Cartel versus Merger[*]

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Abstract

Procurement practices are affected by the nature of competition and uncertainty regarding collusion among suppliers. Buyers dissatisfied with bids of incumbent bidders can cancel their procurements and resolicit bids after qualifying additional suppliers. Recent cartel cases show that cartels devote considerable attention to avoiding such resistance from buyers. We show that in a procurement setting with the potential for buyer resistance, the payoff to a cartel exceeds that of a merged entity. This is consistent with behavior described in the landmark *Addyston Pipe* antitrust case, where firms facing buyer resistance had the option to merge but chose instead to collude.

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1   Introduction

As described by George Bittlingmayer (1985, p.77),

> "Perhaps as much as one-half of U.S. manufacturing capacity took part in mergers during the years 1898 to 1902. These mergers frequently included most of the firms in an industry and often involved firms that had been fixing prices or that had been operated jointly through the legal mechanism of an industrial trust. ... The Sherman Antitrust Act was passed in 1890, and the first crucial decisions making price fixing illegal – *Trans-Missouri* (1897), *Joint Traffic* (1898), and *Addyston* (1899) – occurred just before or during the first stages of the merger wave. Merger of competing firms remained unchallenged until 1904."

As this describes, in the late 1800s, although neither mergers nor cartels were illegal,[1] many firms chose to form a cartel rather than merge. Although cartels in this period did not need to hide their existence to avoid prosecution, they operated in a clandestine manner to disguise their presence from their customers.[2] This suggests that a key benefit of cartel formation versus merger is that a cartel can take advantage of customer beliefs that the policing action of competition is still in place.

Procurement practices commonly include an element of "buyer resistance," whereby buyers that are concerned that the policing action of competition is not adequate can resist high prices. The presence of buyer resistance is evident in cartel cases, where colluding firms faced buyer resistance that limited their ability to implement collusive price increases.[3] As shown through a review

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[1] Mergers as an effort to monopolize were not recognized as a violation of the law until the resolution of *Northern Securities v. U.S.* (197 U.S. 400) (hereafter *Northern Securities*) in 1904. The operation of a cartel was not recognized as a violation until decisions of 1897 and thereafter (Bittlingmayer, 1985, p.77).

[2] See Hylton (2003, pp.30–37).

[3] In the Vitamins Cartel, which included firms BASF, Roche, and Daiichi, "When BASF's customers resisted the increase, Roche supported the rise by also announcing an increase.... According to Daiichi, the concerted increase was unsuccessful because of customer resis-

of municipal procurements (see Appendix B.3), which are typically organized as sealed-bid competitive procurements, buyer resistance to high prices often comes in the form of buyers rejecting all bids in an initial procurement and then after some dely holding a new procurement with additional bidders present.[4]

Considering the tradeoffs between merger and cartel formation, a merged entity does not incur costs associated with disguising its existence from its customers, and a merged entity does not have to overcome the difficulties faced by cartels associated with incentives for cartel members to secretly deviate from the terms of a collusive agreement (see Stigler, 1964). Thus, in the absence of agency problems and transaction costs inherent in large firms as in Williamson (1985) or Coase (1937), one might expect a merged entity to be able to duplicate any actions that a cartel can undertake and also potentially take additional actions that a cartel cannot. However, a clandestine cartel may be able to take advantage of customer beliefs that the policing action of competition might is still in place, and thus may face reduced buyer resistance.

In this paper, we examine whether one can understand the decision by firms to form a cartel rather than merge as an equilibrium response to buyer resistance. We consider a model in which firms have an opportunity to merge, collude, or remain noncooperative and in which there is a procurement process with the possibility for buyer resistance, which we model as the ability of the buyer to reject initial bids and hold a new procurement after inviting additional

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tance...." (EC Decision in *Vitamins*, par. 325) In the Cartonboard Cartel, where colluding firms sold product to packaging manufacturers referred to as converters, "The converters have on some occasions resisted a proposed price increase for cartonboard on the ground that their own customers would in their turn refuse to accept a price increase for packaging ...." (EC Decision in *Cartonboard*, par. 19)

[4]The ability of federal procurement officials to reject all bids is formalized in the U.S. Federal Acquisition Regulations, which state: "Invitations may be cancelled and all bids rejected before award but after opening when ... (6) All otherwise acceptable bids received are at unreasonable prices, or only one bid is received and the contracting officer cannot determine the reasonableness of the bid price; (7) The bids were not independently arrived at in open competition, were collusive, or were submitted in bad faith." (U.S. Federal Acquisition Regulations, Section 14.404 Rejection of bids, https://www.acquisition. gov/Far/reissue/FARvol1ForPaperOnly.pdf)

bidders to participate.[5]

As we show, firms may find a cartel structure to be more profitable than a merger when customers are uncertain as to whether nonmerged firms are operating as a cartel or not. We show that in an environment where buyers are strategic, the expected payoff to bidders is greater if they form a cartel rather than merge.

We consider a procurement setting with a buyer, two incumbent sellers, and one potential new seller. We consider two coordination regimes, one in which sellers must compete noncooperatively and another in which sellers may form a cartel or merge. If the sellers merge, this is observed by all players. If the sellers do not merge, then the sellers observe whether a cartel has been formed, but the buyer does not and so is uncertain about the existence of a cartel. The cost environment for the sellers is either low cost or high cost. The cost environment determines the distribution from which sellers' individual costs are drawn. The cost environment is observed by the sellers but not by the buyer. Sellers' individual costs are their own private information. The buyer purchases through a competitive procurement, but the buyer retains the right to suspend the procurement and invite the potential new seller as a bidder.[6] It is costly to the buyer to do this, but it may allow the buyer to obtain a better price.

After observing the initial bids, the buyer forms beliefs about the cost state and whether there is collusion. There is a cost to the buyer of reprocurement, so if cost state were known to be high, then there is little to be gained from reprocurement and the buyer is better off accepting high initial bids. But if cost state is low, then the buyer may prefer to reject high initial bids because of the potential for obtaining a lower price through reprocurement. Firms would like to submit high bids but are disciplined by the threat that the buyer might reject the bids and qualify additional sellers in response.

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[5]In Section 2, we discuss the details of one such episode that received attention in the landmark Addyston Pipe conspiracy. For additional examples, see Appendix B.3.

[6]For other approaches to modeling buyer resistance, see Harrington and Chen (2006) and Marshall, Marx, and Raiff (2008).

We show that in this model, the two incumbent sellers are able to obtain higher profits if they form a cartel than if they merge. Relative to the case of merged firms, when nonmerged firms submit high bids, the buyer, who is uncertain about the existence of the cartel, attaches a greater probability to high bids being the result of high costs. Thus, given that the new seller only reduces the buyer's expected payment in a low-cost environment, the buyer is less likely to incur the cost to invite the new seller when a cartel (whose existence is not observable to the buyer) submits a high bid compared to when a merged entity submits a high bid. As a result, a cartel is more profitable than a merged entity.

While cartels and horizontal mergers have been widely studied in the past,[7] there is not much work that addresses these two forms of industrial organization as potential alternatives for incumbent firms.[8] An exception is Bittlingmayer (1985), which directly addresses why many firms preferred colluding over merging in the past. Building on Sharkey (1973), Bittlingmayer (1985) emphasizes the role of fixed costs in industries with a small number of firms and uncertain demand. Akin to the natural monopoly case, when demand is low the operation of, say, two small plants is more expensive than the operation of a single large plant, and coordinating production (by perhaps operating one plant below its capacity) is necessary to recover costs.[9] Bittlingmayer argues that a cartel may be a cheaper form of organization than a merger in cyclical industries, where costs can be recovered during periods of high demand and cooperation between firms is required only occasionally when demand is low.[10]

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[7]On cartels, see the survey article by Levenstein and Suslow (2006) and the references therein. On mergers, see the survey article by Mookherjee (2006) and the references therein.

[8]One could offer a Coasian (1937) explanation for the choice between a cartel and a merged entity. The trade-off between the costs of maintaining and operating a cartel versus the cost of running a large merged entity due to, say, diseconomies of scale or agency problems, is likely to influence the "merge or cartelize" decision for firms. See Nocke and White (2007) for the effects of vertical mergers on incentives to collude and Kovacic et al. (2009) for effects of horizontal mergers.

[9]For a more detailed discussion and illustrative examples, see Bittlingmayer (1982, 1985).

[10]Bittlingmayer (1985) also argues that early antitrust decisions against cartels raised the cost of maintaining cartels, which left firms with merger as the next best option and resulted in the first large-scale merger wave in the U.S. between 1898 and 1904. Stigler (1950) suggests that firms in the past might have preferred to cartelize rather than merge

There are also strategic considerations external to incumbent firms that influence their merger decisions. One such consideration is the threat of post-merger entry, which directly affects the incumbent firms' profitability. Gelman and Salop (1983) show that when an entrant can commit to serving a small enough portion of the demand, an incumbent monopolist (merged entity) may find it optimal to accommodate the entrant rather than fight it. For the monopolist to accommodate the entrant, the monopolist must expect a payoff at least equal to what it would obtain by matching the entrant's price. In the model of Gelman and Salop (1983), the entrant ensures this by committing to serve a sufficiently small portion of the market, leaving the residual demand for the monopolist.[11]

In a durable goods environment, Ausubel and Deneckere (1987) show that a cartel has the commitment power to maintain static monopoly prices while a monopolist lacks this ability. Thus, industry profits are higher when incumbent firms collude rather than merge.[12]

The remainder of this paper is organized as follows. Section 2 provides a historical overview of the *Addyston* cartel and other cartels/mergers of the same period. Section 3 presents our model. Section 4 provides our results. Section 5 considers the issue of equilibrium selection. Section 6 considers a number of extensions. Section 7 concludes.

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due to the obstacles posed by large capital requirements for mergers. Stigler argues that mergers became feasible because of the development of a sound market for securities by the New York Stock Exchange at the end of the 19th century and the removal of restrictions on the formation of large corporations after 1880.

[11]As an example, after the International Paper Company was created in 1898, several small firms entered the business. Instead of attempting to fight these smaller firms, the company let them fill their order books and charged monopoly prices on future orders. (Lamoreaux, 1985, p.129)

[12]In the same paper, Ausubel and Deneckere (1987) also show that the monopolist gains the ability to commit to maintaining future prices at the static monopoly level if there is a potential entrant at each time period.

2 Background

The *U.S. v. Addyston Pipe and Steel Co.*[13] case of 1898 is considered to be a landmark event in antitrust history.[14] In 1894, six southern manufacturers of cast iron pipes,[15] which are used to transport water and gas by cities and municipalities, entered into a conspiracy. The cartel divided the U.S. into two territories, Pay Territory and Free Territory. For every ton of pipe shipped into the Pay Territory by a member, the member made a payment, referred to as a bonus payment, into a pool. For shipments into the Free Territory, no bonus payments were necessary. The cartel "reserved" certain cities for particular cartel members, which meant that other cartel members would not meaningfully compete for any contract with the designated cartel members in those cities. At the end of every month, the bonus payments made by the members were tallied and divided among the members based on their capacities.[16]

Before a procurement, the cartel members would participate in a pre-auction knock-out and bid on the per-ton bonus payment they would make into the pool. The winner – the firm that bid the highest per-ton bonus payment – would represent the cartel in the actual procurement and bid an amount fixed by the "representative board" of the cartel.[17] The other cartel members would "protect" this bid by submitting phantom bids.[18]

After about two years of operation, suspicion about the existence of the cartel was raised when at a procurement in Atlanta, cartel members that were within a hundred miles of the city bid one to two dollars higher than a noncartel

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[13] *U.S. v. Addyston Pipe and Steel Co.,* 85 Fed. 271 (6th Cir. 1898) (hereafter *Addyston*). See also *U.S. v. Addyston Pipe & Steel Co.*, 175 U.S. 211 (1899).

[14] See Bittlingmayer (1982).

[15] The firms involved were: Addyston Pipe and Steel Company, Dennis Long & Co., Howard-Harrison Iron Company, Anniston Pipe and Foundry Company, South Pittsburgh Pipe Works, and Chattanooga Foundry and Pipe Works.

[16] Transcript of Record of the Supreme Court of the United States, October Term 1899, No. 51, *Addyston Pipe and Steel et al. vs The United States* (hereafter *Addyston* Transcript of Record), p.296.

[17] *Addyston* Transcript of Record, p.70.

[18] *Addyston* Transcript of Record, p.296.

company (R.D. Wood & Co.) that was a thousand miles away. All bids were rejected as being too high and a new procurement was held. Anniston (for whom Atlanta was reserved) then bid considerably lower than its original bid, suggesting that bids were not competitive in the first instance.[19]

An initial civil suit against the defendants in 1896 was decided in favor of the cartel, but in a landmark 1898 verdict, Howard Taft declared the cartel illegal.[20] The *Addyston* case, along with the railroad cartel cases involving the Trans-Missouri Freight Association and the Joint Traffic Association,[21] was instrumental in defining illegal collusion under Section 1 of the Sherman Act (Bittlingmayer, 1985).

Cartels were not illegal under the common law that existed before the Sherman Act,[22] although agreements among cartel members may have been deemed unenforceable if their primary function was restraint of trade.[23] The Sherman Act of 1890 made cartel agreements criminal offenses and thus a matter for public enforcement authorities.[24]

While the *Addyston*, *Trans-Missouri,* and *Joint Traffic* verdicts set precedents for *collusion* being a criminal offense under the Sherman Act, in 1904 the *Northern Securities* verdict set a precedent for *merging* to form a monopoly being a criminal offense under the Act.[25] Thus, collusion was deemed illegal under the Sherman Act before merging to monopolize was. In fact in 1895, in *U.S. v. E.C. Knight,* the Supreme Court decided in favor of the American

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[19] *Addyston* Transcript of Record, p.299.

[20] The Supreme Court upheld the decision in 1899 in the first unanimous decision in a Sherman Act case (Whitney, 1958).

[21] *U.S. v. Trans-Missouri Freight Association*, 166 U.S. 290 (1897) (hereafter *Trans-Missouri*); *U.S. v. Joint Traffic Association*, 171 U.S. 505 (1898) (hereafter *Joint Traffic*).

[22] According to Hylton (2003, p.37), "no common law action for conspiracy to restrain trade existed." Thorelli (1954, p.53) argues that "the vast majority of cases at common law were private suits between parties to restrictive arrangements." For a more detailed discussion see Torelli (1954, pp.36–53).

[23] Jones (1921, p.17), also Hylton (2003, pp.30–37).

[24] See Hylton (2003, pp. 90-104) for a detailed discussion of the Sherman Act and the common law principles.

[25] *Northern Securities v. U.S.*,197 U.S. 400, was an historic Supreme Court case under the Sherman Act involving the merger of major railroad companies, which lead to the creation of Northern Securities. In 1904, the merged entity was dissolved.

Sugar Refining Company, which was a virtual monopoly formed through the consolidation of sugar refineries.[26] Thus, there was a period between 1898 and 1904 when a large consolidation was not deemed illegal by the Supreme Court, but a cartel was.

Operating prior to 1904, in a legal environment in which there was no legal prohibition against mergers, and where cartel agreements may have been deemed unenforceable by courts, it is noteworthy that the pipe manufacturers formed a cartel rather than merging. But even more interesting is the fact that in a little more than a year after the antitrust decision against the *Addyston* cartel by the Sixth Circuit in 1898, the cartel members merged in 1899 to form the United States Cast Iron Pipe and Foundry Company (USCIP&F).[27] The firms initially chose collusion over merging, and only upon being prosecuted for collusion did they decide to merge. In fact, prior to the first wave of industrial mergers, which happened between 1898 and 1904, the chosen form of cooperation among firms in a wide range of industries seems to have been collusion rather than merger.[28]

A review of the ten largest (in net value) manufacturing industry groups according to U.S. census of 1900,[29] shows that at least eight of those ten industry groups include industries in which firms that had previously cartelized went on to merge. This review is summarized in Table B.1 in Appendix B.1.[30] Included in this list is the meat packing industry, and remarkably the cartel members agreed to merge just ten days after their cartel was disrupted by a Department of Justice investigation.[31] As an example from the "chemicals

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[26] *U.S. v. E.C. Knight*, 156 U.S. 1 (hereafter *E.C. Knight*).

[27] Whitney (1958, vol. 2, p.7). The event involved the merger of more than two firms and so might also be referred to as a consolidation.

[28] Jones (1921, p.6).

[29] U.S. Census Office (1902, p.325). The Twelfth Census classified industries into fifteen groups. The industry groups absent in our sample from the Census classifications are (i) clay, glass, and stone products, (ii) vehicles for land transportation, (iii) shipbuilding, (iv) miscellaneous industries, and (v) hand trades. Our sample includes the ten most valuable groups excluding miscellaneous industries and hand trades.

[30] The list is not exhaustive. We provide representative examples of the observed phenomenon.

[31] Whitney (1958, vol. 1, p.33).

and allied products" industry group, there was a cartel in gunpowder manufacturing called the Gunpowder Trade Association from 1872 to 1902 (by which time 95% of the industry was in the association). In 1902, Du Pont Co. took over the second-largest manufacturer, Laflin & Rand, which was also part of the association. This and subsequent mergers were consistent with the advice of Du Pont's lawyers, who cited *Addyston* as an example of collusion being perceived as illegal and cited *E.C. Knight*, where consolidation resulting in a virtual monopoly was allowed, as an example of a merger being less likely to be prosecuted (Bittlingmayer, 1985).

3   Model

There is one buyer that wishes to procure a single item by means of a first-price procurement. We assume the buyer has value greater than 1 for the item. There are three potential sellers: two incumbent sellers, which we label seller 1 and seller 2, and one new potential seller, which we label seller 3. We assume that with probability $\rho \in (0, 1)$, the cost state is low and each seller i draws its cost x_i independently from the uniform distribution on zero to one, and that with probability $1 - \rho$, the cost state is high and all sellers' costs are equal to 1. All sellers and the buyer know the distributional source of costs conditional on the low or high-cost state. Sellers observe whether they are in the low-cost or high-cost state, but the buyer does not. The buyer knows that costs are bounded above by one and so does not accept bids greater than 1.

We assume that with probability $\xi \in (0, 1)$, sellers 1 and 2 are able to form a cartel or merge if they so choose. However, with probability $1 - \xi$, communication costs or other organizational impediments prevent sellers 1 and 2 from being able to form a cartel or merge. The sellers observe whether the environment permits them to form a cartel or merge, but the buyer does not, although if the sellers choose to merge, that is observed by the buyer. We model both a merged entity and a cartel as a bidder that draws two costs and then bids to maximize its payoff based on the minimum of those two costs.

We assume that the buyer can qualify seller 3 to participate as a bidder

and reconduct the procurement at cost k to the buyer.[32]

The timing and information in the model is as follows:

Stage 0 (industry structure): The cooperation state determining the ability of the sellers to form a cartel or merge is realized and observed by the sellers but not by the buyer: cartel or merger is possible with probability ξ and not possible with probability $1 - \xi$. If the formation of a cartel or merger is possible, then sellers 1 and 2 choose between merging and acting as a cartel.[33] A decision to merge is observed by all players. A decision by sellers 1 and 2 to form a cartel is observed by the sellers, including seller 3, but not by the buyer. The state of sellers' costs is realized and observed by the sellers but not by the buyer: low with probability ρ and high with probability $1 - \rho$.

Stage 1 (initial bidding): The buyer announces a procurement and all players observe the buyer's reprocurement cost k. The sellers draw independent costs x_1 and x_2 from the cost distribution, which is determined by the cost state. A merged entity or cartel bids based on its cost being $\min\{x_1, x_2\}$. The sellers submit bids.

Stage 2 (evaluation of bids): After observing the bids, the buyer decides either to make an award to the low bidder at the amount of its Stage-1 bid or to void the initial bids and incur cost k to reconduct the procurement with seller 3 as an additional qualified bidder, in which case Stage 3 is reached.

Stage 3 (post-entry bidding): Sellers draw new costs from either the low-cost or high-cost distribution according to cost state and submit bids. The buyer makes an award to the low bidder at the amount of its Stage-3 bid.

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[32]In many industries potential suppliers have to be pre-qualified before they are allowed to participate in the procurement. Supplier qualification process is usually costly for the procurer as it typically involves verification of quality and reliability requirements, on-site visits, and verification of insurance coverages and credit-worthiness.

[33]We can also allow the firms to have the option of choosing to remain as noncooperative bidders, but in equilibrium this option is not chosen.

We use Perfect Bayesian Equilibrium (PBE) as our solution concept.[34] In analyzing the equilibria of this game, it will be useful to break it into two separate games. We define the "merger game" to be the game above but with $\xi = 1$ and where the sellers' are required to merge. We define the "cartel game" to be the game above, but where sellers are required to act as a cartel when the cooperation state allows them to do so. This allows us to analyze the tradeoff to sellers between merging and forming a cartel and so identify equilibria of the larger game. In particular, given a PBE of the merger game and a PBE of the cartel game, where the merged entity's expected payoff in the merger game is less than a cartel's expected payoff in the cartel game, then there exists a PBE of the larger game involving the same behavioral strategies and beliefs in which the firms choose to form a cartel when the cooperation state allows them to do so.

To help the reader keep track of notation, we summarize the key notation in Appendix B.2.

4   Results

To analyze the game, consider the stages in reverse order.

4.1   Stage 3: Post-entry bidding

Stage 3 is only reached if seller 3 has entered. Seller 3 knows whether it is competing against a merged entity, cartel, or two other noncooperative bidders.

In the high-cost state, each bidder has a cost of 1 and bids 1. The buyer pays 1 and all sellers have zero surplus.

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[34]If the sellers always choose to collude in the favorable cooperation state, then following the observation of a merged entity, the buyer's beliefs as to the cost state are not pinned down by Bayes' Rule. However, in a Perfect Bayesian Equilibrium, Bayes' Rule is applied even following histories that have probability zero in equilibrium and so the buyer's belief on the low-cost state conditional on observing merged firms is ρ, the prior probability of the low-cost state. See Fudenberg and Tirole (1991, p.332, condition B(ii)).

In the low-cost state, bidding is as in a standard IPV first-price procurement (with asymmetric bidders if sellers 1 and 2 merged or formed a cartel). Given our environment, this equilibrium exists and is unique.[35] Let P_{nc} be the expected winning bid in the low-cost state when bidders are noncooperative and P_m be the expected winning bid in the low-cost state when bidders 1 and 2 have merged or formed a cartel.[36] Note that $P_m > P_{nc}$. Let π_{nc} be the expected surplus to one of three symmetric noncooperative bidders in the low-cost state, and let π_m be the expected surplus to the merged entity or cartel competing against a third bidder in the low-cost state.

In what follows, to avoid uninteresting cases in which the buyer never qualifies seller 3, we assume that $k < 1 - P_m$. If k is greater than $1 - P_m$, then the buyer prefers to accept the maximum bid of 1 in Stage 1 rather than move to Stage 3, where the buyer's expected payment is at most P_m.

4.2   Stage 2: Evaluation of bids

Whether the buyer invites seller 3 to enter depends upon whether the firms merged in Stage 0, the reprocurement cost k, and the buyer's inferences from the observed bids regarding the cost state and collusion.

In the merger game, the buyer's expected payment if it rejects a bid of 1 is P_m times the buyer's posterior belief on the low-cost state plus 1 times the posterior belief on the high-cost state plus k. Thus, the buyer is indifferent between accepting and rejecting the bid of 1 if

$$\Pr\left(\text{low cost} \mid b_m = 1\right) P_m + \left(1 - \Pr\left(\text{low cost} \mid b_m = 1\right)\right) + k = 1.$$

Solving this for k, we get

$$k = \Pr\left(\text{low cost} \mid b_m = 1\right)\left(1 - P_m\right). \tag{1}$$

We let $\alpha_m(k)$ denote the probability with which the buyer accepts the

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[35]See, e.g., Athey (2001), Bajari (2001), and Lebrun (1996, 1999).
[36]The expected winning bid in Stage 3 is the same for a buyer facing a merged entity and one facing a cartel.

Stage-1 bid by a merged entity. Clearly, if $k > \Pr\left(\text{low cost} \mid b_m = 1\right)\left(1 - P_m\right)$, then the buyer's best response is to accept the bid, so $\alpha_m(k) = 1$. Similarly, if $k < \Pr\left(\text{low cost} \mid b_m = 1\right)\left(1 - P_m\right)$, then $\alpha_m(k) = 0$. Thus, it will be useful to define two threshold values for k, denoted k_m and $\bar{k}_m$, where the buyer rejects a bid of 1 if $k \leq k_m$, uses the randomized strategy of accepting a bid of 1 with probability $\alpha_m(k) \in (0,1)$ if $k \in (k_m, \bar{k}_m)$, and always accepts a bid of 1 if $k \geq \bar{k}_m$.

In the cartel game, we focus on equilibria in which noncooperative firms always bid less than 1 in the low-cost state. In this case, if the buyer observes that both bids are equal to 1, it believes it is facing either a cartel in the low-cost state or it is facing bidders in the high-cost state. The buyer is indifferent between accepting and rejecting a bid of 1 if

$$\Pr\left(\text{low cost and cartel} \mid b_1 = b_2 = 1\right) P_m$$
$$+ \left(1 - \Pr\left(\text{low cost and cartel} \mid b_1 = b_2 = 1\right)\right) + k = 1,$$

where the left side is the buyer's expected cost if it rejects the bids, and the right side is the buyer's cost if it accepts a bid of 1. Solving this for k, we get

$$k = \Pr\left(\text{low cost and cartel} \mid b_1 = b_2 = 1\right)\left(1 - P_m\right). \tag{2}$$

As with the case of a merged entity, it will be useful to define two threshold values for k, which we denote by k_c and $\bar{k}_c$, where a buyer facing non-merged firms and receiving two bids of 1 rejects both bids if $k \leq k_c$, uses the randomized strategy of accepting a randomly chosen bid with probability $\alpha_c(k) \in (0,1)$ if $k \in (k_c, \bar{k}_c)$ (and rejects both bids with the complementary probability), and always accepts a randomly chosen bid if $k \geq \bar{k}_c$.

4.3 Stage 1: Initial bidding

In the initial bidding, in the low-cost state, if the merged entity or cartel wins in Stage 1, its payoff is at most $1 - \min\left\{c_1, c_2\right\}$, and if the bid is rejected, its expected payoff is π_m. Thus, in the low-cost state, a merged entity or

cartel whose minimum cost draw is greater than $1 - \pi_m$ prefers to have its bid rejected rather than win in Stage 1 even at the maximum price of 1. We let b^* denote a bid that signals to the buyer that the cost state is low but that firms are not prepared to submit meaningful bids at this time. In what follows, we assume that b^* is a bid less than 1, consistent with the low-cost state, but greater than $P_m + k$, which is the maximum price the buyer would pay in Stage 1 conditional on believing the cost state is low (in the low-cost state the buyer prefers to reject bids above $P_m + k$ bid regardless of whether the suppliers are competing, in a cartel, or merged).[37] As other interpretations, the bid of b^* might represent a qualified bidder choosing not to submit a bid at all, or the bid of b^* could represent a bid of 1 submitted without any justification for the high bid. We discuss the extension of the model to allow price justifications by suppliers in Section 6.1.

In the high-cost state, all bids less than 1 are weakly dominated by a bid of 1. In particular, even though bids of b^* are rejected in equilibrium, in the high-cost state a bid of b^* is weakly dominated by a bid of 1 because the bidder would have negative surplus if the bid of b^* were accepted and always has zero surplus with a bid of 1. We state this as a lemma below.

Lemma 1 *In any PBE involving non-weakly-dominated bids, all bidders bid 1 in the high-cost state.*

4.3.1   Bidding by a merged entity

In Stage 1, a merged entity has the option of bidding $P_m + k$, which is accepted by the buyer in equilibrium.[38] Thus, the merged entity never bids less than $P_m + k$ in equilibrium. Bids greater than $P_m + k$ other than 1 are

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[37] To be more precise, b^* is a function of k, where $b^*(k) \in (P_m + k, 1)$, where the interval is nonempty given our assumption that $k < 1 - P_m$. For ease of notation, we omit the conditioning on k.

[38] Given that we allow continuous bidding increments, there is no equilibrium in which the buyer rejects a bid of $P_m + k$ because then the merged entity's best response would be to bid arbitrarily close to but less than $P_m + k$, and so the merged entity's equilibrium bidding strategy would not be well defined.

rejected by the buyer because the buyer infers the cost state is low (given Lemma 1), and prefers to reconduct the procurement at those prices.

Lemma 2 *In the merger game, in any PBE involving non-weakly-dominated bids, in the low-cost state the merged entity bids $P_m + k$, 1, or a bid that is rejected with probability 1.*

Thus, we consider equilibria in which the merged entity will either bid $P_m + k$, 1, or b^*, where a bid of $P_m + k$ is accepted with probability one, a bid of 1 is accepted with probability $\alpha(k)$, and a bid of b^* is rejected with probability one. We can formulate the merged entity's bidding strategy as a function of its cost, $\min\{x_1, x_2\}$, in terms of two threshold cost values, which we denote $x_m(k)$ and $\bar{x}_m(k)$, as follows:

$$
b_m(x) \equiv \begin{cases} P_m + k, & \text{if } x < x_m \\ 1, & \text{if } x_m \leq x \leq \bar{x}_m \\ b^*, & \text{otherwise,} \end{cases} \tag{3}
$$

where we suppress the conditioning of x_m and $\bar{x}_m$ on k for ease of notation.

Given this bidding strategy, the buyer's posterior belief on the low-cost state following bids of 1 is $\Pr(\text{low cost} \mid b_m = 1) = \frac{(G(\bar{x}_m) - G(x_m))\rho}{(G(\bar{x}_m) - G(x_m))\rho + 1 - \rho}$, where G denotes the cumulative distribution function for the minimum of two random variables drawn from the uniform distribution on zero to one.[39] In what follows, we show that $\bar{x}_m = 1 - \pi_m$, so it will be useful to define the belief function $\gamma_m(x) \equiv \frac{(G(1 - \pi_m) - G(x))\rho}{(G(1 - \pi_m) - G(x))\rho + 1 - \rho}$, which is the buyer's posterior belief on the state being low cost given that a merged entity with low cost bids 1 when its cost is in the interval $[x, 1 - \pi_m]$.

In the following proposition, we provide a characterization of what must be true in any equilibrium of the merger game.

Proposition 1 *If $k < \gamma_m(0)(1 - P_m)$, then in the merger game the buyer does not accept a bid of 1 with probability one in any PBE involving non-weakly-dominated bids.*

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[39]For $x \in [0, 1]$, $G(x) = 2x - x^2$.

Proof. Suppose a PBE involving non-weakly-dominated bids in which for some $k < \gamma_m(0)(1 - P_m)$, the buyer accepts a bid of 1 with probability one. By Lemma 1, the merged entity bids 1 in the high-cost state. In the low-cost state, the merged entity's best response is to bid 1 for all $x < 1 - \pi_m$ and to bid b^* or some other bid that is certain to be rejected for all $x > 1 - \pi_m$. Thus, the buyer's posterior belief on the low-cost state after observing a bid of 1 is $\gamma_m(0)$. In order for the buyer's acceptance of the bid of 1 to be a best response, it must be that $1 \leq \gamma_m(0)P_m + (1 - \gamma_m(0)) + k$, which we can write as $k \geq \gamma_m(0)(1 - P_m)$, which is a contradiction. Q.E.D.

We know from Proposition 1 that the equilibrium must involve the buyer rejecting a bid of 1 with some probability when k is sufficiently small. We can define an equilibrium by reprocurement cost thresholds k_m and $\bar{k}_m$, mixing probability α_m, and bid function b_m, which is defined in terms of cost thresholds x_m, and $\bar{x}_m$. These must satisfy the conditions that the buyer's accept/reject strategy is a best response given the observed bids and posterior beliefs γ_m on the cost state, and that the merged entity's bidding strategy is a best response given the buyer's strategy. We characterize such an equilibrium in the following proposition.

Proposition 2 *In the merger game, there exists a PBE involving non-weakly-dominated bids in which the seller uses bid function b_m given in (3), where $\bar{x}_m \equiv 1 - \pi_m$ and*

$$
x_m(k) \equiv
\begin{cases}
P_m + k - \pi_m, & \text{if } k \leq k_m \\
x \text{ s.t. } k = \gamma_m(x)(1 - P_m), & \text{if } k \in (k_m, \bar{k}_m] \\
0, & \text{otherwise,}
\end{cases}
$$

and the buyer accepts a bid less than or equal to $P_m + k$, rejects a bid strictly between $P_m + k$ and 1, and accepts a bid of 1 with probability α_m, where

$$
\alpha_m(k) \equiv
\begin{cases}
0, & \text{if } k \leq k_m \\
\frac{P_m + k - \pi_m - x_m(k)}{1 - \pi_m - x_m(k)}, & \text{if } k \in (k_m, \bar{k}_m) \\
1, & \text{otherwise,}
\end{cases}
$$

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where $\bar{k}_m \equiv \gamma_m(0)(1 - P_m)$ and k_m is implicitly defined by $k_m = \gamma_m(P_m + k_m - \pi_m)(1 - P_m)$.

Proof: Let the buyer believe the cost state is low if it observes a bid less than 1,[40] and let the buyer believe the cost state is low with probability $\gamma_m(x_m(k))$ when the bid is 1. This belief is consistent with Bayes' Rule given the merged entity's bidding strategy b_m. The definitions of x_m and $\bar{x}_m$ are such that the buyer's strategy is a best response. The definition of α_m is such that the merged entity's strategy is a best response. For a detailed statement of the requirements for equilibrium, see Appendix A. Q.E.D.

The equilibrium is depicted in Figure 1, which shows x_m, $\bar{x}_m$, and α_m as functions of k.[41] By looking up the value of k on the horizontal axis and the value of $\min\{x_1, x_2\}$ on the vertical axis, one can identify the merged entity's equilibrium bidding strategy. It bids $P_m + k$ if the point lies below the line $x_m(k)$, it bids 1 if the point lies above $x_m(k)$ and below $\bar{x}_m$, and it bids b^* if the point lies above $\bar{x}_m$.

As one can see from Figure 1, as k increases from zero to k_m, the merged entity is increasingly likely to bid $P_m + k$. That is because $P_m + k$ is, obviously, increasing in k, so the amount the buyer is willing to pay in the first round of bidding is increasing with k. As the probability that the merged entity bids 1 decreases, the buyer's inference on the probability of the low-cost state conditional on a bid of 1 falls, until at k equal to k_m, the buyer is indifferent between accepting and rejecting a bid of 1. The merged entity's bidding strategy then maintains that indifference for k between k_m and $\bar{k}_m$. As k increases,

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[40]In extensions in which b^* is not defined as a bid less than 1, then the buyer would also believe the cost state is low when it observes b^*.

[41]The equilibrium of Proposition 2 can be constructed by first defining $x_m(k)$ using the condition that the buyer is indifferent between accepting and rejecting a bid of 1, $k = \gamma_m(x_m)(1 - P_m)$. The lowest value of k such that $x_m(k) = 0$ defines $\bar{k}_m$, and $x_m(k)$ is then redefined to be zero for $k > \bar{k}_m$. If $\alpha = 0$, then $x_m(k) = P_m + k - \pi_m$, so k_m is defined by the condition that $k_m = \gamma_m(P_m + k_m - \pi_m)(1 - P_m)$, and $x_m(k)$ is redefined to be equal to $P_m + k - \pi$ for $k \leq k_m$. Finally, for $k \in (k_m, \bar{k}_m)$, α_m is defined by the condition that the merged entity is indifferent between bidding $P_m + k$ and 1 when its cost is $x_m(k)$, i.e., $P_m + k - x_m(k) = (1 - x_m(k))\alpha_m + \pi_m(1 - \alpha_m)$.
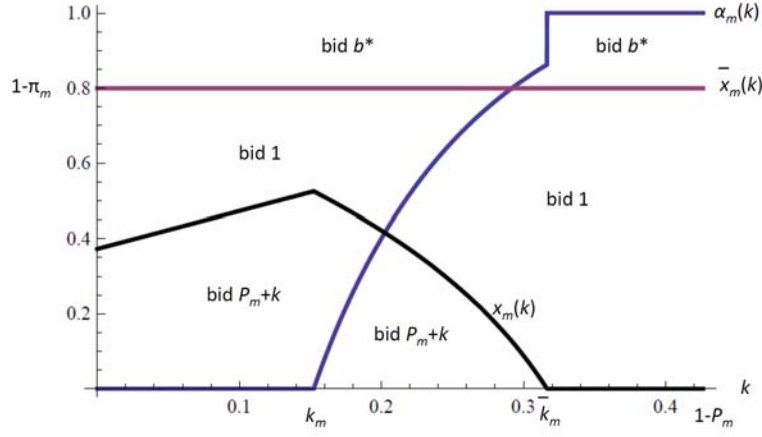
Figure 1: Graph of x_m, $\bar{x}_m$, and α_m as functions of k.

it is increasingly costly for the buyer to reject a bid of 1, so the the range of costs for which the merged entity bids 1 must increase in order to maintain the indifference. At k equal to $\bar{k}_m$, the merged entity is bidding 1 for all costs less than $1 - \pi_m$, so for k beyond that point, the buyer's incentives tip in favor of always accepting a bid of 1.

To focus on the buyer's beliefs, note that the cost to the buyer of qualifying seller 3 is k, and the expected benefit to the buyer from qualifying seller 3 when the Stage 1 bid is 1 is a decrease in price from 1 to P_m when the cost state is low, i.e., $\gamma_m(x_m(k))(1 - P_m)$. For $k \in (k_m, \bar{k}_m)$, the buyer is indifferent between accepting and rejecting a bid of 1 and so it is a best response for the buyer to randomize.

4.3.2 Bidding by nonmerged firms

Similar to the case of a merged entity, in the cartel game there exists an equilibrium in which cartel firms submit identical bids according to the

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following bid function evaluated at $\min\{x_1, x_2\}$:[42]

$$b_c(x) \equiv \begin{cases} P_c + k, & \text{if } x < x_c \\ 1, & \text{if } x_c \le x \le \bar{x}_c \\ b^*, & \text{otherwise.} \end{cases} \tag{4}$$

The analysis is analogous to the case of the merged entity, except that the relevant posterior belief following bids of 1 is $\Pr(\text{low cost and cartel} \mid b_1 = b_2 = 1)$. In what follows, we show that $\bar{x}_c = 1 - \pi_m$, and so it will be useful to define $\gamma_c(x) \equiv \frac{(G(1-\pi_m)-G(x))\rho\xi}{(G(1-\pi_m)-G(x))\rho\xi+(1-\rho)}$, which is the buyer's posterior belief on there being a cartel and the state being low cost given that it observes bids of 1 from a cartel in the low-cost state when $\min\{x_1, x_2\} \in [x, 1 - \pi_m]$ and never observes bids of 1 from noncooperative bidders in the low-cost state.

Proposition 3 *In the cartel game, there exists a PBE involving non-weakly-dominated bids in which noncooperative firms bid less than 1 in the low-cost state, a cartel uses bid function b_c given in (4), where $\bar{x}_c \equiv 1 - \pi_m$ and*

$$x_c(k) \equiv \begin{cases} P_m + k - \pi_m, & \text{if } k \le k_c \\ x \text{ s.t. } k = \gamma_c(x)(1 - P_m), & \text{if } k \in (k_c, \bar{k}_c] \\ 0, & \text{otherwise,} \end{cases}$$

and the seller accepts the lowest bid if it is less than or equal to $P_c + k$, rejects all bids if the lowest is strictly between $P_c + k$ and $P_m + k$, accepts one bid if both are equal to $P_m + k$, rejects all bids if the lowest is strictly between $P_m + k$ and 1, and accepts one bid if both are equal to 1 with probability α_c (rejecting both with probability $1 - \alpha_c$), where

$$\alpha_c(k) \equiv \begin{cases} 0, & \text{if } k \le k_c \\ \frac{P_m + k - \pi_m - x_c(k)}{1 - \pi_m - x_c(k)}, & \text{if } k \in (k_c, \bar{k}_c) \\ 1, & \text{otherwise,} \end{cases}$$

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[42]In the equilibrium we consider, there is no incentive for the cartel to use one of its bids to attempt to disguise its presence. For environments in which this may be the case, see Graham and Marshall (1987) and Graham, Marshall, and Richard (1996).

where $\bar{k}_c \equiv \gamma_c(0)(1 - P_m)$ and k_c is implicitly defined by $k_c = \gamma_c(P_m + k_c - \pi_m)(1 - P_m)$.

Proof: The proof is analogous to that of Proposition 2 with the addition that one must specify the bidding strategy of noncooperative firms and buyer beliefs regarding the existence of a cartel. As described further in Appendix A, there exists an equilibrium in which each noncooperative firm bids according to the equilibrium noncooperative bid function associated with a sealed-bid procurement with a reserve price $P_{nc} + k$ and an outside option of π_{nc}. In that case, a firm with cost zero bids a positive amount less than $P_{nc} + k$ and a firm with cost $P_{nc} + k - \pi_{nc}$ bids $P_{nc} + k$, with continuously increasing bids for intermediate costs. Noncooperative firms with costs greater than $P_{nc} + k - \pi_{nc}$ bid b^*. The equilibrium is supported by beliefs such that if the buyer observes at least one bid less than 1, then the buyer believes the cost state is low.[43] If the buyer observes at least one bid not equal to $P_m + k$, 1, or b^*, then the buyer believes it is facing noncooperative bidders with probability one. If the buyer observes both bids equal to $P_m + k$, the buyer believes it is facing a cartel with probability one. If the buyer observes that both bids are equal to 1, its beliefs are determined by Bayes' Rule and given by $\gamma_c(x_c(k))$. If the buyer observes that both bids are equal to b^*, then the buyer's beliefs are determined by Bayes' Rule, but given that $b^* > P_m + k$, regardless of the belief, the buyer's best response is to reject the bids. Additional details are contained in Appendix A. Q.E.D.

As can be seen from Propositions 2 and 3, the equilibria for the cases of merged and nonmerged firms are similar. However, the key difference is that the posterior beliefs γ_m and γ_c following the observation of bids of 1 differ. For the case of nonmerged firms, bids of 1 could be the result of high costs by a cartel, high costs by noncooperative bidders, or possibly a low-cost cartel attempting to pool with the high-cost bidders. Because the cartel has the possibility to pool with high-cost noncooperative firms as well as high-cost

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[43] In extensions in which b^* is not defined as a bid less than 1, then the buyer would also believe the cost state is low when it observes at least one bid of b^*.

cartels, the posterior belief on costs being low following the observation of bids of 1 is lower in the case of nonmerged firms than in the case of merged firms. That means that the buyer is more likely to accept bids of 1 made by nonmerged firms than a bid of 1 made by a merged firm.

4.4   Stage 0: Cartel versus merger

Consider Stage 0, during which the industry structure for the suppliers is determined. If the state is such that coordination is possible, firms 1 and 2 decide whether to merge or form a cartel. As a preliminary result, consider the four thresholds for the buyer's reprocurement cost defined in Propositions 2 and 3. In Lemma 3, we characterize the relation among these thresholds. As shown in the lemma, the relative ordering of k_m and $\bar{k}_c$ depends on the parameters ρ and ξ.

Lemma 3 *Given ρ, there exists $\bar{\xi} > 0$ such that for all $\xi < \bar{\xi}$ the reprocurement cost thresholds satisfy $0 < k_c < \bar{k}_c < k_m < \bar{k}_m < 1 - P_m$ and for $\xi > \bar{\xi}$ they satisfy $0 < k_c < k_m < \bar{k}_c < \bar{k}_m < 1 - P_m$.*

Proof. See Appendix A.

Lemma 3 shows that for ξ sufficiently close to zero, the reprocurement cost thresholds are ordered as $k_c < \bar{k}_c < k_m < \bar{k}_m$. The range of ξ for which this ordering holds depends on the parameter ρ. For example, for $\rho = 0.75$, the ordering holds for $\xi \in (0, 0.195)$.

For an illustration of the values of k_c, $\bar{k}_c$, k_m, and $\bar{k}_m$ with parameters values $\rho = 0.75$ and $\xi = 0.1$, see Figure 2.
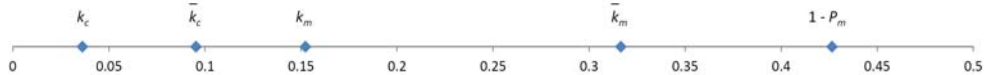

Figure 2: Values of k_c, $\bar{k}_c$, k_m, and $\bar{k}_m$ when $\rho = 0.75$ and $\xi = 0.1$.

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In the low-cost state, the merged entity can potentially bid in such a way that induces the buyer to invite seller 3 to enter, which would give the merged entity an expected payoff of π_m. Thus, we can use Proposition 1 to construct an upper bound on the merged entity's expected payoff in Stage 1 as follows:

Corollary 1 *In the merger game, in any PBE involving non-weakly-dominated bids, in the low cost state if $k < \bar{k}^m$, then the merged entity's expected payoff from the perspective of Stage 1 is less than $\max\left\{1 - \min\{x_1, x_2\}, \pi_m\right\}$.*

Despite the fact that, as shown in Proposition 1, in the low-cost state the merged entity does not win at a price of 1 when $k < \bar{k}^m$, Proposition 3 implies that there is an overlapping range of reprocurement costs for the buyer such that the cartel does win at a price of 1.

Corollary 2 *In the cartel game, there exists a PBE involving non-weakly-dominated bids in which in the low-cost state if $k \geq \bar{k}^c$, then a cartel's expected payoff from the perspective of Stage 1 is $\max\left\{1 - \min\{x_1, x_2\}, \pi_m\right\}$.*

In the low-cost state, given cost draws x_1 and x_2 in Stage 1, an expected payoff of $\max\left\{1 - \min\{x_1, x_2\}, \pi_m\right\}$ is the most that a cartel or merged entity can obtain. To obtain such a payoff requires that the cartel or merged entity wins at the maximum price of 1, except when the cartel or merged entity prefers not to win in Stage 1, but rather to have the buyer invite seller 3 so that the firms can compete in the Stage-3 procurement with expected payoff π_m. Thus, the payoff obtained by the cartel in Corollary 2 is the maximum possible.

We can now state our main results. Proposition 4 states that firms at least weakly prefer to form a cartel when the buyer's reprocurement costs are above threshold $\bar{k}_c$. The result obtains because in this range, as established in Corollary 2, there exists an equilibrium in which the cartel obtains the maximum payoff of $\max\left\{1 - \min\{x_1, x_2\}, \pi_m\right\}$, but, using Corollary 1, for a subset of the range of reprocurement costs, namely $k \in (\bar{k}_c, \bar{k}_m)$, a merged entity has strictly lower payoff in every equilibrium. In this range, the buyer rejects a bid of 1 by the merged entity with positive probability but the buyer accepts bids of 1 from nonmerged bidders with probability 1.

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Proposition 4 *There exists a PBE involving non-weakly-dominated bids of the cartel game such that for all $k \geq \bar{k}_c$, a cartel with cost draws x_1 and x_2 has weakly greater expected payoff from the perspective of Stage 1 than does a merged entity with the same cost draws in any such equilibrium of the merger game, and strictly greater for $k \in (\bar{k}_c, \bar{k}_m)$.*

Proof. The proof follows from Corollaries 1 and 2. Q.E.D.

The result of Proposition 4 is illustrated in Figure 3. It shows that the cartel can obtain the maximum payoff of $\max\{1 - \min\{x_1, x_2\}, \pi_m\}$ for all $k \geq \bar{k}_c$, while a merged entity is always held strictly below that level for $k \in (\bar{k}_c, \bar{k}_m)$.
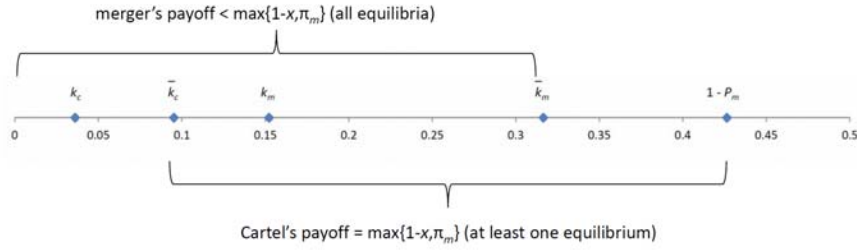

Figure 3: Illustration of Proposition 4.

We have shown that there exists an equilibrium of the merger game such that the sellers' expected joint payoff is weakly greater if they form a cartel than in any equilibrium of the merged game. We have not shown that for *all* equilibria for the nonmerged game, the firms' expected joint payoff is weakly greater if they form a cartel than in *any* equilibrium of the merged game. However, focusing on equilibria in which a merged entity or cartel bids one of three ways, $P_m + k$, 1, or a bid that induces the buyer to invite seller 3, the cartel's expected payoff is weakly greater than the merged entity's expected payoff for any possible cost draws. Proposition 5 completes the argument that for this class of equilibria the sellers are weakly better off choosing a cartel over a merger, and strictly better off for some values of k. This implies that for any prior distribution on the buyer's reprocurement costs with support $(0, 1 - P_m)$,

firms strictly prefer to collude rather than merge when the coordination state allows that.

Proposition 5 *There exist PBEs involving non-weakly-dominated bids for the merger game and cartel game such that for all k, a cartel with cost draws x_1 and x_2 has weakly greater expected payoff from the perspective of Stage 1 than does a merged entity with the same cost draws, and strictly greater for $k \in (k_c, \bar{k}_m)$.*

Proof. See Appendix A.

When the buyer accepts a bid of 1, the cartel or merged entity has expected payoff $\max\{1 - \min\{x_1, x_2\}, \pi_m\}$. When the buyer invites seller 3 when it receives a bid of 1, the cartel or merged entity has expected payoff $\max\{P_m + k - \min\{x_1, x_2\}, \pi_m\}$. When the buyer mixes, the cartel or merged entity has a payoff that is intermediate between the two. As depicted in Figure 4, when ξ is sufficiently small that the reprocurement cost thresholds are ordered as $k_c < \bar{k}_c < k_m < \bar{k}_m$ (Lemma 3), then the cartel's expected payoff is always weakly greater, and is strictly greater for $k \in (k_c, \bar{k}_m)$.
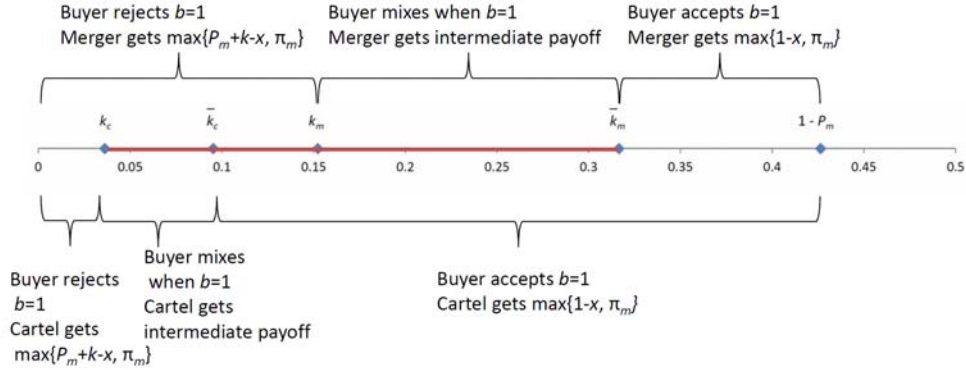

Figure 4: Illustration of Proposition 5 equilibria for sufficiently low ξ.

The result of Proposition 5 also holds when the reprocurement cost thresholds are ordered as $k_c < k_m < \bar{k}_c < \bar{k}_m$. This case is depicted in Figure 5. In this case, the intervals of mixing by the buyer overlap; however, for the

region of overlap, the expected payoff to a cartel is greater than the payoff to a merged entity. This occurs because in that region, the probability with which the buyer accepts a bid of 1 is greater in the cartel game than in the merger game. Intuitively, a buyer facing nonmerged firms places some probability weight on bids of 1 coming from noncooperative firms in the high-cost state, in which case the buyer prefers to accept one of the bids.
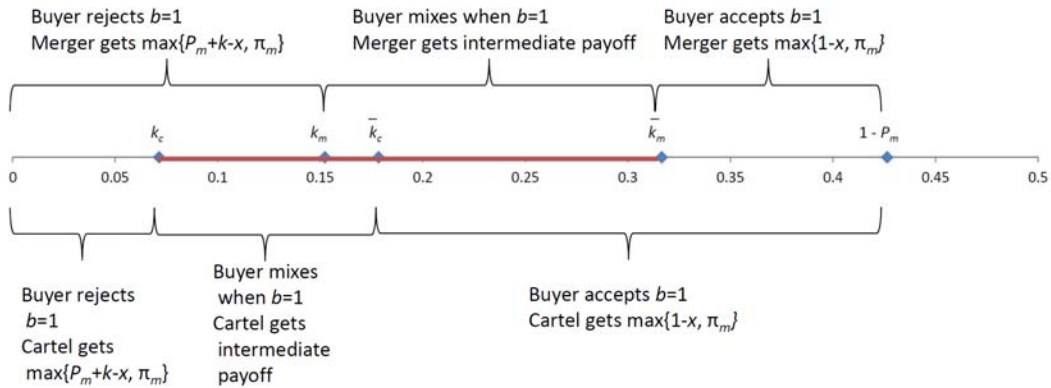

Figure 5: Illustration of Proposition 5 equilibria for sufficiently high ξ.

As we have demonstrated above, a cartel is better able to exploit the buyer's uncertainty about the state to successfully submit high bids when in the low-cost state. Additional uncertainty about the existence of a cartel leads the buyer to be more lenient in terms of accepting higher prices relative to when it faces a merged entity. Stated differently, a merged entity faces greater buyer resistance than firms operating as a cartel when the buyer is uncertain as to whether the firms are in a cartel or acting noncooperatively.

5 Equilibrium selection

In this section, we address the issue of multiplicity of equilibria. We show that under certain restrictions, the payoff to a cartel in any equilibrium of the cartel game is greater than or equal to the payoff to a merged entity in any equilibrium of the merger game.

5.1 Best equilibrium of the merger game

It follows from Lemma 2 that in the merger game, given a PBE involving non-weakly-dominated bids, when the buyer's acceptance function is α, a merged entity with minimum cost draw x has expected payoff

$$\max \left\{ P_m + k - x, \ \alpha(1-x) + (1-\alpha)\pi_m, \ \pi_m \right\},$$

which is nondecreasing in α. Thus, an equilibrium that involves a greater probability of buyer acceptance offers at least weakly greater expected payoff to the merged entity. Because of this, we can identify the "best" equilibrium of the merger game for the merged entity, i.e., the equilibrium that gives the merged entity the greatest expected payoff, by identifying the equilibrium with the greatest probability of buyer acceptance. Similarly, using Lemma 4, we can identify the "worst" equilibrium for the cartel in the cartel game. Then we show that for ξ sufficiently small, the worst cartel equilibrium is better for the sellers than the best merger equilibrium.

To prove this, we first establish that in the merger game the buyer's acceptance probability cannot be higher than α_m (obviously true for $k \geq \bar{k}_m$, where $\alpha_m = 1$). Second, we define a new equilibrium of the cartel game with acceptance probability $\hat{\alpha}_c$ and show that in the cartel game the acceptance probability cannot be less than $\hat{\alpha}_c$. Third, we complete the proof by showing that for ξ sufficiently small $\alpha_m \leq \hat{\alpha}_c$ (with a strict inequality for some values of k).

In Proposition 6, we show that in the merger game there is no equilibrium that has a greater acceptance probability for any k than the equilibrium we identified above. Intuitively, if the buyer accepted with greater probability, the merged entity would bid 1 for a greater range of costs, which would increase the buyer's posterior belief on the low-cost state following a bid of 1, which would cause the buyer to want to accept with lower probability, which provides the contradiction.

Proposition 6 *In the merger game, the equilibrium with acceptance probability α_m provides the merged entity with the greatest expected payoff among all*

PBE involving non-weakly-dominated bids.

Proof. To prove the result, we show that in the merger game, in any PBE involving non-weakly-dominated bids, given k, the probability that the buyer accepts a bid of 1 is less than or equal to $\alpha_m(k)$. See Appendix A.

5.2 Worst equilibrium of the cartel game

Recall that Lemma 2 shows that a merged entity will bid only $P_m + k$, 1, or a bid that is rejected with probability one in equilibrium. A similar result to Lemma 2 is available for the cartel game, except one must specify that the buyer believes identical bids (other than b^*) come from a cartel. In the absence of this assumption, there is an equilibrium in which the cartel bids $P_{nc} + k$ (rather than $P_m + k$), 1, or b^*, supported by the beliefs that any bids greater than $P_{nc} + k$ and less than 1, including $P_m + k$, come from noncooperative bidders, making it a best reply for the buyer to reject those bids.

Definition 1 *An equilibrium of the cartel game has* suspicious *beliefs if the buyer believes different bids come from noncooperative bidders and identical bids less than or equal to $P_m + k$ come from a cartel.*

With this restriction on beliefs, the cartel game also has the feature that the cartel bids only $P_m + k$ or 1 or bids that are rejected with probability 1.

Lemma 4 *In the cartel game, in any PBE involving non-weakly-dominated bids and suspicious beliefs, in the low-cost state the cartel submits identical bids equal to $P_m + k$ or 1 or submits bids that are rejected with probability 1.*

Proof. See Appendix A.

In order to identify the "worst" equilibrium for the cartel game, by which we mean the equilibrium offering the lowest expected payoff to a cartel, we must look for the equilibrium in which the buyer has the lowest probability of accepting a bid of 1.

We define a new equilibrium for the cartel game that differs from the one previously identified in that we allow a cartel with certain costs greater than $1 - \pi_m$ to bid 1 rather than b^*. This increases the buyer's posterior belief on the low-cost state following a bid of 1 and shifts the buyer's response towards rejecting bids of 1. By doing this to the maximal extent possible, i.e., by having the cartel bid 1 when its cost is greater than $1 - \pi_m$ whenever bids of 1 are rejected with probability 1, we can construct the worst equilibrium for the cartel.

Let $\hat{k}_c$ be defined by

$$\hat{k}_c = \frac{\left(1 - G(P_m + \hat{k}_c - \pi_m)\right)\xi\rho}{\left(1 - G(P_m + \hat{k}_c - \pi_m)\right)\xi\rho + 1 - \rho}(1 - P_m).$$

Note that $\hat{k}_c \in (\underline{k}_c, \bar{k}_c)$. We now define the acceptance probability $\hat{\alpha}$ by

$$\hat{\alpha}(k) \equiv \begin{cases} 0, & \text{if } k \leq \hat{k}_c \\ \in (0,1), & \text{if } k \in (\hat{k}_c, \bar{k}_c) \\ 1, & \text{if } k \geq \bar{k}_c, \end{cases}$$

where for $k \in (\hat{k}_c, \bar{k}_c)$, we let $\hat{\alpha}(k)$ be defined by

$$P_m + k - x_c(k) = (1 - x_c(k))\hat{\alpha}_m + \pi_m(1 - \hat{\alpha}_m),$$

where, recall, $x_c(k)$ is defined by $k = \gamma_c(x_c)(1 - P_m)$.

Proposition 7 *In the cartel game, the equilibrium with acceptance probability $\hat{\alpha}_c$ provides the cartel with the least expected payoff among all PBE involving non-weakly-dominated bids, noncooperative bids less than 1 in the low-cost state, and suspicious beliefs.*

Proof. To prove the result, we show that in the cartel game, in any equilibrium satisfying the conditions of the proposition, for all k, the probability that the buyer accepts a bid of 1 is greater than or equal to $\hat{\alpha}_c(k)$. See Appendix A.

5.3 Best merger versus worst cartel equilibrium

Using Propositions 6 and 7, we can show that as long as the probability of the cooperation state that allows collusion is sufficiently small, the sellers prefer to collude rather than merger regardless of the equilibrium selection. Intuitively, it must be sufficiently likely that sellers are noncooperative in order for the sellers to benefit from pooling with them regardless of the equilibrium selection.

Proposition 8 *Focusing on PBE involving non-weakly-dominated bids, non-cooperative bids less than 1 in the low-cost state, and suspicious beliefs, for all $\rho \in (0,1)$, there exists $\bar{\xi} > 0$ such that $\forall \xi < \bar{\xi}$, for all k, the best equilibrium outcome for the seller in the merger game gives weakly lower expected payoff than than the worst equilibrium outcome for the cartel in the cartel game, with strictly lower expected payoff for $k \in (\hat{k}_c, k_m)$.*

Proof. For ξ sufficiently small, by Lemma 3, we can order the reprocurement cost thresholds as $\hat{k}_c < \bar{k}_c < k_m < \bar{k}_m$. For $k \leq k_m$, $\hat{\alpha}_c(k) \geq \alpha_m(k) = 0$ and for $k \geq \bar{k}_c$, $\hat{\alpha}_c(k) = 1 \geq \alpha_m(k)$. Thus, for all k, $\hat{\alpha}_c(k) \geq \alpha_m(k)$, with a strict inequality for $k \in (\hat{k}_c, k_m)$. The result then follows from Propositions 6 and 7. Q.E.D.

Based on Proposition 8, we conclude that even when one considers the issue of multiplicity of equilibria, under natural conditions the expected payoff from forming a cartel exceeds the expected payoff from merging in an environment such as ours with buyer resistance.

6 Extensions

6.1 Justifications by bidders

There is evidence that cartel firms coordinate justifications for price increases.[44] The bidding strategies in the game described above can be extended to allow bids to be accompanied by justifications or not. Then there exists an equilibrium in which competitive firms never offer justifications in the low-cost state but always offer justifications in the high-cost state. When the bidding strategies described above indicate a bid of 1 by a cartel or merged entity, the cartel or merged entity would also submit artificial justifications. When any of the bidding strategies indicate a bid of b^*, the bidder would submit a bid 1 but not offer a justification. The buyer's beliefs are similar to those described above, but with a bid of 1 with no accompanying justification being treated as a bid of b^*. In this model, in order for a cartel to pool with high-cost competitive firms, it must manufacture justifications whenever it chooses to bid 1 in the low-cost state. Thus, the manufacture of justifications by cartels arises as an equilibrium phenomenon.

6.2 Entry cost for seller 3

In our model, seller 3 is assumed to enter even in the high-cost state when its expected surplus from entry is zero. Our results are not affected if we assume that seller 3 must pay a small positive cost to become an eligible bidder. Such a cost might reflect the cost of quality certifications or of making changes to the production process to ensure compatibility with the buyer's

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[44]The EC Decision in *Electrical and Mechanical Carbon and Graphite Products* (Case C.38.359—Electrical and Mechanical Carbon and Graphite Products, Comm'n Decision, Dec. 3, 2003, at ¶108) describes the cartel's manufactured justifications: "With regard to justifications for price increases, a local meeting in the Netherlands on 19 December 1995 came up with the following agreed explanations to 'justify' an impending price increase: 'Explanation for 4% price increase 1. Environmental requirements cost extra. 2. Increase [in price] of raw materials 3. Wages [increased by] 3%.'' The EC Decision in *Cartonboard* (IV/C/33.83—Cartonboard, Comm'n Decision, Jul 13, 1994, at ¶19) states: "Producers of cartonboard have usually attempted to justify a proposed price increase to their customers by reference to increases in the costs of raw material, energy, transport, etc."

requirements. If we assume a positive cost to seller 3 less than π_{nc}, which is incurred before seller 3 learns its cost x_3, then seller 3 always accepts the buyer's invitation in the low-cost state, but rejects the invitation in the high-cost state. In the high-cost state, we can either view the buyer as purchasing from either seller 1 or seller 2 at their Stage-1 bid of 1, or we can assume the buyer reconducts the procurement with just bidders 1 and 2. In that case, since the cost state is high, the bids will once again be equal to 1, giving the same result as in our model.

6.3 Cost draws that carry over from Stage 1

In our model, we assume that bidders draw costs for Stage 1 and then draw new costs for Stage 3 if that stage is reached. One could also assume that sellers 1 and 2 only draw costs once and that those costs carry over to Stage 3 if that stage is reached. In such a model, it seems clear that merged and colluding sellers would still have an incentive to bid 1 in the low-cost state in some cases, with a cartel doing so for a greater range of cost draws than a merged firm, so our main result would continue to hold. However, in the low-cost state, the bidding strategies in Stage 3 and in Stage 1 for bids less than 1 would be more complicated. Seller 3 would make inferences from Stage-1 bids as to the costs of its rivals in the Stage-3 procurement and would bid more aggressively if the inference was that its rivals had lower costs. Given this, bidders in Stage-1 would have an incentive to adjust their bids to affect seller 3's inference. The buyer's incentive to reject Stage-1 bids would depend on how the Stage-3 bidding would unfold should the buyer qualify seller 3 and reconduct the procurement. In constructing the model as we did, it was our assessment that our results were better illuminated by sidestepping this additional complexity.

6.4 Ordering of information

If the cost state is realized prior to the sellers' choice of industry organization, then the sellers are indifferent between merging, colluding, and remaining

independent in the high-cost state. If we suppose some small cost to the sellers to merge or collude, then firms in the high-cost state would prefer to remain independent. In that case, upon observing nonmerged firms, the buyer would believe it is either facing a cartel in the low-cost state or noncooperative firms. The buyer's posterior belief on the low-cost state following a bid of 1 would be the same as described above, so the equilibrium of the continuation game would be unchanged.

6.5 Immediate qualification of seller 3

In our model, it is not a choice for the buyer to immediately qualify seller 3; however, that option can be introduced. In one possible alternative extensive form, if the buyer immediately qualifies seller 3, then it holds a single auction, buying at the lowest bid. In this extensive form, the buyer at least weakly prefers to consider bid from sellers 1 and 2 before potentially qualifying seller 3.

In another alternative extensive form, if the buyer qualifies seller 3 immediately at cost k, then it continues to have the option to reject the bids in the first procurement and reconduct the procurement at no additional cost. In that case, the buyer is able to, in effect, sample six cost draws rather than only five as in our model, which may give the buyer an incentive to immediately qualify seller 3. Nevertheless, in this version of the model, one would expect the same forces to favor cartel over merger, although the ability of either to successfully bid 1 in the low-cost state would be dampened by the presence of seller 3 in the first-stage procurement.

6.6 Cartel detection

As mentioned above, in our model a cartel has no incentive to try to disguise its presence other than using bids that mimic bids in the high-cost state. In the case of nonmerged firms, bids of $P_m + k$ in Stage 1 allow the inference of collusion. In addition, Stage 3 bids that are less than 1 when the Stage 1 bids were equal to 1 also allow the inference of collusion. If a cartel faced penalties

from detection, either from legal enforcement or from lost future profits due to increased buyer resistance in the future (for example, the equilibrium might revert to that associated with a merged entity), then that would affect cartel behavior.

Suppose that such penalties only apply if the observed behavior allows the inference of collusion with probability one. Then cartel firms would have an incentive to randomize over a region below $P_{nc} + k$ rather than bid $P_m + k$, and low-cost cartel firms would bid 1 in Stage 1 for a smaller range of costs. In addition, low-cost cartel firms would never bid 1 when bids of 1 are always rejected. Nevertheless, an equilibrium similar to the one we constructed continues to exist, although it requires an adjustment to the noncooperative bidding strategy.

In the equilibrium we construct for nonmerged firms, when $k < k_c$, noncooperative firms bid b^* when they prefer that their bid be rejected. However, since bids of 1 are also rejected when $k < k_c$, it would also be a best reply for them to bid 1. If cartel firms do not bid 1 when $k < k_c$, then in order to maintain the buyer's incentive to reject bids of 1, low-cost noncooperative firms must bid 1 for some costs. (To see this, note that if low-cost bidders never bid 1, then the buyer's inference from bids of 1 would be that costs are high, and so the buyer would prefer to accept one of the bids.) With the required adjustment to the noncooperative bidding strategy for low reprocurement costs, we retain the result that, as long as penalties for collusion are not too severe, low-cost cartels will sometimes bid 1, pooling with high-cost bidders.

7 Conclusion

It might seem that a merged entity should be able to do anything that a cartel can do, plus more, and so should earn higher profits than a cartel. But in the late 1800s, when firms were relatively unencumbered in the choice between merging or forming a cartel, many chose to function as a cartel. For a more recent example, a steel cartel involving 17 prestressing steel producers operated a global price-fixing and market-sharing cartel between January 1984

and September 2002.[45] In 2002, DWK Saarstahl revealed the existence of the cartel under the EU Leniency Programme introduced that year. The cartel included Mittal Steel and Arcelor, the first and second-largest steel producers in the world, but in 2006, Mittal and Arcelor merged. Thus, although we gloss over many complexities, it appears Mittal and Arcelor chose collusion when a merger was possible.

Whereas a merger is a publicly observed event, a cartel is a clandestine operation (even back in the late 1800s). Other noncartel firms in an industry may be aware of the existence of a cartel, but the buyers that procure from colluding firms are usually uncertain of the existence of the cartel. In a model that parallels buyer procurement practices as well as the informational environment that confronts procurement participants, we show that a cartel can hide behind the possibility that their members might be noncooperative bidders to enhance their profits relative to a merged entity.

In our model, the buyer can invoke additional competitive pressure by inviting a new firm to bid in a reconducted procurement. In practice, reserving the right to void a procurement and resolicit bids is commonplace (see Appendix B.3).

Overall, our analysis highlights the importance of accounting for strategic action by buyers during the procurement process. In practice, buyers are not passive but, rather, actively evaluate the competitive process during a procurement and make profit-enhancing adjustments to increase the policing function of competition as deemed appropriate.

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[45]Statements in this paragraph rely on: Case No COMP/M.4137 - Mittal / Arcelor, Article 6(2) Non-Opposition, 02/06/2006, http://ec.europa.eu/competition/mergers/cases/decisions/ m4137_20060602_20212_en.pdf; and "Corrected Antitrust: Commission fines prestressing steel producers € 458 million for two-decades long price-fixing and market-sharing cartel," Reference: IP/10/1297, 06/10/2010, http://europa.eu/rapid/pressReleasesAction.do?reference= IP/10/1297&format=HTML&aged=0&language =EN&guiLanguage=en.

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A   Appendix: Proofs

Continuation of Proof of Proposition 2*:* First consider the buyer's accept/reject strategy. For $k \leq k_m$, $\alpha_m(k) = 0$, for $k \geq \bar{k}_m$, $\alpha_m(k) = 1$, and for $k \in (k_m, \bar{k}_m)$, the buyer strictly mixes between accepting and rejecting bids of 1 and so must be indifferent. Using (1), this requires:

$$\forall k \in (k_m, \bar{k}_m), \ \ k = \frac{(G(\bar{x}_m(k)) - G(x_m(k)))\rho}{(G(\bar{x}_m(k)) - G(x_m(k)))\rho + (1 - \rho)}(1 - P_m), \quad \text{(A.1)}$$

with the left side being weakly less for $k \leq k_m$ and weakly more for $k \geq \bar{k}_m$. The optimality of the merged entity's bidding strategy requires that

$$\forall x \ \leq \ x_m(k), \ P_m + k - x \geq \max\{\pi_m, \ (1 - x)\alpha_m(k) + \pi_m(1 - \alpha_m(k))\}$$
$$\forall x \ \in \ (x_m(k), \bar{x}_m(k)], \ (1 - x)\alpha_m(k) + \pi_m(1 - \alpha_m(k)) \geq \max\{\pi_m, \ P_m + k - x\}$$
$$\forall x \ > \ \bar{x}_m(k), \ \pi_m \geq \max\{P_m + k - x, \ (1 - x)\alpha_m(k) + \pi_m(1 - \alpha_m(k))\}.$$

One can show that these conditions are satisfied at the values defined in Proposition 2. In addition, one can confirm that $0 \leq x_m \leq \bar{x}_m \leq 1$ and $\alpha \in [0, 1]$.

Q.E.D.

Continuation of the proof of Proposition 3*:* The bid function for a noncooperative bidder with cost x is given by

$$\beta^{nc}(x,k) \equiv \begin{cases} \tilde{\beta}(x), & \text{if } x \leq P_{nc} + k - \pi_{nc} \\ b^*, & \text{otherwise,} \end{cases}$$

where $\tilde{\beta}(x) \equiv \frac{1}{2(1-x)} \left(2(P_{nc} + k)(1 - x_{nc}(k)) + (x_{nc}(k)^2 - x^2) \right)$, which satisfies $\tilde{\beta}(P_{nc} + k - \pi_{nc}) = P_{nc} + k$. This bid function has the feature that a noncooperative bidder with cost $P_{nc} + k - \pi_{nc}$ is indifferent between bidding $\tilde{\beta}(P_{nc} + k - \pi_{nc})$ and winning with probability $1 - F(P_{nc} + k - \pi_{nc})$ and bidding b^* and receiving an expected payoff of π_{nc} in the next stage with probability $1 - F(P_{nc} + k - \pi_{nc})$. Noncooperative bidders with lower costs strictly prefer to bid according to $\tilde{\beta}$ rather than bid b^*, and noncooperative bidders with higher costs strictly prefer to bid b^*. Given the buyer's beliefs, and given that the other noncooperative firm either bids less than or equal to $P_{nc} + k$ or bids b^*, a deviant bid by a noncooperative firm (i.e., greater than $P_{nc} + k$ and not equal to b^*) does not change the buyer's inference that costs are low and bidders are noncooperative. Thus, such deviant bids are not profitable. The remainder of the proof is analogous to that of Proposition 2. Q.E.D.

Proof of Lemma 3. It is clear that $0 < \bar{k}_c, \bar{k}_m < 1 - P_m$. To see that $0 < k_m < 1 - P_m$, note that k_m is defined by $k_m = \gamma_m (P_m + k_m - \pi_m)(1 - P_m)$, where the left side is continuously increasing in k_m as k_m ranges from zero to $1 - P_m$ and the right side is continuously decreasing in k_m as k_m ranges from zero to $1 - P_m$. At k_m equal to zero, the right side is positive. At k_m equal to $1 - P_m$, the right side is equal to zero. Thus, k_m is well defined and $k_m \in (0, 1 - P_m)$. A similar argument holds for k_c. The result that for ξ sufficiently close to zero, $\bar{k}_c < k_m$, follows from $\lim_{\xi \to 0} \bar{k}_c = 0$ and $k_m > 0$. To see that $k_m < \bar{k}_m$ for $\rho \in (0, 1)$, note that when $\rho = 0$, $k_m = \bar{k}_m = 0$, and when $\rho = 1$, $k_m = \bar{k}_m = 1 - P_m$. A numerical evaluation shows that for $\rho \in (0, 1)$, $k_m < \bar{k}_m$. Similar evaluation

shows $k_c < \bar{k}_c$. The inequalities $k_c < k_m$ and $\bar{k}_c < \bar{k}_m$ follow from $\xi \in (0,1)$.
Q.E.D.

Proof of Lemma 4. First, if the buyer observes identical bids less than $P_m + k$, then by the assumption of suspicious beliefs the buyer believes the bids come from the cartel in the low-cost state (using Lemma 1) and so the buyer accepts one of the bids. Because the buyer accepts all identical bids less than $P_m + k$, no bids less than $P_m + k$ are a best response by the cartel because the cartel would have greater expected payoff by submitting identical bids for a slightly higher amount that remains less than $P_m + k$. Thus, in equilibrium the buyer accepts identical bids of $P_m + k$. Second, bids greater than $P_m + k$ and less than 1 are rejected with probability one because, given Lemma 1, the buyer infers that the cost-state is low and so strictly prefers to reject the bids. It follows that in equilibrium the cartel submits identical bids equal to $P_m + k$, identical bids equal to 1, or bids such that both bids are rejected with probability 1.
Q.E.D.

Proof of Proposition 6. For $k \geq \bar{k}_m$, the result follows from $\alpha_m(k) = 1$. Let $k < \bar{k}_m$. Suppose an equilibrium acceptance probability $\alpha > \alpha_m(k)$. One can show that $k < \bar{k}_m$ implies $\alpha < 1$.[46] Using Lemma 2, the merged entity bids 1 if $x \in (\frac{P_m + k - \alpha}{1 - \alpha} - \pi_m, 1 - \pi_m)$, but not if $x < \frac{P_m + k - \alpha}{1 - \alpha} - \pi_m$ or if $x > 1 - \pi_m$. Letting γ be the buyer's posterior belief on the low-cost state following a bid of 1 and using Lemma 1, it follows that

$$
\begin{aligned}
\gamma(1 - P_m) &= \frac{\left(G(1 - \pi_m) - G\left(\frac{P_m + k - \alpha}{1 - \alpha} - \pi_m\right)\right)\rho}{\left(G(1 - \pi_m) - G\left(\frac{P_m + k - \alpha}{1 - \alpha} - \pi_m\right)\right)\rho + 1 - \rho}(1 - P_m) \\
&= \gamma_m\left(\frac{P_m + k - \alpha}{1 - \alpha} - \pi_m\right)(1 - P_m) \\
&> \gamma_m\left(x_m(k)\right)(1 - P_m) \\
&\geq k,
\end{aligned}
$$

[46]If $\alpha = 1$, then the merged entity's best response is to bid 1 for all $x < 1 - \pi_m$ and not for $x > 1 - \pi_m$, which implies a posterior belief of $\gamma_m(0)$. But $\gamma_m(0)(1 - P_m) = \bar{k}_m > k$, which implies the buyer strictly prefers to reject a bid of 1, contradicting $\alpha = 1$.

where the first equality uses Bayes' Rule, the second equality uses the definition of γ_m, the inequality uses the definition of $\alpha_m(k)$, which implies $x_m(k) = \frac{P_m+k-\alpha_m(k)}{1-\alpha_m(k)} - \pi_m$ and $\alpha > \alpha_m(k)$, and the final equality uses for $k \in (k_m, \bar{k}_m)$ the definition of $x_m(k)$, which implies $\gamma_m(x_m(k))(1-P_m) = k$, and for $k \leq k_m$ uses $\alpha_m(k) = 0$, which implies $\gamma_m(x_m(k))(1-P_m) \geq k$. The implied inequality $\gamma(1-P_m) > k$ implies the buyer strictly prefers to reject a price of 1, contradicting $\alpha > 0$. Q.E.D.

Proof of Proposition 7. For $k \leq \hat{k}_c$, the result follows from $\hat{\alpha}_c(k) = 0$. Let $k > \hat{k}_c$. Suppose an equilibrium acceptance probability $\alpha < \hat{\alpha}_c(k)$, which implies $\alpha < 1$. Using Lemma 4, the cartel bids 1 if $x \in \left(\frac{P_m+k-\alpha}{1-\alpha} - \pi_m, 1-\pi_m\right)$, but not if $x < \frac{P_m+k-\alpha}{1-\alpha} - \pi_m$ or if $x > 1-\pi_m$. Letting γ be the buyer's posterior belief on the low-cost state following a bid of 1 and using Lemma 1 and the assumption of noncooperative bids less than 1 in the low-cost state, it follows that

$$
\begin{aligned}
\gamma(1-P_m) &= \frac{\left(G(1-\pi_m) - G\left(\frac{P_m+k-\alpha}{1-\alpha} - \pi_m\right)\right)\xi\rho}{\left(G(1-\pi_m) - G\left(\frac{P_m+k-\alpha}{1-\alpha} - \pi_m\right)\right)\xi\rho + 1 - \rho}(1-P_m) \\
&= \gamma_c\left(\frac{P_m+k-\alpha}{1-\alpha} - \pi_m\right)(1-P_m) \\
&< \gamma_c(x_c(k))(1-P_m) \\
&\leq k,
\end{aligned}
$$

where the first equality uses Bayes' Rule, the second equality uses the definition of γ_c, the inequality uses the definition of $\hat{\alpha}_c(k)$, which implies $x_c(k) = \frac{P_m+k-\hat{\alpha}_c(k)}{1-\hat{\alpha}_c(k)} - \pi_m$ and $\alpha < \hat{\alpha}_c(k)$, and the final equality uses for $k \in (\hat{k}_c, \bar{k}_c)$ the definition of $x_c(k)$, which implies $\gamma_c(x_c(k))(1-P_m) = k$, and for $k \geq \bar{k}_c$ uses $\hat{\alpha}_c(k) = 1$, which implies $\gamma_c(x_c(k))(1-P_m) \leq k$. The implied inequality $\gamma(1-P_m) < k$ implies the buyer strictly prefers to accept a price of 1, contradicting $\alpha < 1$. Q.E.D.

B Appendix For Online Publication

B.1 Appendix: Historical evidence of collusion followed by merger

The information provided in Table B.1 is not exhaustive. We provide representative examples of the observed phenomenon.

Table B.1: Evidence of the pattern of collusion followed by merger

Census industry group	Industry with cartel followed by merger	Merger year	References for existence of cartel and merger year
Food and kindred products	Meat packing	1903	Whitney (1958, vol. 1, pp.31,34)
	Sugar refining	1887	Genesove and Mullin (1998, p.358)
	Corn refining	1897	Whitney (1958, vol. 2, p.258)
Textiles	Cordage	1887	Thorelli (1954, p.78)
	Cotton yarn	1899	Dewing (1914, pp.307-308)
Iron and steel and their products	Wire nails	1898	Lamoreaux (1985, pp.69-74), Jones (1921, p.194)
	Tin cans	1901	Whitney (1958, vol. 2, p.197)
	Tin plates	1898	Lamoreaux (1985, pp.14-15,115)
Paper and printing	Newsprint	*	Whitney (1958, vol. 1, pp.334-335)
	Strawboard	1889	Weeks (1916, pp.305-306)
	Wallpaper	1898	Whitney (1958, vol. 1, p.356)
Chemicals and allied products	Gun powder	1902	Whitney (1958, vol. 1, p.192)
	Cottonseed oil	1889	Thorelli (1954, p.79)
Metals and metal products, other than iron and steel	Farm machinery	1902	Jones (1912, p.232)
Liquors and beverages	Whiskey	1891	Ripley (1916, pp.27,31)
Leather and its finished products	Sole leather (tanning)	1893	Dewing (1914, p.18)
Lumber and its remanufactures	**		
Tobacco	***		

* Some cartel members merged with the Union Bag and Paper Co. The date is uncertain.

** In the lumber industry it was common for manufacturers to participate in price fixing associations. In at least one case the association subsequently attempted to merge, but decided against it due to legal barriers (U.S. Department of Commerce, 1914, pp.256, 274).

*** The five largest tobacco product manufacturers merged in 1890. They merged after considering and deciding against forming a cartel (Porter, 1969).

B.2 Appendix: Notation

Table B.1: Summary of notation

Notation	Explanation
ρ	Probability of the low-cost state
ξ	Probability that formation of cartel or merger is possible (with prob. $1 - \xi$ bidding is necessarily noncooperative)
G	Distribution of the minimum of two random variables drawn from the uniform distribution on $[0, 1]$
P_m, P_{nc}	Stage 3 expected winning bid in the low-cost state when sellers 1 and 2 have merged or formed a cartel or when bidding is noncooperative
π_m, π_{nc}	Stage 3 expected surplus to a merged entity/cartel or noncooperative seller in the low-cost state
k	Cost to the buyer to invite seller 3 to bid in Stage 3 in the interval $(0, 1 - P_m)$
$k_m, \bar{k}_m, k_c, \bar{k}_c$	Threshold values for k
$x_m, \bar{x}_m, x_c, \bar{x}_c$	Threshold values for seller costs used in defining bid functions
b^*	An arbitrary bid in the interval $(P_m + k, 1)$ rejected by the buyer

B.3 Appendix: Bid rejections and reprocurement in practice

In order to seek the best value when acquiring products or services, firms typically use competitive procurements. Governments, whether local, state or federal, are typically required by law to use competitive procurements. In order to participate in a procurement, a seller must either be directly invited by the buyer or satisfy a qualification process to be included in the bidding. For example, a seller with inadequate financial resources to ensure completion of a contract, or one that has performed poorly in the past, may be excluded

from participation in a current procurement. In addition, a potential bidder that does not expend resources to qualify and that is unknown to the buyer may be excluded. For any typical competitive procurement, it is common for there to exist potential suppliers that are either not invited to bid or that do not seek qualification as a bidder.

Almost all procurement rules allow for the buyer, after receipt of all bids, to make no award and void the procurement. During the course of a procurement, a buyer may observe actions by the bidders, including their actual bids, that cause the buyer to believe that they are not obtaining the best value. In that case, a buyer may undertake some incremental action to invigorate the policing action of the competitive process and reconduct the procurement with this new competitive pressure in place. One such action is to invite and seek qualification of sellers that did not participate in the initial round of bidding. If one or more new sellers can be identified, then the procurement may be reopened and new bids solicited.

Overall, a common sequence for procurements in private industry and the public sector is as follows.

1. **Initial bidding**. Invite qualified sellers to participate and obtain initial bids.
2. **Evaluation**. If the initial bids are "reasonable," then make an award. If the bids provide the buyer with less surplus than expected, then consider voiding the initial procurement.
3. **Possible additional bidding**. If the initial procurement was voided, consider seeking additional competitive pressure, conducting a new procurement, and making an award based on the new bidding.

These common procurement practices guide our modeling framework.

In what follows, we provide a review of public procurements conducted by U.S. cities and towns, which generated the observations above. As background, in these procurements the bid specifications typically indicate that the city has the right to award the contract to the lowest responsive bidder, or to reject any and all bids.

In Table B.3.1, we summarize twenty recent examples of procurements in which all initial bids were rejected by the relevant government decision maker because the lowest responsive bid was unacceptably high for the buyer.[47]

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[47]The right to reject all bids can be exercised by government purchasing authorities for other reasons as well, e.g., bids are found to be non-responsive, bid documents are defective and/or incomplete, or there is evidence of inadequate competition.

Table B.1: Bid rejections and reprocurement[48]

City	Project	Industry	Number of Bidders	Date	Reason for Rejection
Belmont	Overhaul and upgrade Sewer and Pump Station pumps, holding tanks, and consultants	Construction / Renovation	4	01.09.07	Not sufficient funding in project budget to award to low bidder
Belmont-2	Sanitary Sewer Rehabilitation Ralston Avenue Pipe Bursting and Pipelining	Construction / Renovation	2	09.14.04	Two received bids exceed the anticipates costs. The City will redesign and re-advertise the project
Clinton	Install water and sewer infrastructure for Sampson Square Apartments	Construction	3	02.16.10	Lowest bid greater than grant funding
Des Moines	Golf Course Repairs – damaged from erosion and slope failure	Construction	2	10.11.10	Lowest bid was 53% over project estimate and exceeded project budget
Folsom	Revitalization Project	Construction	2	07.20.09	Low bid exceeded engineer's estimate
Fresno	Delivery of Ortho Poly Phosphate Blend to the Surface Water Treatment Facility	Ortho Poly Phosphate Blend Delivery	1	05.01.07	Want to obtain greater bidder participation and lower pricing
Fresno-2	Landscaping around City Hall and Santa Fe Depot	Landscaping	4	10.02.07	There is a reasonable expectation that additional bids will be received through a future rebid, thereby, reducing the cost of this item
Lacey	Construct a treatment facility and booster station at reservoir site	Construction	5	05.24.07	Low bidder withdrew because of data errors and next apparent low bidder's value higher than engineer's estimate
Missoula	Stripping and stockpiling topsoil, and large rocks, rough grading, earth moving, landscape contouring and removal of excess granular materials	Construction	2	6.3.09	Both bids were above the anticipated budget for this project
Piedmont	Build children's play area	Construction	3	07.19.04	Large discrepancy between architect's estimate for the base bid work versus the low bid

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[48]We refer to the procurements by the name of the city. The full citations are provided at the end of this appendix.

City	Project	Industry	Number of Bidders	Date	Reason for Rejection
Pinole	Information Network Technology Support Services	IT Support	2	06.15.10	Both responses were for more than double the budgeted amount
Plant City	Furnishing and Installing a 12,000 Gallon Diesel Tank	Fueling	13	8.24.09	Lowest bid was above City's budget for project
San Rafael	Tennis and Basketball Court Renovation	Construction	4	08.02.10	Lowest bid exceeded Engineer's Estimate
Shasta Lake	Build Native American Cultural Resource Center	Construction	7	09.08.10	Low bid exceeds available funding
Silver City	Re-roof library and replace HVAC units in library	Construction/Roofing	4	11.10.09	Town issued bid up to $185,000 from fund but all bids exceeded this amount
Suisun City	Landscaping along Bikeway	Landscaping	7	09.07.10	Lowest bid exceeded engineer's estimate
Tracy	Fire Department wants to purchase Triple Combination Fire Pumper	Fire Apparatus Manufacturers	6	08.05.08	The low bid with tax was higher than the authorized budgeted amount
Villa Park	Mesa Drive Widening & Guard Rail Project	Construction	9	12.16.08	The lowest qualified bid was approximately 44% higher than the engineer's estimate of the project.
Woodinville	Build bridge	Construction	2	06.13.05	The lowest bid exceeded engineer's estimate by approximately 30%
Woodinville-2	Install Fire Detection and Alarm System at City Hall Annex Building	Maintenance	2	07.02.01	The lowest bid was higher than the project funding.

In the cases we reviewed, it is common for the buyer (the city) to have comprehensive cost estimates of the project before soliciting bids. However, usually no formal reserve price is announced prior to bidding. It can happen that all received bids are beyond initial cost estimates or the cost limits established by the purchasing authorities. When the lowest received bid substantially exceeds the cost estimates or limits, the city councils may void the initial bids and announce reprocurement.

For example, in September 2006, the City Council of Belmont procured a contract for pump station rehabilitation. The contract was to be awarded to the lowest responsible bidder for an amount up to the engineer's estimate of $520,000. Four general contractors submitted bids as follows: $695,000,

$724,000, $787,000 and $859,000. After evaluation, the city council rejected all bids and re-advertised the project in Spring 2007.[49]

Bids may be rejected with the expectation of lower future bids. For example, Fresno's reason for rejecting the bid it received in March 2007 was that: "There is a reasonable expectation that additional bids will be received through a future rebid, thereby, reducing the cost of this item."[50] Lacey identified the possibility of seeking more competitive bids as a key reason for rebidding its contract.[51]

In many of the examples listed in Table B.1, all bids were rejected because they were above what buyer believed to be a reasonable level. For example, Piedmont received three bids for its project, but there was a large discrepancy between the architect's cost estimate for the project and the lowest bid. According to the staff report, "the difference between the base bid architect's estimate and base bids actually received is obviously disappointing and troubling."[52] The city council rejected all bids, re-worked the project specifications, and re-conducted the procurement. Folsom rejected all bids because "the lowest responsive bid was received from McGuire and Hester for $3,737,259.80 and was $1.55 million over the engineers estimate."[53] San Rafael rejected all bids because "the lowest bid of $161,232.50 is $36,232.50 more than the Engineer's Estimate."[54] Villa Park rejected all bids due to the high cost of the lowest bid, which was above the engineer's estimate.[55] Woodinville rejected all bids because "the low bid amount for this project exceeded the engineer's estimate by approximately 30%."[56]

In other examples, the stated reason for rejection includes the low bid being above the approved budget for the project.[57]

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[49]Belmont, pp.1–2.

[50]Fresno, p.4.

[51]Lacey, paragraph 5.

[52]Piedmont, p.1.

[53]Folsom, p.3.

[54]San Rafael, p.1.

[55]Villa Park, p.1.

[56]Woodinville, p.1.

[57]See, e.g., Clinton, Des Moines, Missoula, Pinole, Plant City, Shasta Lake, Tracy, and Woodinville-2.

To summarize, a review of procurement examples reveals the following phenomena: 1. When the buyer is uncertain about the cost environment, it can infer information from the observed bids. 2. If the initial bids are viewed as reasonable, then the buyer makes an award to the lowest bidder. 3. If the initial bids are viewed as too high, the buyer may void the initial procurement and seek additional bidders to participate in a new procurement. 4. Budget-constrained buyers may reject bids even if there is no expectation of obtaining more favorable bids through reprocurement.

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