# Contracting for Information under Imperfect Commitment*

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**Abstract**

Mechanism design theory rests critically on the assumption that the principal can fully commit to the workings of the mechanism (or contract), yet in many situations of economic interest, this assumption is clearly false. We study optimal contracting between an uninformed principal and informed agent where the principal can commit to compensation schemes but not to other aspects of the contract. Although the standard revelation principle is not valid in this setting, we derive a limited version that nevertheless allows us to identify all feasible contracts. We then find the optimal contract under imperfect commitment and show that the principal should (a) never induce the agent to fully reveal what he knows—even though this is feasible—and (b) never pay the agent for imprecise information. We compare the optimal contract with imperfect commitment to that under full commitment as well as to several "informal" institutional arrangements and find that gains from contracting are greatest when the misalignment of preferences between the principal and the agent is moderate.

JEL Classification D23, D82.

## 1 Introduction

Mechanism design theory is the central tool employed to study how and to what extent problems of asymmetric information can be resolved. Much of the underlying methodology was developed in the study of optimal auctions, taxation, public goods, nonlinear pricing and bilateral trade. More recently, its use has spread to diverse fields such as monetary theory and development economics, where certain economic

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institutions are best viewed as mechanisms. The mechanism design approach relies essentially, however, on the assumption that the planner or principal can fully commit to follow the workings of the mechanism. In contract theory terms, this means that all of the aspects of the contract (or mechanism) proposed by the principal are written in "pen" and cannot be erased.

If, however, the principal's ability to commit is altogether absent, then information provided by agents has no direct payoff consequences and is just "cheap talk." In practice, many situations of economic interest lie between these two extremes. The principal is capable of contracting, but at least some clauses of the contract are written in "pencil" rather than in pen.

For instance, suppose a CEO is considering what tender offer to make in an acquisition. The CEO might consult an investment bank to aid in this decision, but it may well be that the investment bank is biased toward the acquisition. Standard contract theory suggests that the CEO can solve the incentive problem with the bank by simply offering a schedule of tenders with appropriately structured bank fees and then letting the bank make the tender offer on her behalf. The problem, of course, is that this prescription is unpalatable for the CEO. After all, the CEO is ultimately answerable to a board of directors for key corporate decisions like tender offers and so would be extremely reluctant to cede such authority to an outsider. In other words, the CEO would be unable to or be unwilling to fully contract with the bank in the manner assumed in standard contract or mechanism design theory. Thus, the central question remains—how should the bank be compensated for its services even though the CEO is unable to contract on how the bank's advice will be acted upon?

Many other situations have similar features. For instance, managers at retail stores (like Wal-Mart) may offer advice on the optimal square footage for the store at a particular location to the corporate headquarters and their actual compensation may be tied to the resulting sales via a compensation contract. In the case of Wal-Mart, where such decisions are notably centrally controlled, the ultimate decision on retail square footage at a location is retained by headquarters in Arkansas. Similarly, Bajari and Tadelis (2001) note that construction contracts between commercial developers and general contractors consist of compensation schedules based on the project ultimately undertaken (with the advice of the contractor) but with authority ultimately retained by the developer.

In general, little is known about the form of optimal contracts in situations like these. The dilemma that researchers face in such environments is the fact that a central tool of contract theory, the "revelation principle," fails when commitment is imperfect (Bester and Strausz, 2001). Without the revelation principle, there is no systematic way to determine the set of feasible contracts let alone the optimal contract—the class of contracts one may consider is necessarily *ad hoc*.

In this paper, we study contracting in a simple bilateral setting in which a principal must rely on the information provided by an informed agent to select a project. Information and project choices are both continuous. Tension in the model stems from the fact that the agent is biased—given the same information the principal and agent would select different projects. In addition, commitment on the part of the principal is imperfect: while the principal can contract on a schedule of transfer payments, she is unable to contract on a schedule of project choices. In other words, the principal retains the authority to make decisions regarding the project.

We first establish that a limited form of the revelation principle—sufficient for our needs—continues to hold even though commitment is imperfect.[1] We show that direct contracts—in which the agent provides possibly noisy information to the principal—span the set of all feasible contracts. However, because this limited form of the revelation principle does not allow one to restrict attention to truth-telling strategies, the problem of characterizing the optimal contract is considerably more involved than under full commitment. We are able to identify several key properties, described below, that optimal contracts must share. These enable us to ultimately characterize an optimal contract.

Our findings regarding contracts with imperfect commitment are as follows:

1. *Full revelation is always feasible but never optimal.* Even when commitment is imperfect, contracts remain powerful tools for eliciting information from the agent. We demonstrate that, regardless of biased the agent is, there exists a contract that induces the agent to fully reveal all of his information. That is, even under imperfect commitment, there exists a contract that fully aligns the agent's incentives with those of the principal. We show, however, that despite the fact that full revelation is always feasible, it is never optimal.

2. *Never pay for imprecise information.* In a leading case of the model—the so-called uniform-quadratic case—optimal contracts can be explicitly characterized. A notable feature of optimal contracts is that they never entail any payment for imprecise information—the agent is compensated only in those states that he is willing to fully reveal.

3. *Contracts are most valuable for intermediate levels of bias.* When the degree of bias is small, the incremental value to the principal obtained in the optimal contract is also small. When the degree of bias is large, incremental value provided by the optimal contract is also small. Indeed, for extreme levels of bias, the optimal contract is no contract at all.

While the features above are derived in the context of direct contracts, we show that even when commitment is imperfect, a version of the "taxation principle"—that derives an equivalent indirect contract—still applies. An application of this allows us to reinterpret the optimal contract as one in which the principal uses limited delegation. By this is meant that decision-making authority is transferred to the

[1] The positive result of Bester and Strausz (2001) cannot be applied to our model.

agent with the proviso that the principal reserves the right to override the agent's choice.

We then use the structure of the optimal contract—especially, an indirect version—to shed some light on organizational design. One may view the indirect optimal contract as a manifestation of "real versus formal" authority. While the principal reserves the right to override the agent's project choice (that is, she retains formal authority), in equilibrium, the agent's choice of projects will be "rubber-stamped" by the principal (that is, the agent holds real authority). Aghion and Tirole (1997) explore issues related to contracting with real versus formal authority in a moral hazard framework where transfers are effectively absent. Our work complements theirs by deriving properties of optimal allocation of authority in adverse selection settings where transfers are possible.

For purposes of comparison, we also derive the structure of contracts with perfect commitment—that is, situations in which the contract specifies both how the agent will be compensated and how project choices will be made. The main difference between optimal contracts under imperfect commitment and those under perfect commitment is the absence of compromise in the former. Under perfect commitment, the optimal contract specifies either a compromise project or the project most favored by the agent. Compromise is optimal since it economizes on transfers to the agent. If, however, one were to try to implement a similar contract under imperfect commitment, it would not be credible since *ex post* the principal would overrule the agent's project choice.

**Related Literature**

Our analysis builds on the classic "cheap talk" model of Crawford and Sobel (1982) which studies the interaction between an informed agent and an uninformed principal. In their setup, the principal effectively has no commitment power whatsoever. In contrast, we allow the principal to commit to transfer payments and, in the perfect commitment section, to projects as well.

Much of the related literature focuses on a particular specification with negative quadratic preferences and a uniform distribution of states. This so called "uniform-quadratic" case is notable for its tractability and has been extensively used in subsequent applications. For instance, the political science literature on the efficacy of legislative rules (see Gilligan and Krehbiel, 1987 & 1989 and Krishna and Morgan, 2001), largely concerns itself with this case.

Baron (2000) studies the effect that "contracting" arrangements have on the interaction between an uninformed legislature and an informed committee for the uniform-quadratic case. His model differs from ours in many respects. First, he restricts the set of contracts to those that either involve full revelation over some interval and no revelation over another (in that case, the committee is said to be "discharged"). Second, the limited liability constraint is replaced by an endogenous participation constraint. This means that transfers between the legislature and the committee could be in either direction. Indeed the contract that is optimal in his class involves transfers from the agent to the principal. Finally, the principal is assumed to be able to commit to a transfer only when information is revealed. This is a critical assumption as it can be shown that the principal can improve her payoff by means of a contract that also involves transfers even when the agent is "discharged."

Ottaviani (2000) also examines how the use of transfers can enhance the amount of information that the agent shares with the principal. Again, for the uniform-quadratic case, he shows the possibility of full revelation contracts (this is a special case of our Proposition 2) and that this contract is dominated by one that delegates authority to the agent directly but involves no transfers. He does not study optimal contracts under either imperfect or perfect commitment.

Dessein (2001) also examines the benefits of delegation in a similar model, again in the uniform-quadratic case.[2] Unlike our setting, Dessein does not allow for the possibility of transfer payments by the principal. Further, the principal is assumed to be able to commit not to intervene in the project chosen by the agent; thus, issues associated with imperfect commitment are also absent. In Section 5, we compare optimal contracts in our setting with delegation contracts along the lines of Dessein. In a similar model with transfers, Krähmer (2004) allows the principal to commit whether or not she wants to delegate authority to the agent depending on the message sent by the agent. He shows that such "message contingent delegation" may be superior both to ex ante or "unconditional" delegation as in Dessein (2002) and to unconditional retention of authority.

A separate strand of the literature is concerned with solving the moral hazard problem of information gathering on the part of the agent or agents (see, for example Aghion and Tirole, 1997 and Dewatripont and Tirole, 1999). In contrast, our primary interest is in the role of contracts to elicit information from already informed agents. In these papers, incentive alignment for efficient information transmission, once the agent has gathered information, is a secondary consideration.

Problems of commitment can also arise when economic agents suffer from self-control problems such as time inconsistency. for instance, O'Donahue and Rabin (1999) investigate optimal incentive contracts in a moral hazard setting where a time-inconsistent agent would like to commit to a future path of efforts but cannot. Our model may be viewed as complementary to this line—we explore optimal contracting in an adverse selection setting where the principal suffers from problems of commitment.

Finally, our paper is somewhat related to questions addressed in Bester and Strausz (2001). That paper seeks to extend the revelation principle to settings where the principal is unable to commit to one or more dimensions of the contracting space—as in the question we consider. They show that when the set of states is *finite*, any *incentive efficient* outcome of that mechanism—that is, an equilibrium outcome not Pareto dominated by another equilibrium outcome—can be replicated by an equilib-

[2] Dessein also looks at cases where the preferences are concave functions of the quadratic loss specification.

rium of a direct mechanism. Their result, however, does not apply to our model since it has a *continuum* of states. We derive a revelation principle in Proposition 1 that in the context of our model, applies to all *incentive feasible* outcomes, not only those that are also incentive efficient.

The remainder of the paper proceeds as follows: In Section 2 we sketch the model. Section 3 presents results on full revelation contracts and shows that these contracts are never optimal. We then derive various structural properties of optimal contracts and then uses these to characterize in closed form the optimal contract under imperfect commitment. Section 4 does the parallel exercise for perfect commitment. Section 5 compares the value of contracting with several alternative schemes. Finally, Section 6 concludes.

# 2 Preliminaries

In this section we sketch a simple model of decision making between an informed agent and an uninformed principal. The principal may write binding contracts concerning some aspects but not others. If the ability to commit is absent altogether, the model reduces to the well-known "cheap talk" setting of Crawford and Sobel (hereafter 'CS'). Under full commitment, the model is similar to others in standard contract theory, although the analysis of the optimal contract has some new aspects.

Consider a *principal* who has authority to choose a project $y \in \mathbf{R}$, the payoff from which depends on some underlying state of nature $\theta \in \Theta \equiv [0, 1]$. The state of nature $\theta$ is distributed according to the density function $f(\cdot)$. The principal has no information about $\theta$, but this information is available to an *agent* who observes $\theta$.

The payoff functions of the players, not including any transfers, are of the form $U(y, \theta, b_i)$ where $b_i$ is a bias parameter which differs between the two parties. The bias of the principal, $b_0$, is normalized to be 0. The bias of the agent, $b_1 = b > 0$. In what follows we write $U(y, \theta) \equiv U(y, \theta, 0)$ as the principal's payoff function. We suppose that $U$ is twice continuously differentiable and satisfies $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$. Since $U_{13} > 0$ the parameter $b$ measures how closely the agent's interests are aligned with those of the principal and it is useful to think of $b$ as a measure of how *biased* the agent is, relative to the principal. We also assume that for each $i$, $U(y, \theta, b_i)$ attains a maximum at some $y$. Since $U_{11} < 0$, the maximizing project is unique. The biases are commonly known.

These assumptions are satisfied by "quadratic loss functions." In this case, the principal's payoff function is

$$U(y, \theta) = -(y - \theta)^2 \tag{1}$$

and the agent's payoff function is

$$U(y, \theta, b_i) = -(y - (\theta + b))^2 \tag{2}$$

where $b > 0$.

Define $y^*(\theta) = \arg\max_y U(y, \theta)$ to be the *ideal* project for the principal when the state is $\theta$. Similarly, define $y^*(\theta, b) = \arg\max_y U(y, \theta, b)$ be the ideal project for the agent. Since $U_{13} > 0$, $b > 0$ implies that $y^*(\theta, b) > y^*(\theta)$.

Notice that with quadratic loss functions, the ideal project for the principal is to choose a project that matches the true state exactly: for all $\theta$, $y^*(\theta) = \theta$. The ideal project for an agent with bias $b$ is $y^*(\theta, b) = \theta + b$.

In the basic CS model, upon learning the state $\theta$, the agent is assumed to offer some "advice" to the decision maker. This advice is in the form of a costless message $m$ chosen from some fixed set $M$. Upon hearing the advice offered by the agent, the principal chooses the project $y$.

We augment the CS model and allow the principal to contract with the agent and perhaps make transfer payments. We suppose that preferences of the two parties are quasi-linear. Thus, if a payment $t \geq 0$ is made to the agent, then the payoff of the principal from project $y$ in state $\theta$ is

$$U(y, \theta) - t$$

while the payoff of the agent is

$$U(y, \theta, b) + t$$

We assume that only nonnegative transfers ($t \geq 0$) from the principal to the agent are feasible. In effect, the agent is protected by a "limited liability" clause and cannot be punished too severely.[3]

Two contracting environments are studied.

1. *Imperfect commitment.* In this case, the principal commits to transfer payments but retains ultimate authority over the choice of the project. That is, the principal cannot commit not to *intervene* in the choice of projects.

2. *Perfect commitment.* In this case, the principal contracts in advance on both the project choice and the transfer.

While the precise form of these contracts is specified below, throughout this paper we suppose that the two parties cannot contract on the state of nature $\theta$. Contracts are not allowed to depend directly on the realized state of nature since even after the fact, it may be difficult to verify for a third party.[4] With this one exception we allow the contract to depend on any other variable that is mutually observable and can be verified by a third party. For instance, the contract could specify how the

[3]While the precise characterizations of the optimal contract make use of this assumption, many of the qualitative features of the model are unaffected if we replace this with an interim participation constraint. For instance, Proposition 3 below continues to hold.

[4]See Prendergast (1993) for a variety of other reasons why contracting on the realized state (or equivalently on realized payoffs) may be problematic.

compensation will depend on the project actually undertaken by the principal. This, of course, allows an indirect dependence of the contract on the state of nature but is based on something that is verifiable.

# 3 Contracts with Imperfect Commitment

We first study a situation in which the ability of the principal to commit is imperfect. By this we mean that the principal is unable or unwilling to bind herself to choose a particular project as a result of the advice offered by the agent—that is, she retains executive authority.

Without loss of generality, any mechanism in this setting can then be represented as a pair $(M, T)$, where $M$ is an arbitrary set of *messages* and $T(\cdot)$ is a transfer scheme that determines the compensation $T(m) \geq 0$ that the agent will receive if he sends the message $m$. The idea of imperfect commitment is captured by assuming that of the two instruments available to the principal, project choices $y$ and transfers $t$, she can contract on, and commit to, only one. The purpose of the contract is, of course, to align the interests of the agent more closely with those of the principal.

While the specification that compensation is based on the message ("advice") alone seems unnatural; so it is useful to examine how, exactly, such contracts are, indeed, without loss of generality. The key is to notice that more "realistic" contracts, such as those described in the introduction, are all accommodated within this framework. For instance, the compensation contract for an investment bank may be of the form $T(y)$. The investment bank's advice, $m$, ultimately leads the CEO to undertake a project, $y(m)$, and hence to compensation $T(y(m))$. Thus we may suppose that $T$ depends on the message $m$ itself.

When the principal can perfectly commit—that is, to both a project $Y(m)$ and a transfer $T(m)$—then the *revelation principle* may be invoked. Specifically, for any (full-commitment) mechanism $(M, Y, T)$ and any equilibrium of this mechanism, (i) there exists a direct mechanism—in which the agent reports his private information, so that $M = \Theta$—such that (ii) truth-telling is an equilibrium that is outcome equivalent to the given equilibrium of the original mechanism. The underlying argument is very simple. The original mechanism can be composed with the strategies that constitute an equilibrium of the said mechanism to obtain a direct mechanism. It is then easily verified that truth-telling is an equilibrium of the direct mechanism—it is *incentive compatible*—and that the resulting outcomes are the same as in the given equilibrium of the original mechanism. Put another way, incentive compatible outcomes of direct mechanisms span the set of equilibrium outcomes resulting from *all* mechanisms.

The statement above highlights the fact that the standard revelation principle has two components. First, it allows one to restrict attention to direct mechanisms. Second, only truth-telling equilibria of direct mechanisms need be considered. The revelation principle is a powerful tool because, rather than searching over the space of all possible indirect mechanisms, an impossible task, it allows the analyst to restrict attention to direct mechanisms—that is, it is the first component that renders the task of finding an optimal contract feasible.

When the principal's ability to commit is imperfect, the second component of the revelation principle clearly fails in general. This is because if the principal is not committed to act in prespecified ways, when the agent truthfully reveals, the principal will use this information to her own advantage. Knowing this, the agent will, in general, be better off not fully revealing what he knows—that is, some loss of information is likely. To see this in the context of the model of the previous section, suppose that the principal can commit to neither decisions not transfers. In that case, the model is identical to CS, who showed that full revelation cannot be an equilibrium.

It remains to see if the first component continues to hold. For the reasons described above, knowing even this component would be extremely helpful. But, as we demonstrate below (our example is similar to one by Bester and Strausz, 2001), with imperfect commitment, even the first component of the revelation principle may fail—there may be equilibrium outcomes of an indirect mechanism that cannot be replicated by a direct mechanism.

Suppose that the principal and agent have quadratic loss functions where the agent's bias is $b = \frac{1}{6}$, but, in a departure from our model, the state space is *binary*, that is, $\theta \in \Theta = \{\theta_1, \theta_2\}$ where $\theta_1 = \frac{1}{4}$ and $\theta_2 = \frac{3}{4}$. Each state is equally likely. Consider a contract in which the set of messages has three elements so that $M = \{m_1, m_2, m_3\}$ and the associated transfer scheme:

$$T(m_1) = \tfrac{1}{6},\ T(m_2) = \tfrac{7}{48} \text{ and } T(m_3) = 0$$

Suppose the agent follows the reporting strategy:

$$\begin{array}{l}\text{in state } \theta_1\text{, send either } m_1 \text{ or } m_2 \text{ with probability } \tfrac{1}{2}\\ \text{in state } \theta_2\text{, send either } m_2 \text{ or } m_3 \text{ with probability } \tfrac{1}{2}\end{array}$$

The message $m_1$ is sent only in state $\theta_1$ and thus reveals to the principal that the state is $\theta_1$. Similarly, $m_3$ reveals that the state is $\theta_2$. The message $m_2$, however, is sent in both states and so the principal is still unsure as to which state has occurred. Thus the posterior beliefs of the principal after hearing message $m_i$ are

$$p(\theta_1|m_1) = 1,\ p(\theta_1|m_2) = \tfrac{1}{2}, \text{ and } p(\theta_1|m_3) = 0$$

Given these beliefs, the optimal project choices of the principal following $m_i$ are

$$y(m_1) = \theta_1,\ y(m_2) = \tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2, \text{ and } y(m_3) = \theta_2$$

It is routine to verify that, given the $y(m_i)$ as above, it is a best response for the agent to behave in the manner specified. Thus when the set of messages is $M =$

$\{m_1, m_2, m_3\}$, there is a contract $T$ and a perfect Bayesian equilibrium (PBE) of the resulting game, in which the principal chooses *three* possible project with positive probability.

In a direct mechanism, that is, if $M = \Theta$, then following any report $\theta \in \Theta$, at most one project would be optimal for the principal. Thus, the principal chooses *two* possible projects. This means that a direct mechanism cannot replicate the workings of the indirect mechanism specified above; nor can it replicate the resulting payoffs. Without imperfect commitment, both components of the revelation principle may fail. Thus, it is not clear how one should proceed to find an optimal contract—one that is best for the principal. The set of outcomes depends on the number of messages that the agent may use to convey information and that the number required messages may be more than the number of states. What is the "right" number of messages?

In the example above, there were only two states. In our model, there is a *continuum* of states and, as we show below, this restores the first component of the revelation principle: even with imperfect commitment, *any* equilibrium outcome of an indirect mechanism can be replicated by an equilibrium of a direct mechanism.[5]

## 3.1 A "Revelation Principle" with Imperfect Commitment

Consider a contract $(M, T)$ in which the agent sends messages $m$ in some set $M$. Given a message $m$, the principal transfers $T(m)$ to the agent. After that she is free to choose any project $y$ that she wishes. This defines a game between the principal and agent.[6]

A perfect Bayesian equilibrium $(\mu, Y, G)$ of this game consists of (i) a strategy for the agent $\mu : \Theta \to \Delta(M)$ which assigns for every state $\theta$, a probability distribution over $M$; (ii) a strategy for the principal $Y : M \to \mathbf{R}$; and (iii) a belief function $G : M \to \Delta(\Theta)$ which assigns for every $m$ a probability distribution over the states $\theta$. It is required that following any message $m$, the principal maximizes her expected utility given her beliefs; $G$ is derived from $\mu$ using Bayes' rule wherever possible; $\mu$ is optimal given $Y$.[7]

In what follows, an "equilibrium" is always understood to mean "perfect Bayesian equilibrium."

**Proposition 1 ("Revelation Principle")** *Consider a contract $(M, T)$ with imperfect commitment and any equilibrium under this contract. Then there exists an equilibrium under a direct contract $(\Theta, t)$ which is outcome equivalent.*

[5]Following Bester and Strausz (2001), in what follows we refer to this conclusion—the first component—as a "revelation principle without commitment."

[6]We restrict attention to mechanisms in which (a) there is only one stage of communication; and (b) there is no mediator through whom the agent communicates.

[7]Because of the assumption that the principal's utility $U(\cdot, \theta)$ is strictly concave, it is unnecessary to allow for strategies in which the principal randomizes.

**Proof.** Suppose that $(\mu, Y, G)$ is a perfect Bayesian equilibrium under the contract $(M, T)$. Given any state $\theta$, define $\overline{Y}(\theta) = \max\{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta)\}$ and $\underline{Y}(\theta) = \min\{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta)\}$. These are well defined since in equilibrium, the principal plays a best response.

Consider two states $\theta_1 < \theta_2$. Then we claim that $\overline{Y}(\theta_1) \leq \underline{Y}(\theta_2)$. Suppose to the contrary that $\overline{Y}(\theta_1) > \underline{Y}(\theta_2)$. If $T_1$ and $T_2$ are the transfers associated with $\overline{Y}(\theta_1)$ and $\underline{Y}(\theta_2)$, respectively, then by revealed preference of $\overline{Y}(\theta_1)$ in state $\theta_1$ we have that $U\left(\overline{Y}(\theta_1), \theta_1, b\right) - U\left(\underline{Y}(\theta_2), \theta_1, b\right) \geq T_2 - T_1$. Since $U_{12} > 0$, we have that $U\left(\overline{Y}(\theta_1), \theta_2, b\right) - U\left(\underline{Y}(\theta_2), \theta_2, b\right) > T_2 - T_1$ which is a contradiction since this means that it is better to induce action $\overline{Y}(\theta_1)$ and transfer $T_1$ in state $\theta_2$. Thus, $\overline{Y}(\theta_1) \leq \underline{Y}(\theta_2)$ and so in equilibrium, any two states have at most one project in common. Moreover, this also implies that the function $\overline{Y}(\cdot)$ is monotone.

Next, suppose that $\theta$ is such that $\underline{Y}(\theta) < \overline{Y}(\theta)$. Then from the previous paragraph, for all $\theta' \leq \theta$, we have $\overline{Y}(\theta') \leq \underline{Y}(\theta) < \overline{Y}(\theta)$ and so the function$\overline{Y}(\cdot)$ is discontinuous at $\theta$. But a monotonic function can be discontinuous only on a countable set and this implies that $\underline{Y}(\theta) < \overline{Y}(\theta)$ for at most a countable number of points $\theta$. To summarize, we have so far shown that, in any equilibrium of any indirect mechanism, the agent induces a *unique* project $y(\theta)$, and hence a unique transfer $t(\theta)$, in almost every state.

Suppose that under the contract $(M, T)$, the project $y(\theta)$ and transfer $t(\theta)$ are equilibrium outcomes in state $\theta$. Now consider the direct contract $(\Theta, t)$ and the following strategy for the agent: Suppose that in state $\theta$, the equilibrium calls for project $y(\theta)$ to be induced. Define

$$Z(\theta) = \{\sigma : y(\sigma) = y(\theta)\}$$

to be the set of states in which the project induced is the same as that induced in state $\theta$. By the monotonicity property, $Z(\theta)$ is an interval, possibly degenerate.

To complete the proof, let the equilibrium strategy of the agent in the direct contract, $\mu^*(\cdot \mid \theta)$ be the uniform distribution over the elements of $Z(\theta)$. This strategy leads the principal to hold posterior beliefs $G^*$ identical to those in the equilibrium of the indirect contract, and so the project chosen by the principal in state $\theta$ will be the same in the two equilibria. Thus, the direct contract $(\Theta, t)$ is outcome equivalent to the contract $(M, T)$ and this completes the proof. ■

Like the standard revelation principle, Proposition 1 allows us to restrict attention to direct mechanisms—bypassing the plethora of possible indirect contracts. It should be contrasted with a similar result of Bester and Strausz (2001) because (a) it concerns situations with a continuum of states and, as a result, (b) it applies to all incentive feasible contracts, not only those that are incentive efficient.

## 3.2 Full Revelation Contracts

Organization and management texts suggest that the provision of incentives and delegation of authority are complementary (see, for instance, Milgrom and Roberts, 1992, p. 17). It is argued that the principal should delegate decision making authority to the agent only after providing incentives such that the agent's objectives are aligned with those of the principal. In this subsection, we examine the extent to which this applies in our model. In particular, we examine two related questions: First, with imperfect commitment, is it even possible for the principal to design a contract that completely aligns the agent's interests with her own and gets him to fully reveal what he knows? Second, and more importantly, if it is possible, under what circumstances is this the best contract for the principal? Proposition 1 allows us to restrict attention to direct contracts.

In the absence of any contracting ability whatsoever, CS have shown that it is impossible for the principal to induce the agent to fully reveal his private information. However, we show below that when the principal can contract, albeit imperfectly, this is no longer the case—full revelation is always implementable. We then show that, despite the fact that such a contract always leads to the principal obtaining her most desired project in every state, it is never cost effective. That is, full revelation contracts are *never* optimal.

To see that full revelation contracts are always feasible, first notice that under such a contract the agent offers truthful advice; that is, $\mu(\theta) = \theta$. Further, the principal anticipates that this will be the case; hence $y(\theta) = y^*(\theta)$. For truth-telling to be a best response requires that in every state $\theta$

$$U(y^*(\theta), \theta, b) + t(\theta) \geq U(y^*(\theta'), \theta, b) + t(\theta')$$

for all $\theta' \neq \theta$. The first-order condition for the agent's maximization problem results in the differential equation

$$t^{*\prime}(\theta) = -U_1(y^*(\theta), \theta, b)\, y^{*\prime}(\theta)$$

Since $U_1(y^*(\theta), \theta, b) > 0$ and $y^{*\prime}(\theta) > 0$, a contract that induces full revelation is *downward sloping*. Thus among all contracts that induce full revelation and satisfy limited liability, the least-cost one is:

$$t^*(\theta) = \int_\theta^1 U_1(y^*(\alpha), \alpha, b)\, y^{*\prime}(\alpha)\, d\alpha \tag{3}$$

It is routine to verify that the contract in (3) indeed induces full revelation—that is, no nonlocal deviations are profitable either. To summarize:

**Proposition 2** *Under imperfect commitment, full revelation contracts are always feasible.*

Proposition 2 can also be derived as follows. A standard result in contract theory (see Salanié, 1997, p. 31) is that with full commitment every monotonic project choice can be implemented via a truthful direct mechanism with an appropriate transfer scheme. This implies that $y^*$ can be so implemented. But since $y^*$ is ex post optimal for the principal under truth-telling, no commitment is needed to ensure that the principal will in fact, choose $y^*(\theta)$ in state $\theta$. Thus $y^*$ can be implemented even without commitment.

Now we show that a full revelation contract is never cost-effective. For states near the highest possible state, the direct contracting costs of inducing truth-telling are relatively inexpensive ($t^*(\theta)$ is close to zero when $\theta$ is close to 1); however the indirect effect of obtaining this revelation is to increase the information extraction costs for *all* of the lower states. The alternative contract shows that the informational benefits of additional revelation in the high states never justifies these increased costs. The principal can *locally* give up a small amount of information by inducing pooling for the highest states, but more than recovers this in the *global* reduction in the costs of information extraction for lower states. Formally,

**Proposition 3** *Under imperfect commitment, full revelation contracts are* never *optimal.*

**Proof.** See Appendix A. ■

The trade-off between distorting the incentives of some set of agents to save on information rents paid to others is reminiscent of a similar trade-off in monopolistic screening settings, say, which forms the rationale for a price discriminating monopolist to exclude buyers with low values. However, the optimality of exclusion relies on substantial commitment on the part of the seller. Absent such commitment, exclusion is no longer possible in that setting. For instance, if a price discriminating monopoly cannot commit to the quantity associated with a given non-linear price, then consumers will optimally choose not to buy regardless of their type. Thus, when commitment is imperfect, "the exclusion principle" is no longer a general feature of optimal contracts and so the conclusion of Proposition 3 does not follow simply as a consequence of this.

**The "Taxation Principle" under Imperfect Commitment** A common objection to direct mechanisms in the perfect commitment setting is that they are unrealistic—one never sees "direct message games" played between principals and agents to determine economic outcomes. A standard rejoinder to this criticism is the so-called "taxation principle" which points out a variety of realistic indirect mechanisms which are equivalent. For instance, a direct mechanism in a monopolistic pricing setting is equivalent to a nonlinear tariff schedule.

Since a similar objection may be leveled in our context, one may wonder how, if at all, the taxation principle operates in a setting where commitment is imperfect. We now argue that much like the extension of the revelation principle given in Proposition 1 above, a version of the taxation principle also applies with imperfect commitment, but does require some modification compared to the case of perfect commitment since we would like the indirect contract to reflect the same commitment possibilities available in the direct contract. Consider the following indirect mechanism: The principal commits to a transfer (or tariff) schedule as a function of project choices and allows the agent to select the project. A key difference compared to the case of perfect commitment is that, while the principal is bound by the transfer schedule, she is free to "overrule" the agent's choice of project, if necessary, substituting her own most preferred project after hearing the choice of the agent. That is, the principal delegates real decision making authority downstream to the agent but retains formal decision making authority for herself. To see how the taxation principle works in this setting, suppose that the principal wished to implement the full revelation outcome given in Proposition 2. The "modified" taxation principle indicates that she could do so by offering the following transfer schedule associated with the various projects

$$T^{*}(y) = \int_{\phi(y)}^{1} U_1(\gamma, \phi(\gamma), b)\, d\gamma$$

where $\phi(\gamma) \equiv y^{*-1}(\gamma)$ denotes the state in which project $\gamma$ is optimal for the principal. Notice that, even though the project choice in this contract is written in "pencil" (that is, the principal can use her formal authority to overrule the agent), she will never find it optimal to do so under this scheme.

## 3.3 Characterizing Optimal Contracts

What is the structure of optimal contracts under imperfect commitment? To obtain an exact characterization requires placing more structure on the distribution of states and the payoff functions of the actors. In this section, we offer an explicit characterization for the *uniform-quadratic* case (in which the preferences are given in (1) and (2) and the distribution of states is uniform).

We begin by establishing some structural properties of optimal contracts. Notice that, because equilibrium projects are nondecreasing in the state, the state space may be delineated into intervals of *separation*—where the agent fully reveals his private information—and intervals of *pooling*—where the agent discloses only that the state lies in some interval.

**No separation to the right of pooling** We first establish that inducing separation by fully aligning the interests of the agent with those of the principal is only cost-effective in low states. That is, once a contract calls for a pooling interval over a set of states, it never pays to induce separation for higher states. Specifically,

**Proposition 4** *The optimal contract under imperfect commitment involves separation in low states and pooling in high states.*

**Proof.** See Appendix B. ■

The property derived above implies that an optimal contract consists of separation for some set of low states, say for $\theta$ below some threshold $a$, followed by a number of pooling intervals that subdivide $[a, 1]$.

**No payment for imprecise information** In the absence of contracts and monetary compensation, the information an agent can credibly convey is coarse—as CS showed, the agent will reveal only that the state is in one of a finite number of subintervals. Contracts, however, enable the principal—at some cost, of course—to tailor incentives in a way that the agent is induced to reveal more than he would otherwise. Indeed, as we showed in Proposition 2, it is feasible to contract with the agent to induce full revelation, but, as we showed in Proposition 3, this is never cost-effective for the principal. The trade-off highlighted in these two propositions suggests that it is possible that an optimal contract would induce the agent to provide additional but still not fully precise information. Our next proposition shows that, in fact, this is never the case—the principal should never contract for *partial* revelation. The optimal contract is of the "bang-bang" variety—in low states, the principal pays the agent to fully reveal what he knows; in high states, the principal does not pay the agent at all, and, consequently, the agent reveals what he knows only imprecisely. Thus the optimal contract combines the fully revealing contract in low states with the null contract in high states. Formally,

**Proposition 5** *In an optimal contract under imperfect commitment, the principal* ***never*** *pays for imprecise information.*

**Proof.** See Appendix B. ■

Proposition 5 sheds light on an important aspect of the organizational theory literatures. That literature stresses that incentives and delegation are complements. That is, if the principal is going to effectively push decision making authority downstream, then she must provide incentives to the agent to act in a manner consistent with the organizational objectives. Of course, this is problematic in the case of imperfect commitment since the principal cannot irreversibly transfer decision making power. Thus, a key contracting question is how the principal should resolve this tension. Proposition 5 illustrates that "compromise" in the form of incentives that somewhat align the agent's preferences with those of the principal are never optimal. Depending on the realized state, the contract either aligns the interest of the agent perfectly or dispenses with monetary incentives completely.

Propositions 4 and 5 together imply that, under the optimal contract, the agent is induced to reveal up to some state $a$ and not compensated thereafter. Further, for any value of $a$, it can be shown that the number of pooling intervals, $K$, is uniquely determined—it is the no contracting outcome that maximizes the principal's expected payoffs. (For a formal statement, see Lemma 1 in Appendix B.)

Thus, the optimal contract can be completely characterized as the solution to the problem of choosing $a$ to maximize

$$EV = -\int_0^a (2b(a-\theta) + t(a))\, d\theta - \sum_{k=1}^{K} \int_{x_{k-1}}^{x_k} (y([x_{k-1}, x_k]) - \theta)^2\, d\theta$$

where $K$ is determined as in Lemma 1 in Appendix A.

Finally, we show that the interval over which separation takes place and contractual payments are made is "relatively small." In particular, the optimal contract never involves paying for information more than one-fourth of the time.

**Proposition 6** *The optimal contract under imperfect commitment involves: (i) positive payments and separation over an interval* $[0, a^*]$ *where* $a^* \leq \frac{1}{4}$*; (ii) no payments and a division of* $[a^*, 1]$ *into a number of pooling intervals.*

**Proof.** See Appendix B. ■

**Some comparative statics of optimal contracts** One may reasonably expect that as the bias of the agent increases, the interval in which payments occur, $[0, a^*]$, shrinks. This is because the costs of aligning the interests of the agent increase with his bias and it seems that the principal would economize on these costs. However, a consequence of Proposition 6 is that this simple intuition is not borne out. By examining equations (20) and (21), one may easily verify that $a^*$ may, in fact, *increase* with the bias of the agent. For instance, when the agent's bias $b = \frac{1}{8}$, the principal should contract for precise information only up to $a^* = 0.191$. In contrast, as preference divergence increases, say to $b = \frac{1}{4}$, the principal should contract for precise information more often—up to $a^* = 0.25$.

Similarly, one might expect that the total cost of the contract increases with the bias of the agent since aligning the interests of a more biased agent is clearly more costly. Another implication of Proposition 6, is that the cost of the optimal contract is not necessarily increasing in the bias of the agent. For instance, when the agent's bias increases from $\frac{1}{3}$ to $\frac{2}{5}$, the total (expected) payment under the optimal contract decreases by more than 30%.

Why is this? The intuition above misses the interaction between the length of the interval in which incentives are perfectly aligned and the amount of information attainable elsewhere. By reducing the length of the separating interval, $[0, a^*]$, the principal obtains more information via a better partition of $[a^*, 1]$. Moreover, the principal gets this information for free since, as Proposition 5 showed, it is not optimal to pay for the imprecise information obtained in $[a^*, 1]$.

Thus, the principal saves by reducing the length of the separating interval for two reasons: First, the better the information to the right of $a^*$, the less expensive is the compensation contract to the left of $a^*$ since this creates a parallel shift downward in the transfer schedule. At the same time, there is a direct cost savings since information to the right of $a^*$ is obtained for free. These savings, however, are offset by the information loss associated with reducing the length of the separating interval. The relative balance of these costs and benefits shifts, in a non-monotonic way, as a function of the agent's bias, thus accounting for the seemingly counterintuitive results described above.

# 4 Contracts with Perfect Commitment

In the face of the commitment problems implied by intervention in the choice of projects by upper management, strategic management guides often counsel that managers invest in a reputation for a consistent style of handling intervention. This investment can involve setting well-established routines before upper management can exercise authority in intervening in project choice. Alternatively, the firm can seek to develop a culture for non-intervention or highlight prescribed circumstances for intervention in its mission statement. What is the value of this commitment? How does the ability to commit not to intervene affect the structure of optimal contracts? Does it now benefit the firm to employ full alignment/full revelation contracts more extensively?

We study these issues in the context of our model. Adding the power of perfect commitment means that the principal can now commit to *both* instruments—transfers $t$ and project choices $y$. When perfect commitment is possible, the standard revelation principle applies, and it is sufficient to consider direct contracts—that is, those in which $M = [0,1]$—which satisfy incentive compatibility. A *direct* contract $(y,t)$ specifies for each message $\theta \in [0,1]$, a project $y(\theta)$ and a transfer $t(\theta)$. A direct contract $(y,t)$ is *incentive compatible* if for all $\theta$, it is best for the agent to report the state truthfully, that is, if $\sigma = \theta$ maximizes $U(y(\sigma), \theta, b) + t(\sigma)$. Standard arguments show that, under perfect commitment, necessary and sufficient conditions for incentive compatibility requires that: (i) $y(\cdot)$ is nondecreasing; and (ii) $t'(\theta) = -U_1(y(\theta), \theta, b)\, y'(\theta)$ at all points $\theta$ where $y(\cdot)$ is differentiable (see, for instance, Salanié, 1997).

One might be tempted to apply standard techniques for analyzing this class of problems; however, there are several features of the CS model that prevent the application of standard techniques. Specifically, a usual assumption about the agent's utility is that $U_2 > 0$; that is, a given project yields higher utility in higher states (see, for instance, Sappington, 1983). This guarantees that the agent's payoff in any incentive compatible contract is non-decreasing in the state the limited liability constraint (or a participation constraint) is indeed met for all $\theta$ if it is met for the lowest type. In the CS model, however, the agent's payoff is nonmonotonic—$U(y, \theta, b)$ is maximized at $y = y^*(\theta, b)$. Hence, it is not enough to ensure the limited liability constraint only for extreme types and the analysis becomes non-standard.

**Full Revelation Contracts** First, we revisit the optimality of full revelation contracts. When commitment was imperfect, we saw that these contracts were feasible but not optimal (Propositions 2 and 3). Clearly any contract that is feasible under imperfect commitment is feasible under perfect commitment. And since the full revelation contract was never optimal in the former circumstances, it is never optimal under the latter either. Thus it immediately follows that:

**Corollary 1** *Under perfect commitment, full revelation contracts are always feasible but never optimal.*

Thus, even when the principal can perfectly commit not to intervene, the best policy is still not to align the incentives of the agent and delegate decision making responsibility fully to the informed party.

**Optimal Contracts** The optimal contract is the solution to the following control problem

$$\max \int_0^1 \left(U\left(y,\theta\right) - t\right) f\left(\theta\right) d\theta$$

subject to the law of motion

$$t' = -U_1\left(y,\theta,b\right) u \tag{4}$$

and the constraints

$$\begin{aligned} y' &= u \\ t &\geq 0 \end{aligned}$$

where $y$ and $t$ are the state variables and $u$ is the control variable. Notice that local incentive compatibility constraints are captured in the law of motion, which says that either: (i) $y$ is locally strictly increasing, and in that case $y$ and $t$ are related according to (4); or (ii) $y$ and $t$ are both locally constant. That local incentive compatibility implies global incentive compatibility follows from standard arguments.

Necessary conditions that the optimal contract must satisfy can be obtained using standard methods of control theory and some salient features of the optimal contract under perfect commitment can be inferred from these. Appendix C contains the detailed analysis and shows that the optimal contract under perfect commitment involves:

1. *Compromise*. The principal (almost) *never* fully aligns the agent's objectives with her own. Instead, in states in which the principal compensates the agent, she does so in such a way that the chosen project lies between her most preferred project and that of the agent. (Lemmas 2 and 4 in Appendix B.)

2. *"Caps" on project choice.* The principal places a "cap" on the highest project that the agent can induce and does not compensate the agent if he selects this highest project. As a consequence, for high states, projects are unresponsive to the state. Put differently, the optimal contract always involves pooling in high states. (Lemma 5 in Appendix C.)

3. *No strategic "overshooting".* The principal can set incentives in such a way that in some states the agent induces a project that is higher than his most preferred action. While this is clearly undesirable for the principal in a local sense, such "strategic overshooting" could, in theory, reduce the principal's costs of obtaining more preferred projects in other states. While this kind of contract is feasible, it is never optimal.[8] (Lemma 2.)

**Comparing Optimal Contracts under Perfect and Imperfect Commitment**

What features do optimal contracts under perfect commitment share with those under imperfect commitment? How are they different? The surprising fact that the principal should never reward the agent for conveying imprecise information is a feature shared by optimal contracts under both types of commitment. With imperfect commitment, this results in project choices that jump discontinuously with the underlying state. Under perfect commitment, this feature manifests itself in the form of delegation with caps on project choice. That is, under perfect commitment, the selected project is responsive to the underlying state (up to the cap), but reflects the optimal choice from the perspective of the agent rather than the principal.

The critical respect in which contracts under the two types of commitment differ is in the role of compromise on project choice given the information of the principal. With imperfect commitment, the principal never compromises on project choices—she always chooses her optimal project given the available information. Indeed, there cannot be compromise without commitment. Under perfect commitment, the principal chooses a scheme whereby, in some regions, a compromise project is selected. That is, the project contracted for lies between principal's ideal and the agent's ideal. Compromising is valuable because it lets the principal save on transfer payments to the agent. Under imperfect commitment, the principal cannot help but use the information provided by the agent to select her own ideal project making this type of economy simply unavailable.

**"Capped" Delegation and Optimal Contracts** While our characterization concerns the direct contract, an outcome equivalent indirect contract is as follows: The principal *delegates* the decision about project choice to the agent, but restricts the agent to select from a menu of projects which is "capped"—the highest project available to the agent, $\bar{y}$ (say) is less than $y^*(1)$. Further, the principal specifies a

[8] In contrast, strategic overshooting is a feature of contracts in the class studied by Baron (2000).

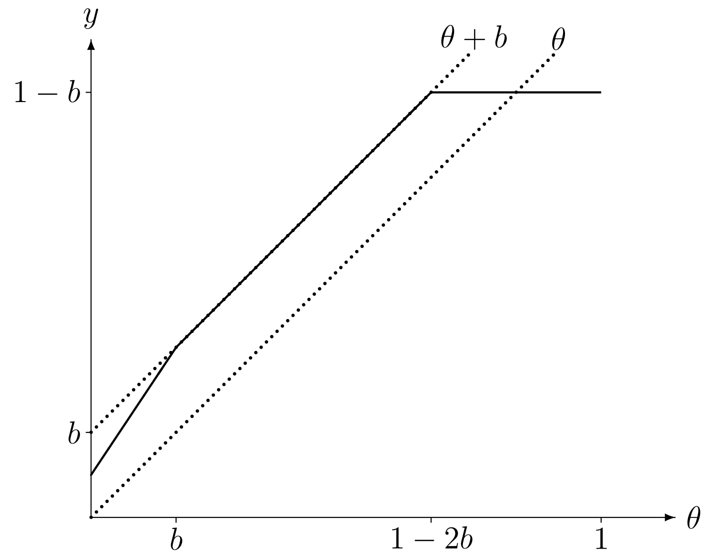


Figure 1: Optimal Contract with Perfect Commitment, $b \leq \frac{1}{3}$

compensation schedule as a function of the project chosen by the agent. This schedule entails higher levels of compensation for low projects and no compensation when $\bar{y}$ is selected. Since the principal is committed not to intervene, the agent simply chooses his most preferred project (taking into account the compensation scheme and the cap) for the given state.

Notice that the optimal delegation scheme does not entail a full alignment of objectives; nor does it entail giving the agent complete freedom in project choice.

**Uniform-Quadratic Case** We conclude this section with an explicit characterization of the optimal contract for the uniform-quadratic case under perfect commitment. The qualitative features of the contract when the bias is low differ somewhat from those when the bias is high.

When the bias is low, that is, if $b \leq \frac{1}{3}$, the optimal contract has three separate pieces (see Figure 1). In low states, that is when $\theta \leq b$, the project $y(\theta) = \frac{3}{2}\theta + \frac{1}{2}b$ lies between that optimal for the principal ($y^*(\theta) = \theta$) and that optimal for the agent ($y^*(\theta, b) = \theta + b$). As $\theta$ increases, the project chosen tilts increasingly in favor of the agent, with a commensurate decrease in the transfer payments. For states between $b$ and $1 - 2b$, the project that is best for the agent ($y^*(\theta, b) = \theta + b$) is played and no transfers are made. It is as if the project choice were delegated to the agent. The set of feasible projects is "capped" at $\bar{y} = 1 - b$. For states above $1 - 2b$, the project is unresponsive to the state—that is, the agent always chooses project $\bar{y}$ and there is, effectively, pooling over this interval.

When the bias is high, that is, $\frac{1}{3} < b \leq 1$, the optimal contract consists of only two pieces (see Figure 2). In low states, the project again lies between the project

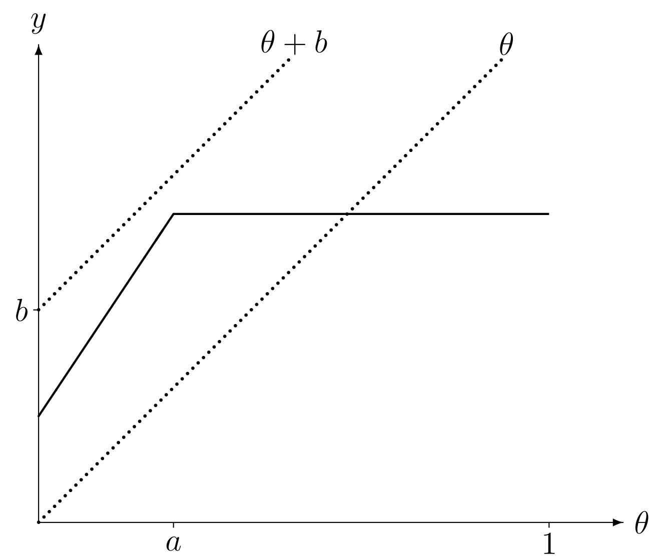


Figure 2: Optimal Contract with Perfect Commitment, $b > \frac{1}{3}$

ideal for the principal and that ideal for the agent. As in the case when the bias is low, the choice tilts in favor of the agent as the state increases with a corresponding decrease in the transfer payments. The set of feasible projects is again capped, but at a lower level. Indeed, as the agent becomes more biased, the cap decreases; that is, the agent becomes more constrained in his choice of projects. For high states, the agent always chooses the highest feasible project and there is, effectively, pooling over this interval. Unlike the case of low bias, there is no region in which the principal effectively delegates authority to the agent.

For very high biases, that is when $b > 1$, contracting is of no use—the optimal contract is no contract at all.

# 5 The Value of Contracting

It is argued (e.g., Coase, 1937, Williamson, 1975) that creating formal contracting arrangements between principals and agents is inherently costly. Further, as contracts become more complex, as in the case of perfect commitment, these contracting costs might increase. In this section, we compare the value of contracting under full and imperfect commitment with two, arguably less costly regimes: no contracting and full delegation. When contracting is costly, when is it worthwhile to write contracts? How does the value of contracting differ depending on the degree of commitment? When is full delegation worthwhile? In the uniform-quadratic case explicit answers to these questions may be obtained.

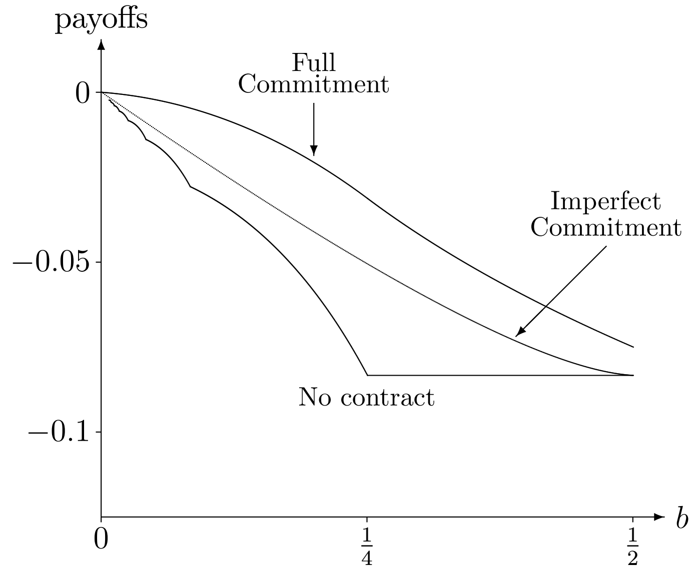


Figure 3: Value of Contracting

**Gains from Contracting** Figure 3 depicts the expected payoffs from three alternative arrangements: the optimal contract under imperfect commitment, the optimal contract under perfect commitment, and finally, no contract at all.[9] The key thing to notice about the figure is that the gains from contracting—with or without perfect commitment—are *nonmonotonic* in the degree of bias. Clearly, when the preferences of the agent and the principal are closely aligned the latter's payoff is close to her first-best level. In this case, the potential upside from contracting is quite limited. As the bias increases, the informational losses to the principal become more severe and there is more scope for contracting to "fix" the incentive problem. For cases of severe bias, $b \geq \frac{1}{4}$, absent contracts, the agent can credibly reveal no information. Resorting to contracts improves the situation, but the cost of aligning the agent's preferences increases until, at $b \geq \frac{1}{2}$, it becomes prohibitively costly for the principal. Thus, when the agent's preferences are extreme, the gains from contracting are also limited. This suggests that if there were some costs associated with "formalizing" the exchange of information between principals and agents by writing contracts, one would expect to see contracts in cases of intermediate bias, but not when incentives are relatively closely aligned nor when the agent being consulted is an extremist.

**Contracting versus Full Delegation** Strategic management texts often suggest that, for businesses faced with decentralized information, delegation (or a flat organizational structure) is the appropriate response. For example, Saloner, Shepard, and Podolny (2001, pp. 79-80) write: "One basic principle of organization design is

[9] To compute the payoffs under no contracting, we select the equilibrium maximizing the decision maker's payoffs in the CS game.

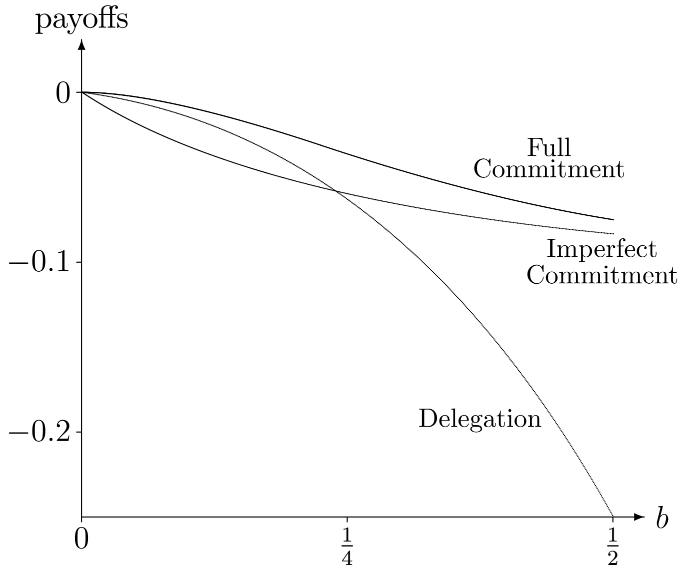


Figure 4: Contracts vs. Delegation

to assign authority to those who have information."

In the context of our model, the validity of the delegation principle may be examined by comparing *full delegation*—the unconditional assignment of authority to the person with information—to the optimal contracting arrangement with imperfect commitment. By "full" delegation we mean that the principal commits not to exercise any discretionary authority and so no longer has the freedom to intervene ex post. Specifically, there are no "caps" on what project the agent may choose. In that case, the agent will, of course, choose his favorite project $y^*(\theta, b) = \theta + b$ in each state, and the payoff of the principal is simply $-b^2$.

Figure 4 compares the principal's expected payoffs from the optimal contract under imperfect commitment with those from full delegation.[10] As the figure shows, contracting under imperfect commitment is superior to full delegation only when the bias of the agent is high, $b > 0.244$. Recall that the optimal contract lies between the principal's favorite project and that of the agent. This arises because it is more cost-effective for the principal to economize on transfers by compromising on projects. Full delegation is an extreme version of this idea—the principal pays no transfers but instead of a compromise, in effect concedes to the agent, giving him the freedom to choose his preferred project. When the preferences of the two parties are relatively closely aligned, the complete transfer of authority is more cost-effective for the principal than aligning incentives via transfers and retaining authority. As the bias increases, the transfer of authority becomes increasingly costly for the principal and transfers start to become more cost-effective.

[10] In an important paper, Dessein (2002) has shown, again for the uniform-quadratic case, that delegation is superior to no contracting when the bias of the agent is not too extreme.

The same figure also depicts the value derived from the optimal perfect commitment contract of the previous section. As is obvious, the perfect commitment contract dominates full delegation and the gains from the optimal contract increase as the bias increases. Notice that full delegation already implies that the firm is able to commit not to intervene in the project choice of the agent. As the figure shows, when the principal has such commitment power, she is better advised to impose caps on the set of feasible projects and to partially align incentives. Indeed, as the preference divergence between herself and the agent grows large, the upside from a creating a more nuanced delegation relationship (rather than the blunt instrument of full delegation) becomes considerable.

Our results suggest that appropriate organizational design needs to account for the degree of preference misalignment between a subunit with relevant information and the overall objectives of the business. But if the power to perfectly commit is available, it is *never* optimal to simply "assign authority to those who have information."

# 6 Conclusions

While the standard tool for studying optimal contracts in the case of full commitment, the revelation principle, is not valid when commitment is imperfect, we have shown that in an important class of problems, a limited version of the principle continues to hold. Using this tool, we have studied optimal contracting in environments where an agent possesses information that is important to project choice by the principal, where the principal can commit to compensation schemes but not other aspects of the contract, and where the objectives of principal and agent do not coincide.

We have shown that, despite the principal's limited commitment ability, it is always feasible for the principal to design a fully-revealing contract; however, such contracts are *never* optimal. Indeed, in the uniform-quadratic case, full revelation contracts are always worse for the principal than offering no contract at all. Instead, the optimal contract has the following property: in some states, the agent is compensated in a way that induces him to fully convey what he knows, while in other states, no payment is made and the agent conveys noisy, but still informative, messages. In other words, the optimal contract never involves any payment for imprecise information.

Finally, we studied the gains from contracting under imperfect commitment as well as perfect commitment and compared the payoffs under these schemes to the case where no contracts are possible as well as to the case where the principal simply delegates the decision to the agent. In general, gains from contracting are greeatest when the bias of the agent is moderate.

In studying optimal contracts, we have focused on the role of contracts in improving information transmission and abstracted away from their role in providing the right incentives for information acquisition. In many instances, the two problems—

information transmission and information acquisition—can be effectively decomposed and our analysis is directly relevant. In other cases the problems cannot be considered separately. It remains for future research to study how our conclusions about the nature of optimal contracts change in cases where effort incentives are also important.

# A Appendix

**Proof of Proposition 3**. We exhibit a contract that is superior to the best full revelation contract. Consider a contract $t(\cdot)$ that induces the following: the agent reveals any state $\theta \in [0, z]$ where $z < 1$ and pools thereafter. No payment is made if the reported state $m > z$. At $\theta = z$, the agent must be indifferent between reporting that the state is $z$ and reporting that it is above $z$. If we denote by $t_z$ the payment in state $z$, then we must have

$$U(y^*(z), z, b) + t_z = U(y([z, 1]), z, b) \tag{5}$$

where $y([z, 1]) = \arg\max E[U(y, \theta) \mid \theta \in [z, 1]]$ is the optimal project conditional on knowing that $\theta \in [z, 1]$. Since for $z$ close to 1, $U(y^*(z), z, b) < U(y([z, 1]), z, b)$, it follows that $t_z > 0$.

It is routine to verify that

$$\left.\frac{dt_z}{dz}\right|_{z=1} = U_1(y^*(1), 1, b)\left(\left.\frac{d}{dz}y[z, 1]\right|_{z=1} - y^{*\prime}(1)\right)$$

Incentive compatibility over the interval $[0, z]$ requires that

$$t(\theta) = t_z + \int_\theta^z U_1(y^*(\alpha), \alpha, b)\, y^{*\prime}(\alpha)\, d\alpha$$

which is again always greater than zero, so this alternative contract is also feasible.

It is useful to note that:

$$\frac{dt(\theta)}{dz} = \frac{dt_z}{dz} + U_1(y^*(z), z, b)\, y^{*\prime}(z)$$

That is, on the interval $[0, z]$, the new contract $t$ is parallel to the full revelation contract $t^*$. Indeed, for all $\theta \leq z$ we have,

$$t(\theta) - t^*(\theta) = t_z - t^*(z)$$

The expected utility of the principal resulting from the new contract is

$$V = \int_0^z (U(y^*(\theta), \theta) - t(\theta))\, f(\theta)\, d\theta + \int_z^1 U(y[z, 1], \theta)\, f(\theta)\, d\theta$$

Differentiating with respect to $z$, we obtain

$$\begin{aligned}\frac{dV}{dz} &= \left(U\left(y^*\left(z\right),z\right)-t_z\right)f\left(z\right)-U\left(y\left[z,1\right],z\right)f\left(z\right)\\&\quad-\int_0^z\left(\frac{dt\left(\theta\right)}{dz}\right)f\left(\theta\right)d\theta\\&= \left(U\left(y^*\left(z\right),z\right)-t_z\right)f\left(z\right)-U\left(y\left[z,1\right],z\right)f\left(z\right)\\&\quad-\int_0^z\left(\frac{dt_z}{dz}+U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\right)f\left(\theta\right)d\theta\end{aligned}$$

When $z=1$, we have

$$\begin{aligned}\left.\frac{dV}{dz}\right|_{z=1} &= -\left.\frac{dt_z}{dz}\right|_{z=1}-U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\\&= -\left(U_1\left(y^*\left(1\right),1,b\right)\left(\left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1}-y^{*\prime}\left(1\right)\right)\right)-U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\\&= -U_1\left(y^*\left(1\right),1,b\right)\left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1}\\&< 0\end{aligned}$$

where the inequality follows from the fact that $U_1\left(y^*\left(1\right),1,b\right)>0$ and $\frac{d}{dz}y\left[z,1\right]>0$. Thus we have shown that for $z$ close enough to 1, the alternative contract $t\left(\cdot\right)$ yields a higher expected utility for the principal than the full revelation contract $t^*\left(\cdot\right)$.

# B Appendix

This appendix contains proofs of some results pertaining to the structure of optimal contracts under imperfect commitment in the uniform-quadratic case.

**Proof of Proposition 4** Suppose there is pooling in the interval $[w,x]$ and revelation in the interval $[x,z]$. In the interval $[x,z]$ the contract must satisfy

$$t\left(\theta\right)=2b\left(z-\theta\right)+t\left(z\right) \tag{6}$$

Then the indifference condition at $x$ is

$$-\left(\frac{w+x}{2}-\left(x+b\right)\right)^2+t_{wx}=-b^2+t\left(x\right) \tag{7}$$

Notice that $t_{wx}>0$. Otherwise, at $x$, both the projects $\frac{w+x}{2}$ and $x$ are too low for the agent.

At $w$, the agent must be indifferent between some equilibrium project $y$ together with some transfer $t_y$ ,and the project $\frac{w+x}{2}$ together with the transfer $t_{wx}$. Hence, we have

$$\begin{aligned} t_y &= (y-(w+b))^2 - \left(\frac{w+x}{2}-(w+b)\right)^2 + t_{wx} \\ &= w^2+2zb+y^2-2yw-2yb+t(z) \end{aligned}$$

using (7) to substitute for $t_{wx}$. It is important to note that the transfer $t_y$ does not depend on $x$.

Hence, the principal's utility in this interval

$$\begin{aligned} EV &= \int_w^x \left(-\left(\frac{w+x}{2}-\theta\right)^2 - t_{wx}\right) d\theta - \int_x^z (2b(z-\theta)+t(z))\, d\theta \\ &= wx^2 - xw^2 + t(z)w - w^2 b - \frac{1}{3}x^3 + \frac{1}{3}w^3 + 2bzw - bz^2 - t(z)z \end{aligned}$$

Now consider a small change in $x$, keeping fixed all projects and transfers not in the interval $[w,x]$ . As noted above, this does not affect the transfer $t_y$ associated with the project $y$ to the left of $w$. Moreover, since $t_{wx} > 0$, a small change in $x$ is feasible. The change in expected utility from an increase in $x$ is:

$$\frac{dEV}{dx} = -(w-x)^2$$

and this is negative provided $x > w$. This means that no contract in which there is pooling over some nondegenerate interval $[w,x]$ followed by separation over some interval $[x,z]$ can be optimal.

The following lemma is a first step in establishing Proposition 5.

**Lemma 1** *Suppose that a contract calls for revelation on $[0,a]$ and pooling with no payment thereafter. Such a contract is feasible if and only if the no-contract equilibrium that subdivides $[a,1]$ into the maximum number of pooling intervals is played.*

**Proof.** First, suppose that with no contracts, a size $K$ partition of $[a,1]$ is possible, then the "break-points" of the partition are

$$a_j = \frac{j}{K} + \frac{K-j}{K}a - 2bj(K-j)$$

for $j = 1, 2, ..., K$.

For a size $K$ partition to be feasible ($a_1 > a$) and a size $K+1$ partition to be infeasible ($a_1 \leq a$) together requires that:

$$\frac{1-a}{2K(K+1)} \leq b < \frac{1-a}{2K(K-1)} \tag{8}$$

In state $a$, incentive compatibility implies that the agent is indifferent between the project $a$ and the project $\frac{1}{2}(a+a_1)$,

$$-b^2+t_0=-\left(\frac{a+a_1}{2}-(a+b)\right)^2$$

where $t_0$ is the transfer associated with a report $\theta=a$. Substituting for $a_1$ yields

$$t_0=\frac{1}{4}\frac{(1-a-2K(K-1)b)(2bK(K+1)-(1-a))}{K^2}$$

The condition that $t_0\geq 0$ in any feasible contract is the same as (8), the condition that there be at most $K$ partition elements in the interval $[a,1]$. ■

**Proof of Proposition 5.** Proposition 4 implies that an optimal contract must have separation over some interval $[0,z_0]$ (possibly degenerate) and then a number of pooling intervals (say $n^*$). Suppose that the total expected transfer in this contract is $B^*$. Since the contract is optimal it must also maximize the principal's expected payoffs among all contracts in which the expected expenditure is $B^*$, which one may think of as the "budget" of the principal. We will argue that every solution to a budget constrained problem—and the optimal contract must be a solution to such a problem—has the "no payment for pooling" property.

Choose $n\geq\max(n^*,N(b))$ where $N(b)$ is the maximum number of partition elements of $[0,1]$ with no transfers. Further, let the budget $B$ be arbitrary. Given a budget $B$, we want to construct the equilibrium maximizing the principal's expected utility among those that consist of revealing over the interval $[0,z_0]$ followed by at most $n$ intervals of pooling in a way that the expected transfers add up to exactly $B$. Let the revealing interval be $[0,z_0]$ and let the cut points be denoted by $z_1,z_2,...,z_{n-1}$ with payments $t_i$ over the interval $[z_{i-1},z_i]$. Payments for any $\theta$ in the revealing interval $[0,z_0]$ are $t_0+2b(z_0-\theta)$. For notational convenience, we adopt the convention that $z_n=1$.

For $i=1,2,...,n-1$, incentive compatibility on the part of the agent implies that, in state $z_i$,

$$-\left(\frac{z_i+z_{i-1}}{2}-(z_i+b)\right)^2+t_i=-\left(\frac{z_i+z_{i+1}}{2}-(z_i+b)\right)^2+t_{i+1}$$

and solving this recursively, we obtain

$$t_i=\frac{1}{4}(z_i-z_{i-1})^2-(z_i+z_{i-1})b-\frac{1}{4}(1-z_{n-1})^2+(1+z_{n-1})b+t_n \qquad (9)$$

Incentive compatibility also implies that, in state $z_0$,

$$-b^2+t_0=-\left(\frac{z_0+z_1}{2}-(z_0+b)\right)^2+t_1$$

and, using the solution for $t_1$ obtained in (9) we get

$$t_0 = -2z_0 b - \frac{1}{4}\left(1 - z_{n-1}\right)^2 + \left(1 + z_{n-1}\right) b + t_n \tag{10}$$

Given a budget $B$, the optimal contract under imperfect commitment is the solution to the following:

**Problem 1** *Choose $z_0, z_1, ..., z_{n-1}$ and $t_n$ to maximize*

$$EU = -\frac{1}{12}\sum_{i=1}^{n}\left(z_i - z_{i-1}\right)^3$$

*subject to the constraints that (i) the total expected transfers*

$$z_0\left(bz_0 + t_0\right) + \sum_{i=1}^{n} t_i\left(z_i - z_{i-1}\right) \leq B$$

*and (ii) for $i = 0, 1, ..., n-1$,*

$$t_i \geq 0$$

*where $t_i$ are given by (9) and (10).*

The Lagrangian associated with Problem 1 is

$$L = U + \lambda\left(B - z_0\left(bz_0 + t_0\right) - \sum_{i=1}^{n} t_i\left(z_i - z_{i-1}\right)\right) + \sum_{i=0}^{n-1}\mu_i t_i$$

where $\lambda$ and $\mu_i$ are multipliers. The first-order necessary conditions require that the following expressions equal zero:

$$\frac{\partial L}{\partial z_0} = \frac{1+3\lambda}{4}\left(z_1 - z_0\right)^2 - 2\mu_0 b - \frac{1}{2}\mu_1\left(z_1 - z_0 + 2b\right) \tag{11}$$

for $i = 1, 2, ..., n-2$

$$\frac{\partial L}{\partial z_i} = \frac{1+3\lambda}{4}\left(\left(z_{i+1} - z_i\right)^2 - \left(z_i - z_{i-1}\right)^2\right) + \frac{1}{2}\mu_i\left(z_i - z_{i-1} - 2b\right) - \frac{1}{2}\mu_{i+1}\left(z_{i+1} - z_i + 2b\right) \tag{12}$$

$$\begin{aligned}\frac{\partial L}{\partial z_{n-1}} &= \frac{1+3\lambda}{4}\left(\left(1 - z_{n-1}\right)^2 - \left(z_{n-1} - z_{n-2}\right)^2\right) - \frac{1}{2}\lambda\left(1 - z_{n-1} + 2b\right) \\ &\quad + \frac{1}{2}\left(1 - z_{n-1} + 2b\right)\left(\sum_{i=0}^{n-2}\mu_i\right) + \frac{1}{2}\mu_{n-1}\left(1 - z_{n-2}\right)\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial L}{\partial t_n} &= -\lambda\left(z_0\frac{\partial t_0}{\partial t_n}+\sum_{i=1}^{n}\frac{\partial t_i}{\partial t_n}\left(z_i-z_{i-1}\right)\right)+\sum_{i=0}^{n-1}\mu_i\frac{\partial t_i}{\partial t_n} \\ &= -\lambda+\sum_{i=0}^{n-1}\mu_i \qquad (14)\end{aligned}$$

Notice that the expected cost of full revelation is $b$. Thus, when the budget is large enough, that is, $B \geq b$, then full revelation is feasible and clearly solves the budget constrained problem.

For any $B < b$, we will show that a solution to the budget constrained problem is characterized as follows:

First, for any point $\theta = a$ define $K$ to be the integer satisfying

$$\frac{1-a}{2K\left(K+1\right)} \leq b < \frac{1-a}{2K\left(K-1\right)}$$

We know from CS that there is a partition equilibrium of $[a,1]$ into $K$ intervals with cut points

$$a_j = \frac{j}{K}+\frac{K-j}{K}a-2bj\left(K-j\right)$$

for $j = 0, 1, 2, ..., K$ and no transfers. Clearly since $a \leq 1$, it follows immediately that $K \leq N\left(b\right)$ and from Lemma 1, $t_0 \geq 0$..

Second, let $a$ be the solution to:

$$a\left(ba-\left(\frac{a+a_1}{2}-\left(a+b\right)\right)^2+b^2\right) = B$$

that is, $a$ is such that the entire budget is exhausted in getting the agent to reveal all states $\theta \in \left[0,a\right]$.

**Case 1:** $n = K$. It is useful to begin with the case in which $n = K$.

The solution in this case is: for $n = 0, 1, 2, ..., n-1$,

$$z_j = a_j \qquad (15)$$

where $a_0 \equiv a$. In addition,

$$t_n = 0 \qquad (16)$$

We also need to specify the values for the various multipliers. These are:

$$\lambda = -\frac{\frac{4}{3}K^2\left(K^2-1\right)b^2+\left(1-a\right)^2}{\left(2K\left(K+1\right)b-1\right)\left(2K\left(K-1\right)b-1\right)-4a+3a^2} \qquad (17)$$

which is positive.

$$\mu_0 = 0 \text{ and } \mu_1 = \frac{1+3\lambda}{2}\frac{r_1^2}{f\left(0\right)} \qquad (18)$$

and for $i = 2, ..., n-1$

$$\mu_i = \frac{(1+3\lambda)}{g(i-2)\,g(i-1)}\left(4b\sum_{j=0}^{i-2} g(j)^2 + \frac{1}{2}r_1^2 g(-1)\right) \tag{19}$$

where $r_1 = \frac{1-a}{K} - 2b(K-1)$ and $g(j) = r_1 + 4jb + 2b$.

It may be verified that the values for $z_i, t_n$ together with the multipliers $\lambda$ and $\mu_i$ solve the necessary first-order conditions for Problem 1.

**Case 2:** $n > K$. When $n > K$, a solution to the first-order conditions can be obtained by setting $z_0 = z_1 = ... = z_{n-K} = a$ and for $i = 1, 2, ..., K$, $z_{n-K+i} = a_i$. The indices of the remaining variables are also displaced by $n - K$.

This completes the argument that the solution specified in (15) to (19) satisfies the necessary first-order conditions (11) to (14) associated with Problem 1. We now show that in fact this is an optimal solution. We do this by showing that it satisfies both the necessary and sufficient conditions for an equivalent problem.

Consider the following alternative specification of the budget constrained problem in which the choice variables are the lengths of the intervals $r_i = z_i - z_{i-1}$ rather than their end points $z_i$.

**Problem 2** *Choose $z_0, r_1, ..., r_n$ and $t_n$ to maximize*

$$EU = -\frac{1}{12}\sum_{i=1}^{n} r_i^3$$

*subject to the constraints that: (i) the total expected transfers*

$$z_0(bz_0 + t_0) + \sum_{i=1}^{n} t_i r_i \leq B$$

*(ii) for $i = 0, 1, ..., n-1$,*

$$t_i \geq 0$$

*and (iii)*

$$z_0 + \sum_{i=1}^{n} r_i = 1$$

*where $t_i$ are given by (9) and (10).*

Problem 2 is the same as Problem 1 except for a change of variables. Since they share all local extrema, for every solution to the first-order conditions for Problem 1 there exists a corresponding solution to the first-order conditions for Problem 2. But in Problem 2, the objective function is concave in the choice variables and the constraints are all convex functions, the first-order conditions for Problem 2 are also sufficient. Thus any solution to the first-order conditions for Problem 1 constitutes a global optimum.

We have thus shown that the optimal solution to the budget constrained problem entails that except for $t_0$, all other $t_i = 0$. In other words, in the optimal contract, the principal never pays for pooling. This completes the proof of Proposition 5.

**Proof of Proposition 6** We claim that the optimal value of $a$ is

$$a^* = \frac{3}{4} - \frac{1}{4}\sqrt{4 + \frac{1}{3}\left(3 - 8bK\left(K-1\right)\right)\left(8bK\left(K+1\right) - 3\right)} \tag{20}$$

where $K$ is the unique integer such that

$$\frac{3}{8K\left(K+1\right)} \le b < \frac{3}{8K\left(K-1\right)} \tag{21}$$

It is routine to verify that $a^* \le \frac{1}{4}$.

First, we show that for all $b$, the payoff to the principal from choosing $a > a^*$ is worse than her payoff from choosing $a = a^*$. At $a = \frac{1}{4}$, the most informative partition has $K$ elements where $K$ is the unique integer satisfying (21). For any $a > a^*$,

$$\frac{\partial EV}{\partial a} = \frac{1}{6}\frac{8b^2K^4 - 8b^2K^2 - 6bK^2 + 3\left(1-2a\right)\left(1-a\right)}{K^2} < 0$$

using (20). This shows that all $a > a^*$ are suboptimal since for any such $a$ the most informative partition of $[a, 1]$ can have at most $K$ elements. In particular, $\frac{dU}{da} < 0$ at $a = \frac{1}{4}$.

Next, we show that for all $b$, the payoff to the principal from choosing $a < a^*$ is worse than her payoff from choosing $a = a^*$. For $a < a^*$ and fixed $K$, one may readily verify that

$$\frac{\partial EV}{\partial a} > 0$$

The only thing left to verify is that for $a < a^*$, the utility is lower than at $a^*$ even if the number of elements in the most informative partition of $[a, 1]$ is greater than $K$.

Suppose that when $a = 0$, the maximal size of the partition of $[a, 1]$ is $N$ (as in CS).

For $L = N-1, N-2, ...K+1, K$ define $a_L$ to be the smallest $a$ for which it is *not* possible to make a size $L+1$ partition. That is,

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2\left(1-a_L\right)}{b}} = L$$

The principal's expected payoff function is not differentiable at the points $a_L$ since there is a "regime change" from $L+1$ to $L$ element partitions. We can however, find the right and left derivatives of $EV$ at $a_L$ and $a_{L-1}$, respectively.

The right derivative of $EV$ at $a = a_L = 1 - 2bL(L+1)$ is,

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_L}^{+} = \frac{1}{3}8b(2L+1)(L+1)\left(b - \frac{3}{8L(L+1)}\right) \tag{22}$$

But since for all $a \in [a_L, a_{L-1})$, there does *not* exist a partition of $[a, 1]$ with $L+1$ elements and $a < \frac{1}{4}$, we have

$$b \geq \frac{(1-a)}{2L(L+1)} > \frac{3}{8L(L+1)}$$

and so (22) is positive.

Similarly, the left derivative of $U$ at $a = a_{L-1} = 1 - 2bL(L-1)$

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_{L-1}}^{-} = \frac{1}{3}8b(2L-1)(L-1)\left(b - \frac{3}{8L(L-1)}\right) \tag{23}$$

But since at $a_{L-1}$, there does not exist a partition of $[a_{L-1}, 1]$ with $L$ elements and $a_{L-1} < \frac{1}{4}$

$$b \geq \frac{(1-a_L)}{2L(L-1)} > \frac{3}{8L(L-1)}$$

and so we have that (23) is also positive.

The proof is completed by noting that when $L = K$, we have

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_K}^{+} > 0 \text{ and } \left.\frac{\partial EV}{\partial a}\right|_{a=a_{K-1}}^{-} < 0$$

# C Appendix

This appendix derives properties of the optimal contract under perfect commitment. The optimal contract is the solution to the following control problem

$$\max \int_0^1 (U(y,\theta) - t) f(\theta) d\theta$$

subject to the law of motion

$$t' = -U_1(y, \theta, b) u \tag{24}$$

and the constraints

$$\begin{aligned} y' &= u \\ t &\geq 0 \end{aligned}$$

where $y$ and $t$ are the state variables and $u$ is the control variable.

If we write the generalized Hamiltonian

$$L = (U(y,\theta) - t) f(\theta) - \lambda_1 U_1 (y,\theta,b) u + \lambda_2 u + \mu t$$

the resulting Pontryagin conditions are: there exist non-negative costate variables $\lambda_1, \lambda_2$ and a nonnegative multiplier $\mu$ that satisfy:

$$\lambda_1' = -\frac{\partial L}{\partial t} = f(\theta) - \mu \tag{25}$$

$$\lambda_2' = -\frac{\partial L}{\partial y} = -U_1(y,\theta) f(\theta) + \lambda_1 U_{11}(y,\theta,b) u \tag{26}$$

$$0 = \frac{\partial L}{\partial u} = -\lambda_1 U_1 (y,\theta,b) + \lambda_2 \tag{27}$$

$$0 = \mu t \tag{28}$$

and the transversality conditions are:

$$\lambda_1(1) = 0 \text{ and } \lambda_2(1) = 0 \tag{29}$$

**Lemma 2** *For all* $\theta \in (0,1)$, $y(\theta) \leq y^*(\theta,b)$.

**Proof.** Suppose that the contrary is true, that is, there exists a $\theta$ such that $y(\theta) > y^*(\theta,b)$. Recall that in any optimal contract

$$-\lambda_1 U_1(y,\theta,b) + \lambda_2 = 0$$

and since $\lambda_1(\theta) \geq 0$ and $\lambda_2(\theta) \geq 0$. If $\lambda_1(\theta) > 0$, then the contradiction is immediate since $U_1(y,\theta,b) < 0$. Suppose that $\lambda_1(\theta) = 0$ then $\lambda_2'(\theta) = -U_1(y,\theta) f(\theta) > 0$ and hence $\lambda_2(\theta) > 0$ and again there is a contradiction. ■

An immediate implication of the previous lemma is that the transfers are nonincreasing in the state.

**Lemma 3** $t(\cdot)$ *is nonincreasing.*

**Proof.** The law of motion (24), is

$$t' = -U_1(y,\theta,b) u$$

and from the fact that any incentive compatible $y(\cdot)$ is nondecreasing, we know that $u = y' \geq 0$. Now Lemma 2 implies that $U_1(y,\theta,b) \geq 0$ and so $t' \leq 0$. ■

**Lemma 4** *If* $t(\theta) > 0$, *then* $y^*(\theta) < y(\theta)$.

**Proof.** If $t(\theta) > 0$, then from Lemma 3, for all $\sigma < \theta$, $t(\sigma) > 0$. This means that $\mu(\sigma) = 0$ for all $\sigma \in [0, \theta]$. Now (25) implies that

$$\lambda_1(\theta) = F(\theta) + \lambda_1(0)$$

where $F$ is the cumulative distribution function associated with $f$ and from (27)

$$\lambda_2(\theta) = (F(\theta) + \lambda_1(0)) U_1(y, \theta, b)$$

and differentiating this results in

$$\lambda_2'(\theta) = f(\theta) U_1(y, \theta, b) + (F(\theta) + \lambda_1(0)) (U_{11}(y, \theta, b) u + U_{12}(y, \theta, b))$$

Equating this with the expression in (26), we get

$$U_1(y, \theta, b) + U_1(y, \theta) = -\frac{F(\theta) + \lambda_1(0)}{f(\theta)} U_{12}(y, \theta, b) < 0$$

since $U_{12} > 0$. But since $y \leq y^*(\theta, b)$ this implies that $y > y^*(\theta)$. ∎

Finally, the optimal contract must involve some pooling in high states. Thus, even though the principal has the option of full revelation, this is too expensive and never optimal.

**Lemma 5** *There exists a* $z < 1$, *such that* $y$ *is constant over* $[z, 1]$.

**Proof.** We claim that there exists a $z < 1$, such that $t(z) = 0$. If $t(\theta) > 0$ for all $\theta \in (0, 1)$, then we have that for all $\theta \in (0, 1)$, $\mu(\theta) = 0$. Now (25) together with the transversality condition implies that $\lambda_1(\theta) = F(\theta) - 1$, which is impossible since $\lambda_1(\theta) \geq 0$. ∎

**The uniform-quadratic case.** In the uniform-quadratic case, the Pontryagin conditions (25) to (28) are also sufficient since the relevant convexity conditions are satisfied (see for instance, Seierstad and Sydsæter, 1987). Some qualitative features of the solution differ depending on whether the bias $b$ is less than or exceeds $\frac{1}{3}$. These are depicted in Figures 1 and 2, respectively.[11]

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