, J H Baxter, Stella-jones and Carney) will never be feasible bidders for a non-pole sale.

8 has over 1,000 employees. ("2" is 26 to 75; "3" is 76-150; "4" is 151 to 250; "5" is 251 to 500; "6" is 501 to 750; "7" is 751 to 1,000) [23]

- Lumber price is the producer price index for softwood lumber (WPU081107) FRED St Louis. (Scale: lumber price/100)

The sales are occurring in different regions of Idaho, as shown in Figure 1. The large majority of timber tracts sold by IDL are in the panhandle of Idaho (Clearwater, St. Joe and Pend Orielle Lake). The annual number of sales and volumes are shown in Figure 2.

Figure 1: Map of Timber Regions in Idaho



[23]There are three 2+bidder auctions and one one-bidder auction where the winner's business size is not recorded. We found two winners' business size information based on related documents from Timber Data Company. We checked the other two winners' information online. Based on the number of its employees, we code the business size for each of the two winners. The details can be found in the online appendix (https://www.dropbox.com/s/yic594ycwqbku1a/notes_on_dataprocessing%20final%20version.pdf?dl=0).

Figure 2: Number of Auctions and Total Volume by Year (2 or more bidders, 626 auctions)

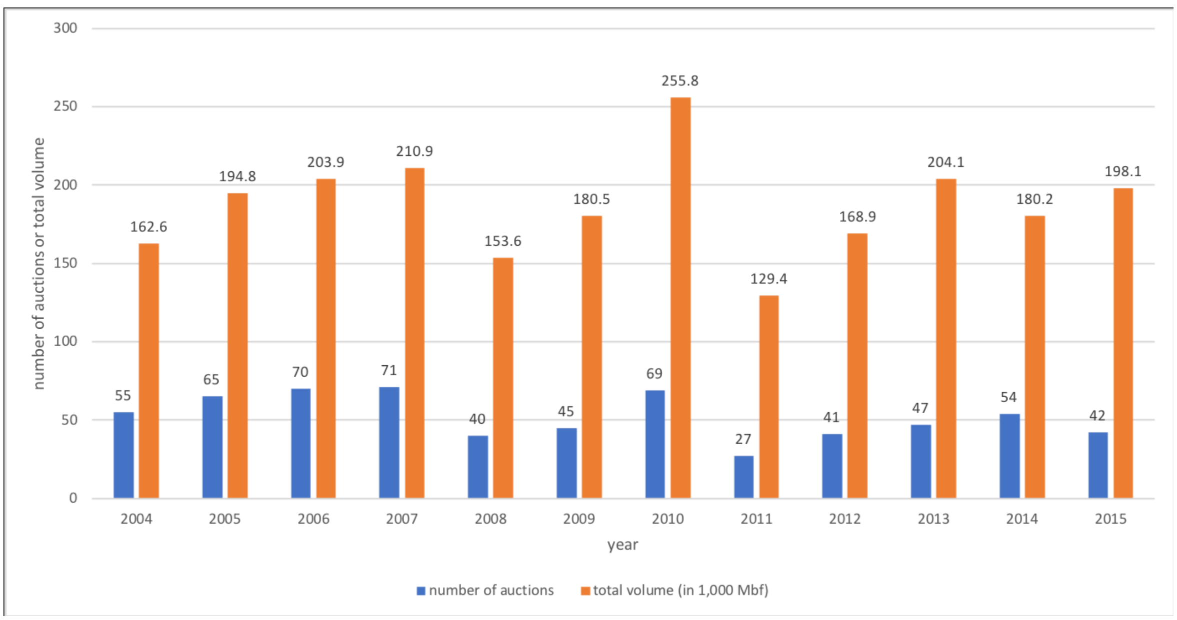
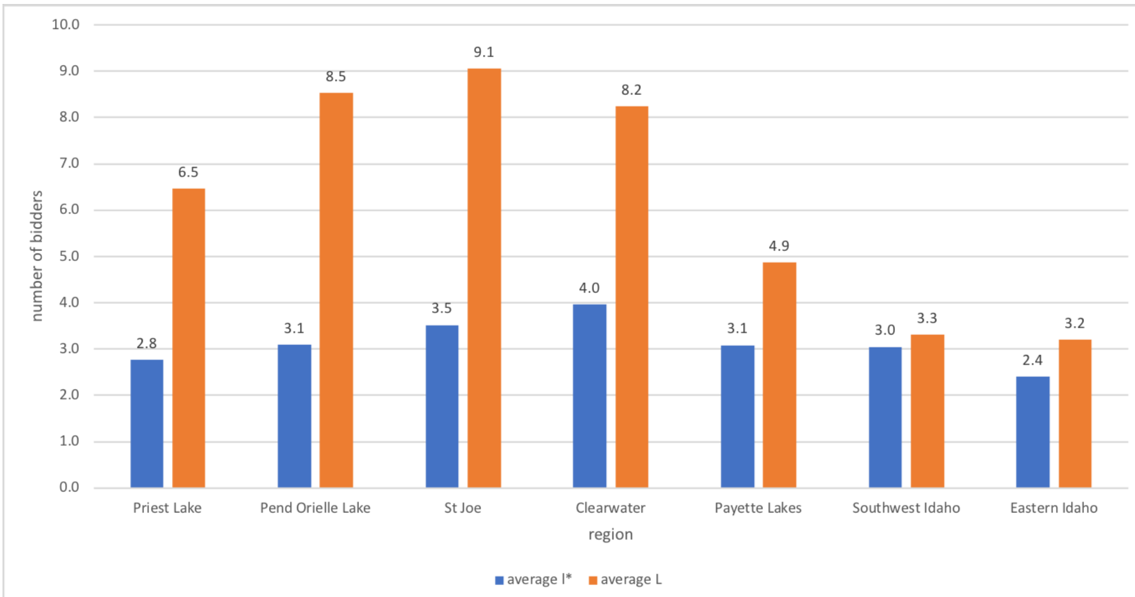


Figure 3 shows average l_* and L by region. In the panhandle, L exceeds l_* by about 5 on average, which is consistent with the summary statistic in Table 1.

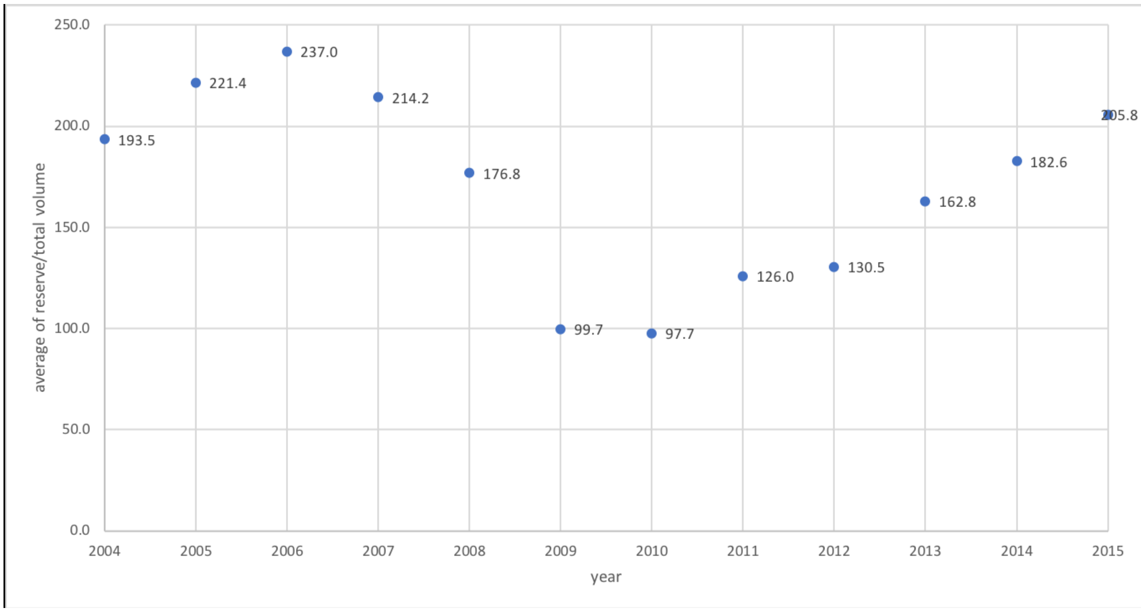
Figure 3: Average l_* and Average L by Region (2 or more bidders, 626 auctions)



In Figure 4, we report yearly reserve prices. To compute reserve prices, IDL relies upon a residual value method, which starts with the price of the logs at the gate of the purchasing mill and works backwards through the processing, transportation, and logging costs. Since log market prices change through time, the residual value per MBF changes accordingly.

When the lumber market is strong the effective radius for a mill to profitably bid on a tract might expand to as much as 200 miles. However, the pro-cyclical market sensitivity of the reserve price implies that the radius for being a feasible bidder is on the order of 150 miles through time. Yet, it is important to note that IDL has less than three no-sales per year, indicating that reserve prices are typically not aggresive.

Figure 4: Average of Reserve/Total Volume by year (2 or more bidders, 626 auctions)

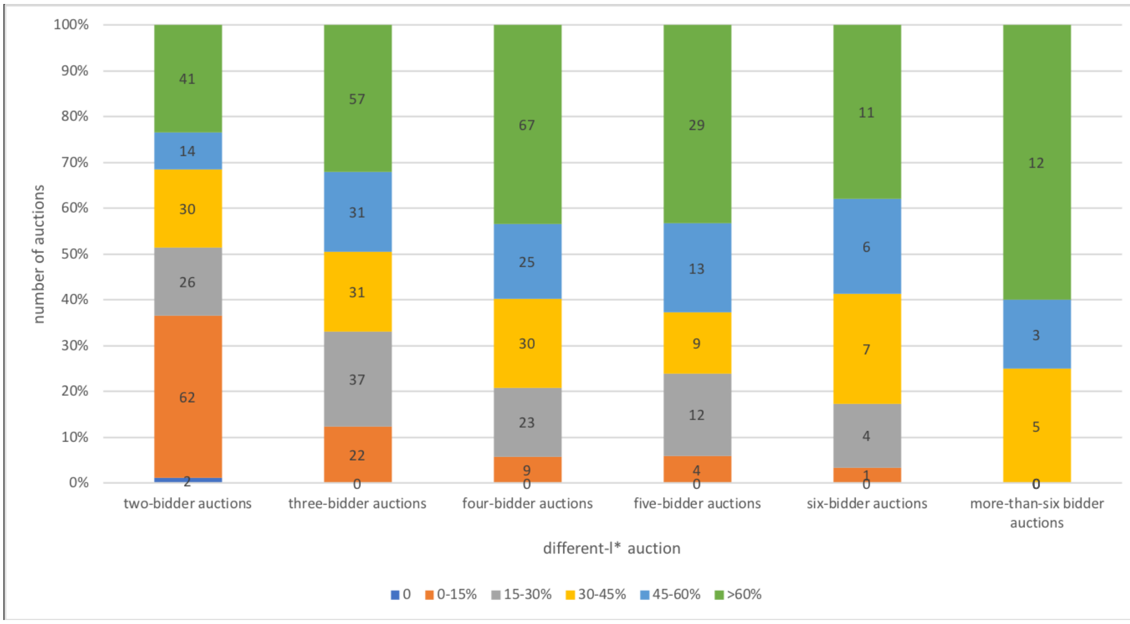


Next, we turn our attention to the winning bid measured as a percentage above the reserve price, which we refer to as the "premium". Figure 5 shows the premium as a function of l_*, the number of registered bidders. [24]

We see that the premium for 62 of the two bidder auctions was greater than zero but less than 15%. For these auctions the average number of feasible bidders (L) is 7.3. For the three bidder auctions there are 22 auctions where the premium is greater than zero but less than 15%. For these auctions the average number of feasible bidders (L) is 7.6. A potential (non-collusive) rationalization of such large differences between L and l_* might be that feasible bidders who did not register were filled with inventories, while those who did register were short on inventories. But, an obvious alternative explanation is that several feasible bidders were part of a ring and were instructed not to register. Similar comments apply to auctions with more

[24] There are only 2 sales, each a 2-bidder sale, where the premium is eactly zero. Premiums at boundaries are counted in the left interval.

Figure 5: Different l_* Auctions by Premium of Winning Bid over Reserve Price (2 or more bidders, 626 auctions)



than three registered bidders. Table 2 provides a histogram of L for the 626 auctions with two or more bidders. It indicates that about 80% of these auctions had between 5 and 11 feasible bidders.

In order to provide additional insight on the potential impact of inventories, we contrast in Table 3 average inventories for the top 2 bidders with those of the additional (not top 2) feasible bidders, as a function of business size.[25] A remarkable feasure of Table 3 is that the average inventories of the not top 2 feasible bidders is uniformly lower than those of the top 2 bidders. This is exactly the opposite of what we would expect from the inventory based non-collusive conjecture presented above.

[25]Business size “1” was excluded by definition of a feasible bidder while there are no mills of business size “7”.

Firm specific inventory is the amount of timber won by the firm, which was sold in the USFS, Washington State, Montana State, and IDL in years prior to the year of the sale, within 100 miles (straight line distance) of the given mill, that does not have to be harvested within the next 12 months.

Table 2: L frequency (2 or more bidders, 626 auctions)

L	number of two-or-more-bidder auctions
2	9
3	34
4	38
5	40
6	58
7	70
8	126
9	99
10	71
11	35
12	18
13	19
14	6
16	1
17	1
18	1

In Tables 4 to 6 we provide similar comparisions for the 82 auctions with a single registered bidder. The average number of feasible bidders for these auctions is 6. Here again average inventories for the feasible bidders that did not register are uniformly smaller than those of the sole bidder. Again, such findings are suggestive of collusive behavior.

Table 3: business size versus bidder's inventory (2 or more bidders, 626 auctions)

business size	number of bids by business size for top2	avg inventory of top2 bidders	number of feasible bids by business size for not top2	avg inventory of feasible bidders, not top2
1	115	737		
2	230	21,208	973	17,785
3	122	25,171	864	14,435
4	183	34,639	276	16,109
5	63	17,818	474	14,495
6	348	80,947	423	28,159
8	191	59,434	595	24,857

Table 4: Descriptive Statistics (82 one-bidder auctions)

	Mean	Stan Dev	Min	Max
winning bid	389,869	377,282	10,405	1,846,527
reserve price	389,862	377,281	10,405	1,846,527
total volume (Mbf)	2,893	2,481	39	13,430
l_*	1	0	1	1
L	6	3	1	14
logging cost	672,524	643,121	6,011	3,591,308
term	32	13	12	60
acres	265	204	4	850
winner business size	4	2	1	8
lumber price	170	23	134	232

Table 5: L frequency (82 one-bidder auctions)

L	number of one-bidder auctions
1	6
2	9
3	6
4	7
5	4
6	6
7	12
8	13
9	11
10	3
11	2
12	1
13	1
14	1

Table 6: business size versus bidder's inventory (82 one-bidder auctions)

business size	number of bids by business size for top1	avg inventory of top1 bidders	number of feasible bids by business size for not top1	avg inventory of feasible bidders, not top1
1	8	1,799		
2	17	14,945	115	13,364
3	4	19,635	76	16,106
4	14	47,433	39	24,140
5	9	29,698	48	11,818
6	21	45,030	64	23,210
8	9	34,200	80	28,913

4.2 Feasible bidders and potential collusion

As noted above, a bidder is feasible if, among other things, "the top three species in terms of volume purchased historically by the bidder are at least 60% of the species, in terms of volume, sold at the tract in question". To illustrate the implications of this criterion for actual bidding, consider the following illustration with four bidders – A, B, C, and D – and four species – S1, S2, S3, and S4. The table below shows the species that each bidder processes, in equal proportions.

	S1	S2	S3	S4
A	x	x	x	
B		x	x	x
C	x		x	x
D	x	x		x

The following table shows five auctions with the corresponding percentage of each species on each tract.

	S1	S2	S3	S4
1	.25	.25	.25	.25
2	.33	.33	.33	0
3	.5	.5	0	0
4	.55	.35	.1	0
5	1	0	0	0

We assume that all potential bidders satisfy the first two criteria. Then all four bidders are feasible for auctions 1 and 2. For auction 3, B and C get only 50% of their species demand satisfied by the available timber on the tract. Therefore, only A and D are feasible for auction 3. Similarly, only A, C and D are feasible for auctions 4 and 5.

The implications for collusion are straightforward. For the first and second auctions each of the four bidders would make a positive contribution to the ring by joining. For the third auction, B and C would not register were they acting non-collusively and would, therefore, not be considered to be potential ring members. The same applies to bidder B in auctions 4 and 5.

In our empirical model, the number of feasible bidders for a given auction will play a central role since infeasible bidders are neither meaningful potential participants at the auction nor potential ring members.

5 Empirical Model and Estimation

5.1 Model

5.1.1 Descriptions

As discussed above, for each of the 708 IDL auctions ending in a sale, we observe the reserve R_*, the number of registered bidders l_*, the price paid P_*, as well as a vector x of auction specific covariates, including the number L of feasible bidders, as constructed in Section 4 above. Since the latter obviously plays a key role in our analysis, we will analyze the sensitivity of our results relative to L.

Let W denote the (individual) WTP, assumed to be *i.i.d.* for each auction. W, R_* and P_* are divided by volume (Vol). Let

$$V = \ln(\frac{W}{Vol}), R = \ln(\frac{R_*}{Vol}), P = \ln(\frac{P_*}{Vol}). \tag{1}$$

We assume that the V's follow a normal distribution with mean μ and variance σ^2. Thus, W is log-normally distributed. Both μ and p are assumed to depend on x, say

$$\mu = \beta' x \text{ and } p = \gamma' x, \tag{2}$$

where γ will be parameterized in such a way that $p \in [0,1]$. Let $f(v)$ and $F(v)$ denote the density function and cdf of v, that is

$$f(v) = \frac{1}{\sigma}\phi(u) \text{ and } F(v) = \Phi(u), \tag{3}$$

where $u = \frac{1}{\sigma}(v - \mu)$, ϕ denotes the density function of a standardized $N(0,1)$ and Φ is its cdf.

5.1.2 The (l_*, P) process

The (potential) collusive mechanism described above implies three baseline probabilities

$$p_3 = F(\frac{R-\mu}{\sigma}), p_1 = p(1-p_3) \text{ and } p_2 = (1-p)(1-p_3) \tag{4}$$

where p_1 is the probability of drawing a WTP above R and joining the ring, p_2 is the probability of drawing a WTP above R and not joining the ring, and p_3 is the probability of drawing a WTP below R. Let (l_1, l_2, l_3) with

$$L = l_1 + l_2 + l_3 \tag{5}$$

denote the corresponding numbers of feasible bidders in each of these three categories. Let the V_i's denote the L order statistics in descending order

$$V_1 > V_2 > \ldots > V_L \tag{6}$$

If $l_1 > 1$, the ring registers only one representative member. Let C denote the ring composition. In order to facilitate the subsequent inference process, we consider five cases:

$$
\begin{aligned}
\text{Case 1} &: \quad l_1 = 0, l_2 = 0 \Longrightarrow \text{no sale} \\
\text{Case 2} &: \quad l_1 = 0, l_2 = 1 \Longrightarrow (l_*, P) = (1, R) \\
\text{Case 3} &: \quad l_1 = 0, l_2 > 1 \Longrightarrow (l_*, P) = (l_2, V_2) \\
\text{Case 4} &: \quad l_1 > 0, l_2 = 0 \Longrightarrow (l_*, P) = (1, R) \\
\text{Case 5} &: \quad l_1 > 0, l_2 > 0 \Longrightarrow (l_*, P) = \begin{cases} (l_2 + 1, V_2) & \text{if } r = 0 \\ (l_2 + 1, V_{r+1}) & \text{if } r > 0 \end{cases}
\end{aligned} \tag{7}
$$

where $r \in (0, l_1)$ is given by

$$
\begin{aligned}
r &= 0, \text{ if } V_1 \notin C \\
r &> 0, \text{ if } \left(\cap_{i=1}^{r} V_i \in C\right) \cap \left(V_{r+1} \notin C\right).
\end{aligned} \tag{8}
$$

As shown in the Appendix (Section 10), we can derive analytical expressions for the probabilities and densities of all feasible pairs (l_*, P) and these will be used in Section 6 to compute (ex-post) collusive gains. However, for reasons to be discussed in Section 5.2, we shall rely upon a Method of Simulated Moments (hereafter MSM) for inference and for which we do not need these analytical expressions. Instead, all we need is to be able to produce Monte Carlo (hereafter MC) simulations of the (l_*, P) process, conditional on tentative values of (μ, σ, p) and on the absence of no sales. In order to do so, we proceed as follows for each auction separately (using an auxiliary vector lb for ring membership).

Step 1: draw (l_1, l_2)

1. Draw L WTPs and rank them in descending order

2. If $V_1 < R$, then (l_*, P)=$(1, R)$ and we proceed to the next auction [26]. Else, initialize $l = l_1 + l_2 = 1$ and $l_1 = 0$

3. Draw sequentially ring membership as follows

   $lb(l) = 1$ with probability p and 0 otherwise

   If $lb(l) = 1$ then $l_1 = l_1 + 1$

   Set $l_{p1} = l + 1$

   If $l_{p1} \leq L$ and $V(l_{p1}) > R$, then set $l = l_{p1}$ and repeat

   Else, set $l_2 = l - l_1$ and proceed to Step 2

Step 2: given (l_1, l_2), (l_*, P) obtains from equation (7) and (8), using the auxiliary vector lb to find r.

5.2 Method of Simulated Moments

In a model like the one we just described, there are obvious concerns relative to potential over-simplication and/or mis-specification, each of which could impact the robustness of estimators.

At an early stage, we did experiment with Maximum Likelihood Estimation (hereafter, MLE) relying upon the analytical distribution of (l_*, P). We encountered (qualitative) identification problems resulting in non-concavity and local extremes.

Instead, we decided to take advantage of the fact that, as discussed above, we can easily simulate the model and, thereby, rely upon the Method of Simulated Moments

[26] As we draw sequentially L (unranked) WTPs, conditioning on $V_1 > R$ is equivalent to drawing the L-th WTP in $(R, \infty($, when the first $L - 1$ all are less than R. It is also equivalent to drawing V_1 in $(R, \infty($ in that case. However, we can dispense with that redraw since (l_*, P) is then equal to $(1, R)$, irrespective of the redrawn V_1.

(hereafter, MSM).[27] While MSM estimators are inherently less efficient than MLEs under correct specification, they are generally more robust against mis-specification. Moreover, as we discuss next, MSM enables us to account for additional sample moments that are not directly linked to the likelihood but easy to simulate and, thereby, to also rely upon MC simulations of the estimated model to produce finite sample standard deviations for MSM estimates.

Let $\widehat{m}_n$ denote a vector of observed sample moments, with n denoting the sample size. Here, n will be either 708 for the full sample or 626 if we exclude 82 auctions for which $l_* = 1$. In one exercise we will eliminate the one bidder auctions to demonstrate that inference on collusion is not driven by the latter. If we could derive an analytical expression for $m(\theta) = p\lim \widehat{m}_n$, then a General Method of Moments (hereafter, GMM) estimator obtains as

$$\theta_{GMM} = \arg\min_{\theta} \left[\widehat{m}_n - m(\theta)\right]' \widehat{W}_n \left[\widehat{m}_n - m(\theta)\right] \tag{9}$$

where $\widehat{W}_n$ denotes a symmetric positive definite (SPD) weight matrix. Consistency obtains for any SPD weight matrix and (relative) asymptotic efficiency obtains when $p\lim \widehat{W}_n = \left[\Sigma(\theta)\right]^{-1}$, where $\Sigma(\theta)$ denotes the asymptotic covariance matrix of $\sqrt{n}\widehat{m}_n$.

In the present case, we cannot produce analytical expressions for $m(\theta)$. Instead, we can rely upon S independent replications of the MC procedure described in Section 5.1 conditionally on any tentative value of θ in order to produce a sequence of simulated moments $\{\widehat{m}_{s,n}(\theta)\}_{s=1}^{S}$.

A (finite sample) estimate of $m(\theta)$ obtains as

[27] See the pioneering contribution of McFadden (1989). Laffont, Ossard, and Vuong (1995) use a simulation technique to compute a non-linear least squares regression of the winning bid on a vector of covariates when the analytic expression of the conditional expectation of the winning bid is not known analytically.

$$\widehat{m}_s(\theta) = \frac{1}{S}\sum_{s=1}^{S}\widehat{m}_{s,n}(\theta) \tag{10}$$

It is important to note that, in order to secure the continuity of the objective fuction in (9), we rely upon Common Random Numbers (CRNs), whereby the baseline $N(0,1)$ draws used for simulations are reused for any tentative values of θ, as produced by the optimization program.[28] Effectively, we do find that under CRNs, the simplex optimizer we rely upon seamlessly converge to a solution $\widehat{\theta}_{MSM}$ without relying upon additional smoothing techniques. We note that replacing $m(\theta)$ by $\widehat{m}_s(\theta)$ in (9) implies an asymptotic efficiency loss ratio of $1+\frac{1}{S}$. This is of no concern here, as we rely upon values of $S \geq 500$. Moreover, we do not compute an asymptotic covariance matrix for $\widehat{\theta}_{MSM}$, such as one obtained by inversion of a numerical Hessian matrix at the optimum. Instead we keep relying upon MC simulations to produce a finite sample covariance matrix for $\widehat{\theta}_{MSM}$. In order to do so, we proceed as follows. We generate 50 independent auxiliary samples for (l_*, P) conditional on $\widehat{\theta}_{MSM}$. To each of these samples we then apply the very same optimization process that was initially used to compute $\widehat{\theta}_{MSM}$ (that means same initial values, same CRNs, same optimizer settings, a.s.o). This produces fifty independent draws from the finite sample distribution of $\widehat{\theta}_{MSM}$ conditional on the estimated "true value" $\theta = \widehat{\theta}_{MSM}$, from which we compute the corresponding finite sample covariance matrix used for inference on $\widehat{\theta}_{MSM}$.

Next, we discuss the selection of $\widehat{m}_n$ in (10). An obvious choice consists of moments associated with the two key variables $(P, \ln l_*)$, conditional on (subsets of) the auction specific variables listed in Table 1 together with our estimated value of L, the number of feasible bidders. Specifically, we estimate a two-equation Seemingly Unrelated Regression Equation (SURE) for $(P, \ln l_*)$. After extensive experimen-

[28] See Hendry (1984).

Table 7: SURE Results

Variable	Dependent variable: ln(winning bid/vol)		Dependent variable: $\ln l_*$	
	coefficient	t-statistics	coefficient	t-statistics
constant	1.3440	7.7430	1.1776	3.1874
ln(reserveprice/vol)	0.6471	28.2154	-0.2036	-4.1706
ln(acres)	-0.0518	-3.3918	-0.0148	-0.4554
ln(loggingcost)	-0.2423	-6.2770	-0.4512	-5.4920
ln(lumberprice)	0.7717	10.6468	0.9083	5.8871
ln(volume)	0.2859	7.0295	0.5299	6.1198
ln(term)	0.0902	2.8144	0.0094	0.1374
ln(L)	0.1504	7.5043	0.3874	9.0822
sigma square	0.0478		0.2164	
number of auctions	708			

tation and elimination of all the covariates that turned out to be insignificant in both equations, we ended with a SURE system with a common set consisting of the following eight regressors: constant, R, $\ln(acres)$, $\ln(loggingcost)$, $\ln(lumberprice)$, $\ln(volume)$, $\ln(term)$ and $\ln(L)$, where the regresssors are scaled as described in Section 4.1. The results are presented in Table 7. Most importantly, all coefficients have the expected signs and are highly significant, with the exception of two coefficients in the $\ln l_*$ equation (we did verify that excluding these two made a very slight difference in the SURE estimation).[29]

With 16 moments, we can estimate up to 16 coefficients, which are a subset of (β, γ) in equation (2), together with σ^2. We expect $\ln(L)$ to play a key role for inference on collusion. However, it turns out that including it as a regressor in both μ and p raises significant (qualitative) identification issues, related to the fact that low values of l_* (relative to L) can be produced by low value of μ (relative to R) and/or high values of p. Therefore, we impose the identification restriction that

[29]Note that the SURE R^2 for $\ln(l_*)$ equation is much lower than that for the P equation, which reflects, in signficant part, the discrete nature of $\ln(l_*)$.

$\ln(L)$ be excluded from the μ equation. In other words, we assume that mills draw their individual WTPs independently from L and that only the decision of whether or not to join a ring depends on L.[30]

We proceeded to an extensive specification search, sequentially eliminating insignificant coefficients and ended with a total 8 regression coefficients consisting of

- 5 regression coefficients for μ: intercept, R, $\ln(loggingcost)$, $\ln(lumberprice)$ and $\ln(volume)$
- 2 regression coefficients for p: intercept and $\ln(L)$

together with the ninth coefficient σ.

For the ease of interpretation, we reparameterized the p equation as

$$p = \gamma_1 + \gamma_2 \ln(L) = \gamma_1^* + \gamma_2(\ln(L) - \overline{\ln(L)}), \tag{11}$$

where $\overline{\ln(L)}$ denotes the sample mean of $\ln(L)$ and

$$\gamma_1^* = \gamma_1 + \gamma_2 \overline{\ln(L)} \text{ at that sample mean.} \tag{12}$$

Similarly, we reparameterize the μ equation as

$$\mu = \beta_1 + \beta_2 R + \cdots = \beta_1^* + \beta_2(R - \overline{R}),$$

where $\overline{R}$ denotes the sample mean of R, as defined in equation (1) and

$$\beta_1^* = \beta_1 + \beta_2 \overline{R}. \tag{13}$$

[30] It becomes increasingly difficult to find non-collusive justifications for a large L relative to l_*, and we find a positive coefficient for $\ln(L)$ in the p equation. How a change in the number of bidders may affect the likelihood of explicit collusion is a focus of "coordinated effects" in the Horizontal Merger Guidelines (2010; Section 7).

6 Collusive Gains

In this section, we describe how to compute moments for (ex-post) collusive gains conditionally on (l_*, P) using Gauss (Jacobi) quadrature in combination with analytical expressions for the relevant probabilities. With reference to equation (7), collusion only occurs in cases 4 and 5. In case 4 ($l_* = 1$), the non-collusive price would be the second highest from l_1 draws above R, instead of R. In case 5 ($l_* > 1$) when $r > 1$, it would have been the second highest from r draws above R, instead of $P = V_{r+1}$.

We proceed in two steps, First, we demonstrate how Jacobi quadrature can be used to compute the moments of case 5 collusive gains, conditionally on (r, P). In Section 6.1 we derive collusive gains given r and P. In Section 6.2 we apply it to case 4 ($l_* = 1$), where (r, P) is replaced by (l_1, R). In Section 6.3 we apply it to case 5 ($l_* > 1$).

6.1 Collusive gains for case 5, conditional on (r, P)

In view of equation (1), the collusive gain when $r > 1$ and $V_2 > P$ is given by

$$\begin{aligned} W_2 - P_* &= Vol \cdot [\exp(V_2) - \exp(P)] \\ &= Vol \cdot [\exp(\mu + \sigma U_2) - \exp(P)] = h(U_2) \end{aligned} \tag{14}$$

where U_2 denotes the second order statistic of r $N(0,1)$ draws larger than $\widetilde{P} = \frac{1}{\sigma}(P - \mu)$.

For $r > 1$, the density of U_2 is given by

$$f_2(u|r,\widetilde{P}) = \frac{r!}{(r-2)!}\frac{[1-\Phi(u)]\,\phi(u)\left[\Phi(u)-\Phi(\widetilde{P})\right]^{r-2}}{\left[1-\Phi(\widetilde{P})\right]^{r}}. \qquad (15)$$

Therefore,

$$E\left[h^s(U_2)|r,\widetilde{P}\right] = \frac{r!}{(r-2)!}\left[1-\Phi(\widetilde{P})\right]^{-r}\int_{\widetilde{P}}^{\infty} h^s(u)\phi(u)\left[1-\Phi(u)\right]\left[\Phi(u)-\Phi(\widetilde{P})\right]^{r-2}du. \qquad (16)$$

In order to compute these integrals by Jacobi quadratures, we need to transform the interval $(\widetilde{P},\infty($ into $(-1,1($. This is done by means of the transformation

$$z = 2\frac{\Phi(u)-\Phi(\widetilde{P})}{1-\Phi(\widetilde{P})} - 1. \qquad (17)$$

It follows that

$$\phi(u)du = \frac{1}{2}\left[1-\Phi(\widetilde{P})\right]dz. \qquad (18)$$

The inverse transformation is given by

$$u = \varphi(z) = \Phi^{-1}\left[\frac{1+z}{2} + \frac{1-z}{2}\Phi(\widetilde{P})\right]. \qquad (19)$$

It follows that

$$E\left[h^s(U_2)|r,\widetilde{P}\right] = \frac{r!}{(r-2)!}2^{-r}\int_{-1}^{+1} h^s(\varphi(z))(1-z)(1+z)^{r-2}dz. \qquad (20)$$

The N points Jacobi quadrature with parameters $(1, r-2)$ produces N nodes and weights $\{z_i, w_i\}_{i=1}^{N}$. Therefore

$$E\left[h^s(U_2)|r,\widetilde{P}\right] \simeq \frac{r!}{(r-2)!}2^{-r}\sum_{i=1}^{N} w_i h^s(\varphi(z_i)). \tag{21}$$

Note that for $s=0$, we have $\sum_{i=1}^{N} w_i = \frac{2^r(r-2)!}{r!}$. Hence

$$E\left[h^s(U_2)|r,\widetilde{P}\right] = \sum_{i=1}^{N} \widetilde{w}_i h^s(\varphi(z_i)) \tag{22}$$

where $\widetilde{w}_i$ denotes the normalized weight

$$\widetilde{w}_i = w_i \cdot \left[\sum_{j=1}^{N} w_j\right]^{-1}. \tag{23}$$

6.2 Collusive gains in case 4: $(l_*, P) = (1, R)$

Under case 4, $(r,\widetilde{P})$ is replaced by $(l_1,\widetilde{R})$ with $\widetilde{R} = \frac{1}{\sigma}(R-\mu)$. The following probabilities are derived in the Appendix (Section 10)

$$\Pr(l_* = 1) = Lp_2p_3^{L-1} + (p_1+p_3)^L - p_3^L \tag{24}$$

$$\Pr(l_1|l_* = 1) = \pi(l_1,0|L)/\Pr(l_*=1), l_1 : 2 \to L \tag{25}$$

with

$$\pi(l_1,l_2|L) = \binom{L}{l_1\ l_2\ L-l_1-l_2} p_1^{l_1}p_2^{l_2}p_3^{L-l_1-l_2}. \tag{26}$$

The marginal moments of case 4 collusive gains are given by

$$vm(s|l_*=1) = E\left[h^s(U_2)|l_*=1,\widetilde{R}\right] = \sum_{l_1=2}^{L} \Pr(l_1|l_*=1)\cdot E\left[h^s(U_2)|l_1,\widetilde{R}\right] \tag{27}$$

and its standard deviation by

$$std = \left[vm\left(2|l_* = 1\right) - vm^2\left(1|l_* = 1\right)\right]^{1/2}. \tag{28}$$

6.3 Collusive gains in case 5: (l_*, P)=$(l_2 + 1, V_{r+1})$

In the case we need not only the probabilities $\pi(l_1, l_2|L)$ with $l_2 = l_* - 1$, but also the probabilities for $r|l_1, l_2$ when both l_1 and l_2 are greater than zero. The following probabilities are derived in Appendix (Section 10)

$$\Pr(l_* = k) = \binom{L}{k} p_2^k p_3^{L-k} + \binom{L}{k-1} p_2^{k-1}\left[(p_1 + p_3)^{L-k+1} - p_3^{L-k+1}\right], k : 2 \to L \tag{29}$$

$$\Pr(r|l_1, l_2) = \frac{1}{b_u}\binom{l_1 + l_2 - r - 1}{l_1 - r}, r : 0 \to l_1 \tag{30}$$

where

$$b_u = \sum_{r=0}^{l_1}\binom{l_1 + l_2 - r - 1}{l_1 - r} = \binom{l_1 + l_2}{l_1}. \tag{31}$$

The marginal moments of case 5 collusive gains are given by (with $l_2 = k - 1$)

$$\begin{aligned} vm(s|l_* &= k > 1) = \frac{1}{\Pr(l_* = k)} \sum_{l_1=2}^{L-l_2} \pi(l_1, l_2|L) \sum_{r=2}^{l_1} \Pr(r|l_1, l_2) E\left[h^s(U_2)|r, \widetilde{P}\right] \quad (32) \\ \text{with } \widetilde{P} &= \frac{P - \mu}{\sigma} = \frac{V_{r+1} - \mu}{\sigma} \end{aligned}$$

Last but not least, we also computed collusive gains as a direct by-product of our MC simulations. These are expected to be less accurate than those derived by

Jacobi quadratures, since the latter covers a much wider range accounting for the most extreme outliers.

Comparing results obtained under 100 Jacobi nodes to those obtained from 10,000 MC simulations, we find respective means that are within 1% of one another and standard deviations within 1 to 3%. This indicates that our Jacobi estimates of collusive gains are numerically very accurate.

7 Results and Robustness Checks

Our baseline case includes all 708 auctions and relies upon the 150 miles distance in the definition of feasible bidders (referred to as (708, L150)). Nevertheless, there are two potential concerns that need to be addressed. One is whether our results are sensitive to the distance of 150 miles. Thus, we also estimate the model under a 125 miles distance criterion. The other is whether the results on collusion might be driven by the 82 auctions with a single registered bidder ($l_* = 1$), since these are highly suggestive of collusion (pending estimation of the WTP distribution). Therefore, we estimated four different scenarios:

Baseline: (708, L150)

Alternative: (708, L125), (626, L150), (626, L125)

The results are reported in Tables 8 to 11, where we provide MSM estimates of the parameters β in μ and γ in p, together with their respective intercepts at the mean (β^*, γ^* as defined in (13) and (12), respectively). We also provide MC means and standard deviations based on 50 replications of the MSM optimization under simulated samples. These results call for several comments.

Table 8: MSM estimators (708 auctions; L150)

		MSM	Monte Carlo (50 reps)	
$\mu = \beta' x$			mean	st. dev
intercept	β_1	2.0218	2.0870	0.1501
ln(reserveprice/vol)	β_2	0.6103	0.5954	0.0305
ln(loggingcost)	β_3	-0.2505	-0.2575	0.0346
ln(lumberprice)	β_4	0.8293	0.8563	0.0610
ln(volume)	β_5	0.3045	0.3149	0.0352
at res mean	β_1^*	5.1070	5.0972	0.0360
	σ	0.1875	0.1894	0.0169
$p = \gamma' x$				
intercept	γ_1	0.4101	0.4249	0.0660
ln(L)	γ_2	0.1262	0.1181	0.0318
at L mean	γ_1^*	0.6559	0.6549	0.0121
collusive gain	cg	42.76×10^6 (2.41×10^6)	48.53×10^6	3.62×10^6

Table 9: MSM estimators (626 auctions; L150)

		MSM	Monte Carlo (50 reps)	
$\mu = \beta' x$			mean	st. dev
intercept	β_1	2.1665	2.1641	0.1008
ln(reserveprice/vol)	β_2	0.5880	0.5884	0.0225
ln(loggingcost)	β_3	-0.1899	-0.1922	0.0257
ln(lumberprice)	β_4	0.7480	0.7515	0.0494
ln(volume)	β_5	0.2274	0.2296	0.0253
at res mean	β_1^*	5.1528	5.1524	0.0347
	σ	0.1513	0.1489	0.0176
$p = \gamma' x$				
intercept	γ_1	0.1642	0.1585	0.0781
ln(L)	γ_2	0.2312	0.2334	0.0383
at L mean	γ_1^*	0.6241	0.6229	0.0094
collusive gain	cg	28.76×10^6 (2.02×10^6)	33.70×10^6	2.40×10^6

Table 10: MSM estimators (708 auctions; L125)

		MSM	Monte Carlo (50 reps)	
$\mu = \beta' x$			mean	st. dev
intercept	β_1	2.0862	2.1507	0.0963
ln(reserveprice/vol)	β_2	0.6059	0.5926	0.0230
ln(loggingcost)	β_3	-0.2428	-0.2558	0.0258
ln(lumberprice)	β_4	0.7685	0.7888	0.0692
ln(volume)	β_5	0.2949	0.3098	0.0276
at res mean	β_1^*	5.1495	5.1465	0.0426
	σ	0.1788	0.1790	0.0166
$p = \gamma' x$				
intercept	γ_1	0.3532	0.3500	0.0530
ln(L)	γ_2	0.1437	0.1439	0.0270
at L mean	γ_1^*	0.6117	0.6088	0.0125
collusive gain	cg	35.49×10^6 (2.14×10^6)	39.85×10^6	3.49×10^6

Table 11: MSM estimators (626 auctions; L125)

		MSM	Monte Carlo (50 reps)	
$\mu = \beta' x$			mean	st. dev
intercept	β_1	2.0647	2.0682	0.0817
ln(reserveprice/vol)	β_2	0.6135	0.6089	0.0180
ln(loggingcost)	β_3	-0.1680	-0.1674	0.0255
ln(lumberprice)	β_4	0.7042	0.7108	0.0392
ln(volume)	β_5	0.2046	0.2055	0.0241
at res mean	β_1^*	5.1647	5.1607	0.0277
	σ	0.1579	0.1584	0.0155
$p = \gamma' x$				
intercept	γ_1	0.0703	0.0637	0.0463
ln(L)	γ_2	0.2684	0.2713	0.0236
at L mean	γ_1^*	0.5651	0.5640	0.0101
collusive gain	cg	23.88×10^6 (1.92×10^6)	28.88×10^6	2.60×10^6

1. The statistical performance of our MSM estimation procedure is impressive. The MC simulation of the finite sample properties of our MSM estimates indicate that they are virtually unbiased and highly significant. Overall they are also largely similar across all four scenarios. These statistical findings suggest that the original

MSM procedure we introduced could prove useful across a wide range of empirical auction models.

2. Our most important results concern the individual probability p of joining a ring. As we might have expected from the tight range of L, the coefficients γ_1 and γ_2 in equation (11) are the ones that are most sensitive to the distance choice. However γ_1^*, which represents the estimate of p of the sample mean of L, lies in the interval (0.565, 0.656), indicative of significant bidder collusion under all four scenarios. Therefore, while the 82 auctions with only one registered bidder ($l_* = 1$) are highly suggestive of collusion, it is all 708 auctions together that provides clear evidence of across the board collusion.

3. The MSM estimates of the coefficients β_2 to β_5 in μ all have the expected signs, are virtually unbiased and highly significant. Moreover, they are fairly similar across all four estimation scenarios.

4. Our estimates of collusive gains are highly significant with a baseline estimate of \$42.76million and vary across the four scenarios for two main reasons:

(i) The expected values of the WTP second order statistic V_2 increases with L. But with an individual probability p around 0.6, the expected number of ring members with WTP's above R (l_1) also increases. The net effect is that decreasing L from 150 to 125 reduces collusive gain by 17%.

(ii) The 82 auctions with $l_* = 1$ imply that the price equal to reserve, irrespective of V_2. Thus, while they represent only 11.6% of all 708 auctions, they contribute on the order of 22.7% of total collusive gain (under both L150 and L125).

5. The fact that the MC estimates of collusive gain are larger than the corresponding MSM estimates (by factors ranging from 12.3% to 20.9%) is due to a key difference between the scenarios under which they are dervied. As explained in Section 6, the MSM estimates of collusive gains are conditional on the observed (P, l_*).

Under the MC simulations, they are unconditional since they rely upon the pairs (P, l_*) produced by simulation.

6. Last but not least, under MSM estimation, we also computed the percentage ratios between the mean WTP ($= Vol * \exp(\mu + \frac{1}{2}\sigma^2)$) and reserve, both individually and aggregated. The aggregate ratios under the four scenarios range from 137% to 141%, fully confirming the notion that IDL reserves are definitely "non-aggressive".

8 Discussion

8.1 Auction Design

It is difficult to imagine an auction design that is less resistant to collusion than the one used by IDL. This is a well recognized fact (see Marshall and Marx 2009). It is ascending bid, which is well known to be more vulnerable (as discussed above). The identities of the bidders are well known at the time of the auction and all bidding is publicly observable in real time. The reserve price is publicly announced well in advance of the auction. This auction design eases all monitoring burdens for a ring. There are auction designs for the sale of timber that have been demonstrated to be highly robust to collusion, such as the one used by the province of Quebec. Idaho should investigate alternative auction designs. Of course, the industry will strongly object, but that is, in part, an objection rooted in the profitability of bidder collusion.

8.2 First price auctions and bidder collusion

It is not uncommon for researchers to assume that first price sealed bid auctions are void of collusion (for example, see Athey, Levin, and Seira, 2011, page 235, "... we assume competitive behavior in the sealed bid auctions as outlined.") and thus they can serve as a benchmark for evaluating the extent of collusion at ascending bid auctions. This a remarkably strong assumption. From the public record, the vast majority of effective cartels have confronted first price sealed bid procurements by buyers and successfully rigged bids to elevate prices paid by buyers.

Second, we looked at the USFS timber sales in Idaho, many of which are first price sealed bid auctions. Over the same period of time as investigated herein for the IDL auctions, in Table 12 we find that L is bigger than l-star for the USFS first price sealed bid auctions by the same relative magnitude as was the case for the IDL sales we have studied. If first price sealed bid auctions are void of collusion then why are so many feasible bidders not attending those sales? Our findings suggest that a leading potential explanation would be bidder collusion. Perhaps bidder collusion at a sealed bid auction, where the auctions repeat and the bidders are largely common across those auctions, is less effective than what we observe at the ascending bid auctions, but that is an empirical question for future research. Our point is that the assumption of non-collusive conduct at the first price sales appears to be a highly questionable maintained hypothesis.

Table 12: l-star and L comparison: IDL auctions and USFS Idaho auctions

	average l-star	average L	number of auctions
IDL	3.2	7.6	708
USFS Idaho ascending	2.7	6.4	31
USFS Idaho first price	2.3	6.3	152

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10 Appendix: probabilities

We first derive the probability $\Pr(r|l_1, l_2)$ for the event "V_{r+1} is the first non-collusive WTP" given l_1 and l_2, with $r : 0 \to l_1$. Let the index 1 denote a ring member and 0 a non-ring bidder (these indices are stored in the auxiliary vector lb introduced in the simulation procedure described in Section 5.1.1). In order for V_{r+1} to be the first non-collusive WTP, we need the first r WTP's to be indexed by 1, the next one by 0, with the remaining $l_1 + l_2 - r - 1$ being any permutation of the $l_1 - r$ one's and $l_2 - 1$ zero's. Moreover, the total number of permutations of l_1 one's and l_2 zero's is given by $\binom{l_1+l_2}{l_1}$. Hence,

$$\Pr(r, l_1, l_2) = \tfrac{1}{b_u}\tbinom{l_1+l_2-r-1}{l_1-r}, r : 0 \to l_1$$
$$\text{where } b_u = \tbinom{l_1+l_2}{l_1}$$

$\pi(l_1, l_2|L)$ is the multinomial probability of a draw of $(l_1, l_2, L - l_1 - l_2)$ elements with respective probabilities (p_1, p_2, p_3). Thus

$$\pi(l_1, l_2|L) = \binom{L}{l_1 \;\; l_2 \;\; L - l_1 - l_2} p_1^{l_1} p_2^{l_2} p_3^{L-l_1-l_2} \tag{33}$$

Finally, we derive the probabilities for l_* given L. First,

$$\Pr(l_* = 0|L) = \pi(0, 0|L) = p_3^L \tag{34}$$

For $l_* > 0$, the pairs (l_1, l_2) that imply $l_* = k > 0$ consist of $(0, k)$ and $\{(l_1, k - 1), l_1 : 1 \to L - k + 1\}$.Therefore,

$$\begin{aligned} \Pr(l_* &= k|L) = \pi(0, k|L) + \sum_{l_1=1}^{L-k+1} \pi(l_1, k-1|L) \\ &= \binom{L}{k} p_2^k p_3^{L-k} + \binom{L}{k-1} p_2^{k-1} \sum_{l_1=1}^{L-k+1} \binom{L-k+1}{l_1} p_1^{l_1} p_3^{L-k+1-l_1} \end{aligned}$$

The sum from $l_1 = 1$ to $L - k + 1$ equals the sum from $l_1 = 0$ to $L - k + 1$ minus the term $l_1 = 0$. Therefore,

$$\Pr(l_* = k|L) = \binom{L}{k} p_2^k p_3^{L-k} + \binom{L}{k-1} p_2^{k-1} \left[(p_1 + p_3)^{L+1-k} - p_3^{L+1-k} \right], \text{for } k > 0 \tag{35}$$

Note: it is useful to verify that

$$\sum_{k=0}^{L} \Pr(l_* = k|L) = p_3^L + \left[L p_2 p_3^{L-1} + (p_1 + p_3)^L - p_3^L \right] + \left[1 - L p_2 p_3^{L-1} - (p_1 + p_3)^L \right] = 1$$