Measuring the Efficiency of an FCC Spectrum Auction

Patrick Bajari and Jeremy T. Fox*
University of Minnesota and NBER
University of Chicago

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Abstract

FCC spectrum auctions sell licenses to provide mobile phone service in designated geographic territories. We empirically measure efficiency in the C block auction, where the continental US was split into 480 licenses, which differs from a common European system of awarding nation-wide licenses. Spectrum auctions can be inefficient because of demand reduction and intimidatory collusion. Unfortunately, there is no one standard model of spectrum auctions. In the spirit of Haile and Tamer (2003), we structurally estimate bidder valuations using a necessary condition for equilibrium behavior known as pairwise stability. Pairwise stability holds across a set of spectrum auction models, including models with intimidatory collusion and demand reduction. Using our valuation estimates, we find that the allocation of licenses in the C block was inefficient compared to awarding four large regional licenses to the largest bidders.

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1 Introduction

The US Federal Communications Commission (FCC) auctions licenses of radio spectrum for mobile phone service. Based on the recommendations of academic economists, the FCC employs an innovative simultaneous ascending auction. We study data from the 1995–1996 auction of licenses for the C Block of the 1900 MHz PCS spectrum band. The C block divided the continental United States into 480 small, geographically distinct licenses. A mobile phone carrier that holds two geographically adjacent licenses can offer mobile phone users a greater contiguous coverage area. One intent of auctioning small licenses is to allow bidders rather the FCC to decide where geographic complementarities lie. Bidders can potentially assemble packages of licenses that maximize the benefits from geographic complementarities. The US practice of dividing the country into small geographic territories differs markedly from European practice, where often nationwide licenses are issued. These nationwide licenses ensure that the same provider will operate in all markets, so that all geographic complementarities are realized.

Economic theory suggests the allocation of licenses in a simultaneous ascending auction need not be efficient. Brusco and Lopomo (2002) demonstrate that bidders may implicitly collude through the threat of bidding wars. For example, a bidder might not add an additional license to a package to take advantage of complementarities because of threats of higher, retaliatory bids on the bidder's other licenses. Ausubel and Cramton (2002) demonstrate that bidders may find it profitable to unilaterally reduce demand for licenses, similar to how a monopolist raises prices and profits by reducing supply. Descriptive empirical evidence supports the concern about intimidatory collusion and demand reduction in FCC spectrum auctions. Cramton and Schwartz (2000, 2002) show that bidders in the AB block did not aggressively compete for licenses and in the later DEF block auction used the last digits of numeric bids to signal rivals not to bid on other licenses.

This paper provides the first structural model of bidding in the FCC spectrum auctions. There is no generally accepted theoretical model that is capable of fully capturing the complexities of a spectrum auction. As a result, we follow the spirit of Haile and Tamer (2003) and work with necessary conditions for equilibrium that will hold across a variety of models that have been proposed in the literature. Our estimator is based on the assumption that the allocation of licenses is pairwise stable, that is, the exchange of two licenses by winning bidders must not raise the sum of the valuations of the two bidders. Pairwise stability is consistent with the collusive equilibria found in Brusco and Lopomo (2002) and under demand reduction as in Ausubel and Cramton (2002).

In our econometric model, bidder valuations are a parametric function of license characteristics, bidder characteristics, and bidder private information. We build a loss function similar to the maximum score estimator of Manski (1975), where the objective function is the number of inequalities that satisfy pairwise stability. We provide sufficient conditions for the consistency of our estimates and discuss the identification of our model under weak functional form assumptions. We estimate the influence of various bidder and license characteristics on bidder valuations. Finally, we measure the efficiency of the observed allocation of licenses and discuss the implications of our estimates for alternative auction designs.

Our research contributes to the literature on spectrum auctions and the empirical analysis of multiple unit auctions in several ways. First, we estimate a structural model of bidding in spectrum auctions. The existing empirical literature on FCC spectrum auctions is primarily descriptive. McAfee and McMillan (1996) provide an early analysis of the AB auction results. Cramton and Schwartz (2000, 2002) report evidence of attempts at coordination through bid signalling. Ausubel, Cramton, McAfee and McMillan (1997) and Moreton and Spiller (1998) present bid regressions showing evidence for complementarities. The structural approach is useful because it formally links the econometric model to economic theory. A structural approach requires the econometrician to formally state the identifying behavioral assumptions that we use to make inferences concerning bidder valuations and the efficiency of the allocation of licenses. These identifying assumptions are not formally stated in the existing reduced form literature.

Second, our estimator contributes to the literature on the structural estimation of multiple unit auctions. Relatively few papers structurally estimate auctions with multiple units (especially multiple heterogeneous units) with the notable exceptions of Hortacsu (2002), Cantillon and Pesendorfer (2003), Fevrier, Préget and Visser (2003), Wolak (2004), Chapman, McAdams and Paarsch (2006), and Kastl (2006). Most previous estimators specify a particular model of equilibrium behavior and invert a bidder's first order condition to recover its valuation. See Donald and Paarsch (1993) and Donald and Paarsch (1996), Elyakime, Laffont, Loisel and Vuong (1994), Guerre, Perrigne and Vuong (2000), Athey and Levin (2001), Campo (2002), Flambard and Perrigne (2002), Hendricks, Pinkse and Porter (2003), Bajari and Ye (2003), Jofre-Bonet and Pesendorfer (2003), Athey, Levin and Seira (2004) and Krasnokutskaya (2004).[1] This first order approach is not possible in our application because there is no generally accepted model of bidding in simultaneous ascending auctions that can capture the complexity of the game. Our approach exploits necessary conditions based on pairwise stability that hold across a range of models. Therefore we avoid a potential specification error by committing to a particular equilibrium as the data generating process. Also, our estimator can be applied to other multiple unit auctions that satisfy pairwise stability.

Third, previous methods for structural estimation in auctions identify bidder valuations from final bids submitted in the auction. Theorists such as Crawford and Knoer (1981), Kelso and Crawford (1982), Leonard (1983), Demange, Gale and Sotomayor (1986), Hatfield and Milgrom (2005), Day and Milgrom (2007), and Edelman, Ostrovsky and Schwarz (2007), among others, have pointed out that a one-to-many two-sided matching game is a generalization of an auction of multiple heterogeneous items. We are the first to use this insight in empirical work. We infer valuations, up to a normalization, based on the match between bidder characteristics and license characteristics. We do not use bid data in our preferred estimator. We argue that the link between bids and bidder valuations may be polluted

[1] See Athey and Haile (2008) and Paarsch and Hong (2006) for surveys of some of this material.

if demand reduction, intimidation or bidder collusion are present, as is likely in our application. We demonstrate that a closely related estimator that uses bid data does not yield reasonable estimates of bidder valuations, consistent with the potential biases mentioned above.

Fourth, our estimator allows for unobserved heterogeneity about the item for sale. Most previous empirical work in auctions assumes that the econometrician observes all of the item characteristics publicly observed by the bidders, which is unlikely to hold in many applications. Our specification, like Berry, Levinsohn and Pakes (1995), allows for a license specific unobserved attribute to enter as an additively separable, vertical characteristic in bidders' structural profit functions. We demonstrate that such license specific attributes difference out in the inequalities implied by pairwise stability. To the best of our knowledge, the only other paper that formally accounts for unobserved heterogeneity is Krasnokutskaya (2004), who studies first price asymmetric auctions. She demonstrates that failing to account for unobserved heterogeneity severely biases estimates of bidder valuations.

Fifth, the effective size of the choice set for bidders in our application is very large. In our application, there are 480 licenses and, as a result, there are more potential packages of licenses than there are atoms in the universe. Any estimator that relies on a direct comparison of the discrete choice between all potential packages will be computationally infeasible. Our estimator, based on pairwise stability, circumvents this computational difficulty by considering a set of necessary conditions for equilibrium behavior.

Finally, this is the first paper to empirically estimate a two-sided, non-search matching game with transferable utility, except for Dagsvik (2000), Choo and Siow (2006), and Weiss (2007), who work with logit-based specifications applied mostly to marriage. We report results, proved in a companion econometrics paper (Fox, 2007), on nonparametric identification that do not rely on the particular functional form assumptions behind the logit model. We then discuss how these identification results matter for our counterfactual analysis.[2] As a bidder can win more than one license, we are the first paper to estimate a many-to-one matching game where the payoffs of bidders are not additively separable across licenses (unlike, say, Sørensen (2007)).

In our results section, we find mixed evidence concerning the efficiency of the observed allocation of the licenses. At least since Coase (1959), the use of airwave auctions has been justified on efficiency grounds. We find that bidders strongly value complementarities between licenses and that bidders with larger initial eligibilities value licenses more. Also, we find that awarding each license to a distinct bidder would dramatically reduce efficiency, justifying spectrum auctions as efficiency enhancing over the prior lotteries regime. However, we find evidence that the observed packages of licenses were too small for an efficient allocation given the presence of complementarities between licenses. Consistent with this finding, many of the bidders in our sample failed to pay for the licenses that they won or were acquired by larger firms.

[2]There are more differences behind the economic / error term properties of the two models than just the logit. Fox (2007, Section 2) argues that the logit models do not enforce that each item in an auction is won by only one bidder for every realization of the error terms.

Our results have implications for future spectrum auctions. In 2006, the United States government auctioned the Advanced Wireless Services (AWS) spectrum block and more auctions of spectrum freed from analog television broadcasting are forthcoming. Our findings suggest that small license territories, together with the possibilities of demand reduction and implicit collusion, can generate an inefficient allocation of licenses.

2 Background for the C block auction

2.1 FCC spectrum auctions for mobile phones

Wireless phones transmit on the publicly owned radio spectrum. In order to prevent interference from multiple radio transmissions on the same frequency, the Federal Communications Commission (FCC) issues spectrum users licenses to transmit on specified frequencies. Wireless phones in the United States transmit on two major regions of radio spectrum. The FCC assigned 800 MHz licenses in the 1980's using comparative worth regulatory hearings, lotteries, and induced partnerships among applicants. In the 1990's, Congress and the Clinton administration decided the mobile phone industry could support more competitors, and so the FCC allocated additional spectrum in the 1900 MHz (PCS) block to mobile phone carriers. The FCC assigned the new PCS spectrum licenses using auctions instead of using lotteries.

There were three initial auctions of mobile phone spectrum between 1995 and 1997. The first auction (the AB blocks) sold 99 licenses for 30 MHz of spectrum for 51 large geographic regions and raised $7.0 billion for the US Treasury. The second auction (the C block) sold 493 30 MHz licenses in more narrowly defined geographic regions to smaller bidders that met certain eligibility criteria. The C block auction closed with winning bids totaling $10.1 billion, although some bidders were unable to make payments, and their licenses were later re-auctioned. The third auction (the DEF blocks) sold three licenses for 10 MHz in each of the same 493 markets as the C block. The bids totaled $2.5 billion in the DEF blocks.

There are a number of reasons to prefer to use data from the C block auction instead of the AB or DEF blocks. First, the number of observations is much larger in the C block: there are 255 bidders in the C block compared to only 30 in the AB blocks and 155 in the DEF blocks.[3] Furthermore, there were two licenses for sale for every geographic region in the AB blocks, and three licenses for every

[3]Moreover, many of the bidders in the AB and DEF blocks were incumbent mobile phone carriers, and for antitrust reasons were ineligible to bid in geographic markets where they already held licenses. In particular, parties owning more than a 40% interest in an existing wireless license in an area could not bid on another license in that area. Imposing the legal choice set of each bidder creates considerable additional complexity in estimation.

The C block, by comparison, featured only potential new entrants, so all bidders could potentially bid on all licenses. The antitrust policy may have lowered competition in the AB auction (Ausubel et al., 1997; Salant, 1997). The FCC limited any one bidder from winning more than 98 total licenses in the C and F entrepreneurs blocks. Only NextWave came close to meeting this limit. Ausubel et al. (1997) point out that because the limit was in total licenses rather than total population, NextWave had incentives to purchase licenses with the highest total population. Our coming pairwise stability condition will be consistent with NextWave favoring licenses with more population.

geographic region in the DEF blocks. An AB or DEF block bidder was thus guaranteed to be competing directly against at least one other winning carrier after the auction ended. This direct externality in the structural payoffs of bidders complicates the analysis of bidding behavior considerably. In the C block, each geographic region had only one license for sale.[4]

The C block auction took 184 rounds, lasting from December 1995 to April 1996. Incumbent carriers did not participate in the C block because of discounts offered to small businesses.[5] The discounts fulfilled a Congressional mandate to encourage smaller companies to offer wireless phone service. Bidding for the C block was more aggressive than in the AB block, with bids (for only half the spectrum sold in the AB blocks) totaling $10.1 billion. Figure 1 is a map of the licenses won by the Top 12 winning bidders. Figure 1 shows that the largest winner in the C block auction was NextWave, whose winning bids totaled $4.2 billion for 56 licenses, including close to $1 billion for the New York City license.

2.2 After the auction: mergers

C block bidders were given an extended payment plan of ten years. Many of the bidders planned to secure outside funding for both their license bids and other carrier startup costs after the auction. Securing licenses first and financing later was an extremely important part of the business plan of what was until the late 1990s the most successful American mobile phone carrier, McCaw Cellular.[6] This strategy was based on McCaw's (correct) forecast of the revenue potential in mobile phones, which was higher than the forecasts of larger companies (Murray, 2001). It is possible that many of the C block bidders were trying to recreate McCaw's strategy. With a scarce license, a small business bidder becomes a relevant player in the mobile phone industry, and can expect to hold serious discussions with financiers.

Compared to McCaw, the C block winners did not have an early-mover advantage. As it turns out, many C block winners were unable to meet their financial obligations to the FCC. These new carriers were unable to secure enough outside funding to both operate a mobile phone company and pay back the FCC. Many C block winners returned their licenses to the FCC, where they were re-auctioned. Others companies merged with with larger carriers (forming a large part of the licenses held by T-Mobile USA, for example) or were able to protect their licenses in bankruptcy court. NextWave is

[4] After the auction, winning C block bidders were much more likely to compete against incumbent mobile phone carriers operating in the same geographic region than against other C block bidders.

[5] Plans to give additional advantages to women and minorities were dropped because of litigation. Small business ownership requirements were not overly strict. Two ownerships structures qualify bidders as small businesses. The first structure is a control group must hold 25% of the businesses' equity. Of that 25%, 15% (or 3/5) of the equity must be held by qualifying entrepreneurs. Of the remaining 75% of equity, no more than 25% can be controlled by any one entity. An alternative structure says the control group can have 50% of the equity, with 30% being qualifying entrepreneurs. This allows the other 50% to be held by one outside entity, which in effect allows the company to partner with a major firm. The most famous case of partnering is Cook Inlet, an Alaskan native corporation that partnered with the incumbent carrier Western Wireless.

[6] McCaw was purchased by AT&T for $17.4 billion and renamed AT&T Wireless in 1993. AT&T Wireless was itself purchased by Cingular in 2004. Cingular was renamed AT&T in 2007.

the most famous case of bankruptcy protection. NextWave was eventually able to settle with the FCC and sell some of its licenses to other carriers for billions of dollars. Ex-post, the C block bidders, who were accused of bidding too aggressively at the time, underpredicted the eventual market value of the licenses. However, much of this value was to larger carriers, not small business entrants who could not secure the financing to operate as a mobile phone carrier. Twelve years later, in 2007, only a few C block winners, such as GWI/MetroPCS, remain true independent carriers marketing service under their own brand.

Aside from NextWave, the legal form of most license transfers was a merger, not a resale. Many of these acquisitions of C block carriers took place years afterward, and many involved one firm that was an incumbent / non-C block bidder. The most important instance of a merger was the creation of T-Mobile USA in 2001 from the takeover of the existing carrier VoiceStream by Deutsche Telekom, as well as mergers with the independent carriers Aerial, Omnipoint and PowerTel.

The resale and merger activity suggests that a bidder's post-auction value for winning licenses was not only a function of the package of territories it planned to serve as a mobile phone carrier. Valuations might be a function of the bidder's beliefs about the expected value from resale of its licenses, from mergers after the auction and the risk of bankruptcy.[7] Valuations also likely reflect the ability to serve traveling customers through roaming agreements and well as signing up new subscribers directly. Therefore, attempting to directly recover a bidder's value from operating a mobile phone carrier will be quite naive in this setting. We favor a nuanced interpretation of the estimates from our structural model that encompasses the possibilities of both merger and independent operation.[8]

2.3 Auction rules and bidder characteristics

Similar rules govern all FCC auctions for mobile phone spectrum. FCC spectrum auctions are simultaneous ascending-bid, multiple round auctions that can take more than a hundred days to complete. Formally speaking, a simultaneous ascending auction is a dynamic game with incomplete information. Each auction lasts multiple rounds, where in each round all licenses are available for bidding. During a round, bidding on all licenses closes at the same time. These auction rules were explicitly designed by academic economists to allow bidders to assemble packages exhibiting complementarities, while letting the bidders themselves and not the FCC determine where the true complementarities lie. If bidders have finite valuations, they will cease bidding after a finite number of rounds, although the length of the auction is not known at the start. Package bidding is not allowed; bidders place bids on each license separately.

Each bidder makes a payment before the auction begins for initial eligibility. A bidder's eligibility is expressed in units of total population. A bidder cannot bid on a package of licenses that exceeds

[7] The FCC's unjust enrichment regulation penalizes resale to carriers that do not qualify as eligible entrepreneurs.

[8] Our policy application focuses on the geographic size of licenses. Larger licenses might actually make resale easier, as carriers need to negotiate with fewer existing owners to secure coverage. The initial 800 MHz licenses that were handed out using lotteries in the 1980s took over twenty years to consolidate.

the bidder's eligibility. For example, a bidder who pays to be eligible for 100 million people cannot bid on licenses that cover geographic areas that together contain more than 100 million residents. Eligibility cannot be increased after the auction starts. During the auction, the eligibility of bidders that do not make enough bids is reduced. By the close of the auction, many bidders are only eligible for a population equal to the population of their winning licenses.

The eligibility payments were 1.5 cents per MHz-individual in a hypothetical license for the C block. Compared to the closing auction prices, these payments are trivial. We use eligibility to control for a bidder's willingness to devote financial resources towards winning spectrum. This paper does not consider strategic motives (such as intimidating rivals) for choosing eligibility levels.

Table 1 lists characteristics of the 85 winning and 170 non-winning bidders in the continental United States.[9] The average winning bidder paid fees to be eligible to bid on licenses covering 10 million people, while the average losing bidder was eligible to bid on licenses covering only 5 million people. Bidders also had to submit financial disclosure forms (the FCC's Form 175) in order to qualify as entrepreneurs for the C block, which was limited to new entrants only. Table 1 shows that the financial characteristics of winners and non-winners are similar, which leads us to believe that these disclosure forms did not represent the true resources of bidders. Hence, in our structural estimator, we use initial eligibility as an individual bidder characteristic instead of assets or revenues.

Table 1 lists the mean number of licenses bid on and won by winners and non-winners. The mean winning bidder won 5 licenses and entered at least one bid on 39 licenses. Although not listed in the table, the top 15 winning bidders, in terms of number of licenses, were active bidders on many licenses. The top 15 winners won an average of 16 licenses and bid on an average of 123 (out of 493) licenses.[10] Most of the major winners and some of the non-winners were investors operating on a national scale. The role of idiosyncratic valuations of licenses due, for instance, to local knowledge seems relatively minor, as the bidding in the C block auction was dominated by national investors that were competing for licenses over the entire country.

2.4 Prices and winning packages

Despite the many potential complications, the C block auction generated closing bids where the underlying characteristics of licenses explain much of the variation in prices across licenses. The most important characteristic of a license is the number of people living in it, who represent potential subscribers to mobile phone service. The population weighted mean of the winning prices per resident is \$40. The second most important characteristic in determining the closing prices was population density. Radio spectrum capacity is more likely to be binding in more densely populated areas. A

[9] The C block also contains licenses for Alaska and Hawaii as well as Puerto Rico and several other island territories of the United States. The potential for complementarities between these licenses and licenses in the continental United States seems limited, so we restrict attention to the contiguous 48 states.

[10] One of the losing bidders submitted bids on all BTAs. This bidder withdrew from the auction because it felt that the prices were too high for its business plan.

regression of a license's winning price divided by its population on its population density gives an R^2 of 0.33.[11] However, prices per resident varied widely across the AB, C and DEF auctions. It is difficult to reconcile this across auction variation with a view that the final bids closely reflect bidder valuations (Ausubel et al., 1997).

Table 2 lists characteristics of winning packages. Only licenses in the continental United States are included in the packages summarized in Table 2. The average winning bidder agreed to pay \$116 million and won a license covering 2.9 million people. The largest winner, NextWave, bid \$4.2 billion for a package covering 94 million people.

Figure 2 plots the log of a bidder's initial eligibility on the horizontal axis and the log of the package's winning population on the vertical axis. A quadratic fit to the data is also included. The R^2 of the quadratic is quite high, at 0.67. This suggests initial eligibility is a useful control for cross bidder differences in the demand for spectrum.

2.5 Suggestive evidence on complementarities

A major justification for the simultaneous ascending auction mechanism used in the FCC spectrum auctions is that it allows bidders to assemble packages of geographically adjacent licenses. In the literature, such adjacent licenses are said to exhibit complementarities or synergies.

One's prior might be that complementarities are not important in the spectrum auctions. The FCC chose market boundaries to be in sparsely settled areas in order to minimize complementarities across markets.[12] Furthermore, 1900 MHz PCS wireless phone service is mainly deployed in urban areas and along major highways, so there might not even be PCS service along the boundaries of two markets.[13] Finally, companies can coordinate with contracts (roaming agreements) if the same company does not own the adjacent licenses.[14]

However, an initial inspection of our data suggests the existence of geographic complementarities. The map of the Top 12 winners (by the number of licenses) in Figure 1 shows several bidders win licenses in markets adjacent to each other.[15] For example, NextWave, the largest winner, purchases

[11] Ausubel et al. (1997) use proprietary consulting data on the population density of the expected build-out areas for C block mobile phone service. They have provided us the same data, which we use here.

[12] Bajari, Fox and Ryan (2007) estimate that consumers do have high willingnesses to pay to avoid roaming surcharges while traveling. So there is evidence that economic primitives do justify complementarities in bidders' structural profit functions.

[13] To some extent, PCS licenses are primarily built out in urban areas because the FCC requires build outs to cover a certain fraction of the population of the market, rather than a fraction of the market's land area. 800 MHz carriers tend to cover both urban and rural areas because the FCC requires coverage as a large fraction of the land area of those licenses.

[14] The Coase Theorem suggests that, in a frictionless world, such contracts will implement the efficient outcome. Our paper uses revealed preference to investigate whether bidders thought the Coase Theorem would be operative in the post-auction mobile phone service industry.

[15] Ausubel et al. (1997) study in part the earlier AB auction and show several bidders win licenses adjacent to markets where the bidder is a mobile phone incumbent, or a landline telephone carrier. For example, Pacific Bell, at the time a California telephone company, won AB block licenses in California. Other bidders, such as the forerunners of Sprint PCS and AT&T Wireless, embarked on a strategy of winning licenses in as many markets as allowed.

clumps of adjacent licenses in different areas of the country. GWI/MetroPCS fits the cluster pattern well, winning licenses in the greater San Francisco, Atlanta and Miami areas.

On the other hand, the majority of winning bidders won only a few licenses. Figure 1 emphasizes this by also plotting the 26 licenses in the continental United States that were the only license won by their winning bidders. We calculate that only 20 out of 89 C block winning bidders won packages of licenses where the population in adjacent licenses within the package was more than 1 million.[16] Aer Force is the prime example of a Top 12 bidder that did not seem overly concerned with complementarities. Figure 1 shows that Aer Force won 12 licenses in the continental United States, but that none of them are adjacent to each other. From the maps alone, it appears some winning bidders cared more about geographic complementarities than others.

Previous researchers have generally concluded that complementarities were important. Ausubel, Cramton, McAfee and McMillan (1997) and Moreton and Spiller (1998) examine whether adjacent licenses exhibited complementarities by regressing the log of winning bids on market and bidder characteristics. Ausubel et al. study the AB and C block auctions and find that the log of winning bids are positively related to whether the runner-up bidders won adjacent licenses, as one might expect in an ascending-bid auction. However, the coefficient in the C block auction is economically small, meaning that prices do not seem to strongly reflect any value of complementarities. Moreton and Spiller have better measures of incumbency, and also find that winning bids are positively related to the runner-up bidder's measures of complementarities. The results are the most statistically significant for the C block auction.[17]

The previous authors also discuss scale economies, the notion that a wireless network involves fixed costs that can be spread out among more customers in a larger carrier. Scale economies can be represented by allowing valuations to be a convex function of package characteristics such as total population. However, because bidders with higher valuations (more initial eligibility) win packages with higher populations, it may be hard to empirically distinguish operating scale economies from heterogeneities in bidder valuations.

The map of winners, Figure 1, suggests that the clusters of nearby licenses in winning packages are possibly too small. If bidder valuations were primarily a function of complementarities, we might expect to see the entire southeast won by one bidder, for example.

2.6 Suggestive evidence about collusion and demand reduction

Milgrom (2000, Theorems 2,3) proves that a simultaneous ascending auction is equivalent to a tatonnement process that finds a competitive equilibrium of the economy, under two assumptions:

[16] This complementarity measure is calculated over pairs of licenses. If a license is adjacent to two others in a package, its population will be counted twice. The 89 winners include four bidders who won licenses only outside of the continental United States.

[17] Ausubel et al. and Moreton and Spiller do not claim their price regressions correspond to hedonic estimates of bidder valuations. Rather, they specify descriptive or in-sample prediction regressions designed to summarize facts about the closing bid prices.

1. The licenses are mutual substitutes for all bidders, and

2. All bidders bid straightforwardly.

Unfortunately, neither one of the assumptions needed to prove that a simultaneous ascending auction is efficient appear to hold in the C block data. The fact that many bidders win clusters of licenses, as seen in the map in Figure 1, is good evidence that licenses are not mutual substitutes for all bidders. Bidding straightforwardly means that a bidder submits new bids each period in order to maximize its structural profit function, rather than some other continuation value in a dynamic game. One violation of straightforward bidding is jump bidding. When making a jump bid, a bidder enters a bid that exceeds the FCC's minimum bid for that round. We define a jump bid to be any bid that is 2.5% greater than the FCC's minimum bid for that license and round. Figure 3 shows that there was a non-trivial level of jump bidding during the C block auction.

When jump bidding, a bidder risks the chance that the jump bid will exceed the valuation of rival bidders, and be the final price. A jump bidder therefore has a nonzero probability of overpaying for a license.[18] However, there are possible strategic advantages from jump bidding. In a single unit, affiliated values model, Avery (1998) demonstrates that jump bidding may signal the jump bidder's intentions to bid aggressively throughout the auction. Since other bidders fear the winner's curse, they may discontinue bidding in order to avoid overpaying conditional on winning the item.[19]

Figure 3 shows jump bidding was prevalent towards the beginning of the auction, where the risk of overpaying is much lower. The number of total new bids dramatically slowed during the second half of the auction, and this slowdown is especially severe for jump bids. The presence of jump bids might represent signals that are attempts at intimidation, but jump bids are not evidence the signals successfully caused other bidders to withdraw. There are anecdotes of actual retaliation. In round 3, Pocket (DCR) placed a large jump bid of 60% more than the minimum for Las Vegas. In round 70, MetroPCS (GWI) outbid Pocket for Las Vegas and PCS2000 for Reno. In round 71, Pocket outbid MetroPCS on Reno and Salt Lake City, the only time Pocket bid on either of those licenses. Further, PCS2000 outbid MetroPCS on Las Vegas, the only time since round 12 PCS2000 had bid on Las Vegas. In round 72, after seeming to retaliate against MetroPCS, Pocket enters the winning bid for Las Vegas, meaning the bid stands until the end of the auction, round 184.

There are other anecdotes of intimidation that do not involve jump bids. Towards the end of the auction, NextWave and AerForce were competing for Fredericksburg, Virginia. NextWave needed Fredericksburg to complete a regional cluster around Washington, DC. In round 162, NextWave outbid

[18]Daniel and Hirshleifer (1998) study a model of an ascending auction of a single item where bidding is costly. An equilibrium involves jump bidding to speed the conclusion of the auction. As bids in the C block auction totalled $10.1 billion, it is unlikely that bidders had a high opportunity cost of time and placed jump bids to speed the conclusion of the auction, at least towards the end of the auction. Serious bidders employed teams of professionals to manage bidding activity.

[19]Theorists have shown that jump bidding can happen for non-collusive reasons. For example, in a simultaneous ascending auction where the bidding on each license can close at different times, Zhèng (2005) shows that jump bidding can alleviate the exposure problem mentioned below. The FCC auction rules have a more direct withdrawal mechanism to mitigate the exposure problem.

AerForce for Fredericksburg. In round 163, AerForce responded not only by bidding on Fredericksburg but also by bidding on Lakeland, Florida. Lakeland is a small population territory that AerForce had not bid on in a long while and that NextWave had been winning. In round 164, NextWave bid again and retook Lakeland, but never bid again on Fredericksburg. By challenging AerForce on Fredericksburg, NextWave only succeeded in paying 10% (two bid increments) more to win Lakeland.

Cramton and Schwartz (2000, 2002) provide many more examples of signalling and implicit collusion through intimidation, especially in the auctions for the AB and DEF blocks. We feel the evidence is strong enough that any estimation method for spectrum auction data must be consistent in the presence of this type of collusive behavior.

3 The Model

3.1 Bidders' profit functions

In this section, we define the economic environment, the equilibrium allocation and prices, and a necessary condition that will hold in a fairly general set of models. In the model, there are $a = 1,...,N$ bidders and $j = 1,...,L$ licenses for sale. We will abuse notation and often let L also denote the set of all licenses for sale. Our environment is a multiple unit auction where bidders may win a package of licenses. We let $J \subset L$ denote such a package of licenses. In the C block auction, the licenses are permits to transmit mobile phone signals in specified geographic territories and there is only one license per territory. There were 255 registered bidders in the C block and 493 licenses for sale. We will limit attention to the 85 winning bidders and the 480 licenses for sale in the continental United States.

In the model, bidder a maximizes its profit

$$\pi_a(J) - \sum_{j \in J} p_j$$

from winning package J at prices $\{p_j\}_{j \in J}$. Bidder a's profit is comprised of two parts. The term $\pi_a(J)$ is a's valuation for the package of licenses J and $\sum_{j \in J} p_j$ is the price that a pays for this package. In our application, we will parameterize $\pi_a(J)$ as

$$\pi_a(J) = \bar{\pi}(w_a, x_J) + \sum_{j \in J} \xi_j + \sum_{j \in J} \varepsilon_{aj}. \qquad (1)$$

The scalar $\bar{\pi}(w_a, x_J)$ is a function of the characteristics w_a of bidder a and of the characteristics x_J of the package of licenses J. In our application, the vector x_J will include the total population covered by the licenses J and measures of geographic or travel complementarities between the licenses in package J. The scalar w_a will be bidder a's initial eligibility, as discussed in Section 2.3.[20] Both x_J and w_a are

[20] Each individual territory is a heterogeneous item. In spectrum auctions, bidders cannot bid on packages, and all bidders do not necessarily place bids on all territories. Therefore, our only feasible estimation strategy is to make $\bar{\pi}(w_a, x_J)$ a function

public information to the bidders and observed by the econometrician.

The term ξ_j is a license j fixed effect, which we assume is publicly observed by the bidders. The fixed effect enters bidders' valuations additively and is meant to capture the characteristics of license j that are observed by the bidders, but which are unobserved to the econometrician.[21] For example, we lack controls for the incumbent phone companies, the winners of the earlier AB auctions and potential merger and roaming partners. However, including ξ_j allows us to account for this license specific unobserved heterogeneity. As is standard in fixed effect models, we cannot identify the effects of elements of x_J that are collinear with the fixed effects ξ_j. We can, however, identify $\bar{\pi}(w_a, x_J)$, which captures the interaction between the bidder and license characteristics observed by the econometrician.

The term ξ_j plays a similar role to the omitted product attributes in the model of Berry, Levinsohn and Pakes (1995). The prior empirical auctions literature typically assumes that there is no unobserved heterogeneity about the objects for sale, i.e. $\xi_j = 0$ for all j. Ignoring ξ_j in this fashion will likely generate biased estimates of bidder valuations. We anticipate that the valuation for elements of x_J that are positively (negatively) correlated with ξ_j will be biased upwards (downwards). The only prior paper to account for unobserved item heterogeneity in auctions is Krasnokutskaya (2004), who considers the case of first price, asymmetric auctions. We extend the literature by accounting for unobserved item heterogeneity in a multiple unit auction.

The unobservables, ε_{aj} , reflect bidder a's private information about license j. Consistent with the independent private values framework, we assume that the ε_{aj} are i.i.d. across bidders and licenses and are independent of all w's, x's and ξ's. These reflect bidder specific costs and benefits from operating in a particular territory. As we will briefly discuss, our approach of using necessary conditions for estimation will not allow us to identify the distribution of the ε_{aj}'s. For the C block, the trade press and the number of licenses bid on by each bidder suggest that many winning bidders were willing to operate in any region of the country. This suggests the variance of ε_{aj} is small. A small variance of ε_{aj} contrasts with the AB blocks, where many bidders were incumbents trying to win territories near their existing service areas.

3.2 Equilibrium and Pairwise Stability

A spectrum auction is a mechanism M that takes as arguments the valuations $\{\pi_a(J)\}_{a=1,\dots,N}^{J\subseteq L}$ and bidder characteristics $\{w_a\}_{a=1,\dots,N}$. This mechanism produces a vector of prices for each license $p^L = (p_1, \dots p_L)$ and an allocation of licenses $A = \{J_1, \dots, J_N\}$. We let $J_a \subset L$ denote the package of licenses

of the observable characteristics of package J and bidder a. Our maximum score identification strategy will not permit us to estimate a random effect distribution of unobserved bidder valuations; we need data on w_a.

[21] In a common values model, ξ_j might be unobserved to the bidders as well. Common values are usually not part of formal models of spectrum auctions because of technical complexity. However, part of the benefit of a simultaneous ascending auction and the eligibility rules is that bidders disclose information about the value of licenses through bidding. Therefore, at the end of the auction a lot of information about ξ_j has been disclosed, possibly mitigating any winner's curse (Hong and Shum, 2003).

won by bidder a. We denote this map by:

$$\left\{p^L, A\right\} = M\left(\{\pi_a(J)\}_{a=1,\ldots,N}^{J\subseteq L}, \{w_a\}_{a=1,\ldots,N}\right).$$

Unfortunately, the theoretical literature has not proposed a unique mapping M that can be used to model equilibrium bidding behavior in the spectrum auctions. Instead, the literature has proposed a variety of alternative models, each with a distinct game form and different equilibrium predictions. Also, any one model may have multiple equilibria. Given the diversity of modeling strategies and equilibrium predictions, we do not believe it is useful to develop an estimator that assumes exactly one of these models is true, while the others are false. Any such model is likely to be misspecified and generate biased estimates of valuations as a result.

Instead, we proceed analogously to Haile and Tamer (2003) and work with necessary conditions for equilibrium that are likely to hold in a number of theoretical models. As discussed in the introduction, an auction of multiple heterogeneous items can be viewed as a one-to-many two-sided matching game.[22] The two sides of the market are bidders and licenses. An item for sale can be won by only one bidder, but a bidder can win multiple items. The exclusivity of each item in an auction makes bidders rivals to match with the item. The results from Section 2 suggest there is important information about valuations that is contained in which bidders win which licenses. For example, the clustering of licenses in Figure 1 suggests that complementarities in licenses may be important. Table 1 and Figure 2 show that bidders with higher initial eligibility win more licenses. This is consistent with bidders with more eligibility having higher valuations for licenses (recall that price per unit of population was fairly uniform across licenses in our sample). We will now define two sets of conditions on the match between bidders and licenses that are necessary conditions for some models of multiple unit auctions, including models of spectrum auctions.

Definition 1. *The outcome $\left\{p^L, A\right\} = \left\{p^L, \{J_1, \ldots, J_N\}\right\}$ is a* ***pairwise stable outcome in prices and matches*** *if, for each bidder $a = 1, \ldots, N$, corresponding winning package $J_a \subset L$, and licenses $i \in J_a$ and $j \notin J_a$, $j \in L$*

$$\pi_a(J_a) - p_i \geq \pi_a((J_a \backslash \{i\}) \cup \{j\}) - p_j. \tag{2}$$

In the above definition, at the closing prices p^L, bidder a must not want to swap one of its winning licenses i for some other bidder's license j.

We also consider the following definition for a stable match.

Definition 2. *An assignment of bidders to licenses $A = \{J_1, \ldots, J_N\}$ is a* ***pairwise stable outcome in matches*** *if, for each pair of winning bidders $a \in N$ and $b \in N$, corresponding winning packages $J_a \subset L$*

[22] Hatfield and Milgrom (2005) present counterexamples for general two-sided matching games that shows that there might not be a static equilibrium when at least one agent has payoffs that feature complementarities across multiple matches. The real C block auction is a dynamic game with possible intimidatory equilibria, so their static counterexample is not applicable to the actual data generating process.

and $J_b \subset L$, $J_a \cap J_b = \emptyset$, as well as licenses $i_a \in J_a$ and $i_b \in J_b$,

$$\pi_a(J_a) + \pi_b(J_b) \geq \pi_a((J_a \backslash \{i_a\}) \cup \{i_b\}) + \pi_b((J_b \backslash \{i_b\}) \cup \{i_a\}). \tag{3}$$

Pairwise stability in matches considers swapping licenses: the total surplus of two bidders must not be increased by an exchange of one license each. Note that unlike Definition 1, the above definition does not involve the prices of the licenses.[23]

Concepts such as pairwise stability with transfers have been used in the matching literature since Koopmans and Beckmann (1957), Shapley and Shubik (1972) and Becker (1973).[24] The matching literature motivates Definition 2 as an implication of Definition 1: adding the inequality

$$\pi_b(J_b) - p_j \geq \pi_b((J_b \backslash \{j\}) \cup \{i\}) - p_i$$

to (2) cancels the license prices and gives (3).[25] We do not rely on this motivation: the two definitions exist separately. A key difference from our definitions of pairwise stability and the most common definitions in the matching literature is that we do not impose a nonnegative profit condition:

$$\pi_a(J_a) - \sum_{i \in J_a} p_i \geq 0.$$

This nonnegativity condition provides little information about geographic complementarities. Further, the condition may be violated because of the exposure problem in simultaneous ascending auctions. We discuss the exposure problem below.

4 Auction Theory and Pairwise Stability

In this section, we discuss the relationship between pairwise stability and existing models of multiple unit auctions . First, both Definition 1 and Definition 2 will hold in many single unit, private value auctions with asymmetric bidders. The second price sealed bid auction and the button auction of Milgrom and Weber (1982) both allocate the item to the bidder with the highest valuation. As a result, the estimator that we propose in the next section could be applied to these environments. An advantage of our estimation method, compared to other approaches for single unit auctions, is that we can estimate

[23]Cramton (2006) interprets the lack of immediate, post-auction resale as evidence that the outcome is efficient. As mobile phone mergers are costly (Fox and Perez-Saiz, 2006) but did take place years later, we argue that the lack of post-auction swapping is evidence that the outcome satisfied the weaker condition of pairwise stability in matches. Consolidation may increase structural profits, but swapping licenses does not.

[24]In an auction, licenses are dummy agents that care only for money. This allows us to interpret $\pi_a(J_a)$ as the structural profit of bidders rather than the total surplus of bidders and licenses.

[25]Both Definitions 2 and 1 utilize the assumption of transferable utility: prices enter additively separably into profits, (1). Hatfield and Milgrom (2005), for example, allow matching in a more general contract space. Our estimation approach, explained below, of using only part of the contract (the physical match but not the monetary transfer) requires additive separability of transfers.

our model by only assuming Definition 2 . Therefore, we do not need to assume bids are informative about valuations. Our estimator would be robust to the presence of efficient collusion as considered in Graham and Marshall (1987). Previous estimation methods for the oral auction that rely on bids being informative of values, such as Haile and Tamer (2003), need not generate consistent estimates under Graham and Marshall and related models of collusion. However, pairwise stability is not satisfied in all single unit models and must be carefully verified. For example, the first price sealed bid auction with asymmetric bidders may be inefficient ex post and hence violate pairwise stability.

Pairwise stability holds in many multiple unit auction models as well. First, when there are multiple identical items for sale, it is trivial to demonstrate that Definition 2 holds simply because the items are identical. Second, in a model where each bidder has declining marginal values, Definition 1 may hold if prices are such that the final allocation of items is efficient. This would occur, for example, if a Vickrey-Clarke-Groves mechanism was used. Third, under some conditions, pairwise stability will hold in the Google keyword auction studied by Edelman, Ostrovsky and Schwarz (2007). Finally, pairwise stability is an implication of the tatonnement conditions for the spectrum auction model of Milgrom (2000).

In the next subsections, we study models of spectrum auctions that allow for the complications of demand reduction and collusion. We demonstrate that both Definitions 1 and 2 are satisfied in a model of demand reduction. However, only Definition 2 is satisfied in a model of intimidatory collusion.[26]

4.1 Demand reduction

Demand reduction is studied by Ausubel and Cramton (2002) for the case of sealed bid auctions of multiple homogeneous items. In a simultaneous ascending auction, demand reduction is consistent with straightforward bidding by forward-looking agents. Kagel and Levin (2001) and List and Lucking-Reiley (2000) find substantial demand reduction in experiments.

Consider bidders a and b competing for two licenses 1 and 2. Use the shorthand notation π_{a12} for $\pi_a(\{1,2\})$. Let the profits of bidders a and b for the three possible packages be as listed in Table 3, case 1. Bidder a has a higher value for all packages. Bidder b has decreasing returns to scale: there is no incremental value from winning both licenses.

If both bidders bid straightforwardly in a simultaneous ascending auction, and ignoring minimum bid increments, a will win both licenses at prices equal to b's values: $p_1 = \pi_{b1}$ and $p_2 = \pi_{b2}$. However, if a reduces its demand and lets b win item 2 at $p_2 = 0$, a can win item 1 at $p_1 = 0$. Bidder b accepts this because it has a demand for only one license and prefers 2 to 1. The demand reduction outcome is inefficient: total structural profit is maximized by having a win both items. However, when a wins 1 and b wins 2, $\pi_{a1} + \pi_{b2} > \pi_{a2} + \pi_{b1}$, so that the total structural profit cannot be increased with license

[26]Day and Milgrom (2007) study the normative problem of developing core-selecting auctions. These auction produce allocations that satisfy the two definitions of pairwise stability, with and without prices, according to the *stated* preferences of bidders. Our goal in studying the simultaneous ascending auction is positive: to examine whether it is likely that an existing mechanism produces an assignment that satisfies Definition 2 according to the *actual* preferences of bidders.

swaps. Definition 2 is satisfied as the bidders disagree on the profit ranking of the licenses. One can use the zero prices to show Definition 1 is satisfied as well.

Now we will argue that the example does not rely on bidder disagreement over the profit ranking. Case 2 in Table 3 changes b's structural profits so that a and b agree on the profit ranking of licenses 1 and 2: $\pi_{b1} \geq \pi_{b2}$. At the beginning of the auction, with $p_1 = p_2 = 0$, bidder b will bid on item 1 as b prefers 1 and has a demand for only one item. Only at a price $p_1^\star$ such that $\pi_{b1} - p_1^\star = \pi_{b2}$ will b accept winning license 2 instead of 1. If $\pi_{a1} - p_1^\star > \pi_{a2}$, then substituting in $p_1^\star = \pi_{b1} - \pi_{b2}$ to $\pi_{a1} \geq \pi_{a2}$ again gives $\pi_{a1} + \pi_{b2} > \pi_{a2} + \pi_{b1}$. Definitions 1 and 2 are satisfied.

What if in case 2, $\pi_{a1} - p_1^\star = \pi_{a1} - (\pi_{b1} - \pi_{b2}) < \pi_{a2}$? If a finds it profitable to reduce its demand, a will reduce its demand on license 1 and win 2, leaving $\pi_{a2} + \pi_{b1} > \pi_{a1} + \pi_{b2}$. Again, $p_1^\star$ is set, by straightforward bidding, to make a coordinate on a pairwise stable outcome. Definitions 1 and 2 are satisfied. The points made in this example are more general.[27]

Theorem 1. *Consider straightforward bidding in a simultaneous ascending auction with demand reduction. Under the tatonnement conditions of Milgrom (2000), the outcome is a pairwise stable outcome to a matching game where the maximum number, or quota, of licenses that a bidder can win is the number of licenses the bidder won in the outcome. Both Definitions 1 and 2 are satisfied.*

Proof. Let the assignment portion of the demand reduction outcome be A, and let bidder a's wining package be J_a. For all bidders a, redefine a's profits for a package J to be negative infinity if J has more licenses than J_a. $\pi_a(J) = -\infty$ for $|J| > |J_a|$. Then Milgrom's tatonnement process theorems (Theorems 2 and 3 in Milgrom) show that the simultaneous ascending auction will find a competitive equilibrium (core outcome) of the economy with the truncated profit functions. Pairwise stability, Definition 1, is implied by being in the core. As the swaps considered in Definition 1 do not change the number of licenses won by any bidder, the profits under the swaps are the same as under the pre-truncated profit functions. So the outcome is pairwise stable under a matching game where bidders cannot add additional licenses to their package. □

Under demand reduction, the outcome may not be efficient, but there is no reason to believe that there exist swaps of licenses that would raise total structural profits.[28] The theorem does not explain how much demand reduction will go on: the unilateral incentive to reduce demand requires knowledge that another bidder has strong decreasing returns to scale.[29] Given unilateral demand reduction and

[27] The conditions for Milgrom's tatonnement process theorem rule out complementarities, to avoid the exposure problem. Definition 2 requires only that total structural profit not be raised by swapping licenses. It is compatible with many forms of the exposure problem. See footnote 34 for more on the exposure problem.

[28] Under demand reduction bidders bid as if they are constrained to only win at most some fixed quota of licenses. This is similar to the constraints on licenses already present in the matching game: each license can be won by only one bidder. Having quotas on both licenses and bidders has analogs in other matching markets such as marriage, where both men and women can have only one spouse at a time.

[29] The initial eligibilities of other bidders are known before bidding starts. Therefore, some forms of decreasing returns are public knowledge. Further, Cramton (2006) interprets the purchase of spectrum in a small, quick, post-auction sale (a bidder did not make its payments) by NextWave as evidence that NextWave was reducing its demand during the auction.

straightforward bidding, the theorem shows that pairwise stability will result. Straightforward bidding under private values is a perfect Bayesian Nash equilibria (Brusco and Lopomo, 2002). If rivals are bidding competitively, only unilateral demand reduction followed by straightforward bidding can raise profits.[30]

4.2 Pairwise stability in matches under intimidation and collusion

Collusive schemes raise bidders' profits and reduce revenue to the auctioneer. Under collusion, it is unlikely that prices will satisfy Definition 1. Without imposing a very specific model of intimidation, it is not clear how data on the auction's closing prices are informative about the structural profit functions of bidders. Consider an example with two bidders *a* and *b* and two licenses 1 and 2. The bidders may split the items and pay 0 each to the seller. In this case, if both bidders *a* and *b* value item 1 more than 2, the bidder *b* who wins item 2 prefers to pay a price of 0 and win item 1 instead of 2. So the outcome will not satisfy Definition 1.

Some collusive models, however, will satisfy Definition 2. In single unit auctions, many models of collusion are efficient in the sense that bidders with higher values win more often. Graham and Marshall model bidder rings in second-price auctions. In equilibrium, the ring member with the highest value always wins if any ring member does. McAfee and McMillan (1992) present similar results for an all-inclusive bidder ring in a first-price auction. Similarly, Athey, Levin and Seira (2004) assume that, in auctions of a single item with an explicitly organized bidding ring, the highest value bidder will represent the ring and compete for the item for sale.

In a spectrum auction, a collusive equilibrium would satisfy Definition 2 if the bidders coordinate on an allocation of licenses that maximizes joint surplus. However, in the C block auction, we have no evidence to suggest that an explicit bidding ring was active. The empirical evidence in Cramton and Schwartz (2000, 2002) suggests that bidders signaled each other through the bidding mechanism, rather than through illegal back-channel communications. There is no evidence that any firms exchanged transfers after the auction as bribes to compensate losing bidders for not winning licenses.

Motivated by this empirical evidence, we consider pairwise stability in matches in the models of Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005). These authors analyze equilibria in simultaneous ascending auctions with two licenses and two or more bidders. Agents signal each other through bids.[31] In Brusco and Lopomo, for example, a typical equilibrium (without complementarities) has each bidder bid on its most preferred item until only two bidders are left. If those bidders are bidding on different items, they stop bidding and walk away with the items at those prices, even if it is efficient for one bidder to win both items. If the last two bidders are bidding on the same item, they continue bidding on that item for a while until one eventually moves to the other

[30]Ausubel and Schwartz (1999) use a Nash concept to compute the unique subgame perfect equilibrium to a complete information, alternating-bid, ascending auction of a single divisible good. The equilibrium involves demand reduction. Bidders have a common valuation for all units of the divisible good, so pairwise stability is satisfied.

[31]The signal is a positive bid on a license.

item. The collusion is sustained by intimidatory threats of resorting to competitive bidding, which is also a symmetric perfect Bayesian equilibrium. The following result summarizes predictions about the assignment of licenses to bidders from propositions in the earlier theory papers. We state it as a theorem although it is a reinterpretation of a set of existing theorems.

Theorem 2. *Brusco and Lopomo (2002) study symmetric perfect Bayesian equilibria to private value, simultaneous ascending auctions of two licenses with $N \geq 2$ bidders and, for some propositions, complementarities. Propositions 0–2 and 4–7 in their paper present equilibria with outcomes that satisfy Definition 2, pairwise stability with matches only. No other equilibria are found.*

Engelbrecht-Wiggans and Kahn (2005) consider cases of two bidders without complementarities in ascending auctions with no jump bidding. Lemma 1 and Theorems 2–4 and 6 present equilibria that satisfy pairwise stability in matches only.

Proof. If bidder a wins item 1 and b wins item 2, then Definition 2 requires $\pi_a(\{1\}) + \pi_b(\{2\}) > \pi_a(\{2\}) + \pi_b(\{1\})$.

First consider Brusco and Lopomo (2002). In Proposition 0 (two bidders, no complementarities), there is no collusion, so the outcome is efficient. In an efficient outcome where bidder a wins item 1 and b wins item 2, then $\pi_a(\{1\}) \geq \pi_b(\{1\})$ and $\pi_b(\{2\}) \geq \pi_a(\{2\})$. Adding the inequalities gives pairwise stability.

In Proposition 1 (two bidders, no complementarities), if license 1 is won by bidder a and license 2 is won by bidder b, $a \neq b$, and the prices are not competitive, then $\pi_a(\{1\}) \geq \pi_a(\{2\})$ and $\pi_b(\{2\}) \geq \pi_b(\{1\})$. Pairwise stability is satisfied by adding the inequalities. Otherwise the outcome is efficient. Proposition 2 is similar to 1 except that additional collusion can occur when the two bidders rank the licenses the same: $\pi_a(\{1\}) \geq \pi_a(\{2\})$ and $\pi_b(\{1\}) \geq \pi_b(\{2\})$. In this case, the bidder a with $\pi_a(\{1\}) - \pi_a(\{2\}) \geq \pi_b(\{1\}) - \pi_b(\{2\})$ wins item 1. Rearranging gives pairwise stability.

Proposition 4 (multiple bidders, no complementarities) uses competitive bidding to weed out the $N-2$ weakest bidders; then the equilibria in Proposition 1 is played. Proposition 5 (multiple bidders, no complementarities) is the same as Proposition 4, with the endgame with the last two bidders using Proposition 2 instead of 3. Proposition 6 (multiple bidders, no complementarities) is competitive, and the outcome is efficient.

In Proposition 7, there are complementarities between the two items, but only the possibility of collusion is a world like Proposition 1 where bidders a and b have high standalone values for one or both items relative to the value of the complementary package. In Proposition 7, there are symmetric regions for each license where bidders are willing to bid on only that one license, so $\pi_a(\{1\}) \geq \pi_a(\{2\})$ and $\pi_b(\{2\}) \geq \pi_b(\{1\})$ without loss of generality. Adding the inequalities gives pairwise stability.

Next consider Engelbrecht-Wiggans and Kahn (2005). In Lemma 1, the outcome is competitive. In Theorems 2 and 3 (two bidders, two items, independent private values), the bidders play strategies where the outcome is only non-competitive when bidders bid on their most preferred item in the first period and split the market if their most preferred items differ, so $\pi_a(\{1\}) \geq \pi_a(\{2\})$ and $\pi_b(\{2\}) \geq$

$\pi_b(\{1\})$. Theorem 4 allows correlated private values for the two items for each bidder; the strategy is the same as Theorem 2. Theorem 6 considers homogeneous goods; pairwise stability in matches automatically applies. □

Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005) do not claim to find all of the symmetric perfect Bayesian equilibria. However, the equilibria they do find are quite natural. In a game with symmetric private values, having a high private value realization for a license tells the bidder little about the valuations of its rivals. There is little to gain from bidding on a subset of licenses that are not the highest private value realizations of the bidder. Because agents have private information, they must signal through bids to find sustainable, implicitly collusive equilibria. This need to signal prevents bidders from using arbitrary punishments to coordinate on arbitrary outcomes, which might occur in a game of perfect information.[32]

All implicitly colluding bidders must win an item for collusion to be successful. The threat of earning almost nothing from a punishment regime of competitive bidding keeps the bidders from entering additional bids.[33] When there are many bidders relative to licenses, the Brusco and Lopomo signalling equilibria require the strong bidders to raise prices to weed out the weak bidders. Also, more bidding is necessary for bidders to sort themselves to licenses if multiple strong bidders initially aim for the same license. So we would not necessarily expect to see very low prices in intimidatory collusive equilibria.[34]

5 The estimator

5.1 Estimator

Following equation (1), we assume that profits take the form:

$$\bar{\pi}_{\beta}(w_a, x_J) + \sum_{j \in J} \xi_j + \sum_{j \in J} \varepsilon_{aj} - \sum_{j \in J} p_j. \tag{4}$$

In this section, we assume that $\bar{\pi}_{\beta}(w_a, x_J)$ can be written as a function of bidder characteristics w_a, package characteristics x_J and a finite dimensional parameter vector β. To make the objective functions more readable, we will sometimes write $\bar{\pi}_{\beta}(a, J) \equiv \bar{\pi}_{\beta}(w_a, x_J)$ for bidder a and package J.

[32] Indeed, Theorem 5 in Engelbrecht-Wiggans and Kahn (2005) allows the distribution of private values to vary across bidder/item pairs. For example, all bidders may know that a has a high private value for item 2 relative to item 1. In that case, there is less need to signal to coordinate and pairwise stability may not occur.

[33] Brusco and Lopomo also mention that complementarities might break implicit collusion. Counterintuitively, it is not the level of complementarities that prevents collusion, but the variability of complementarities across bidders.

[34] An additional concern in simultaneous ascending auctions is the exposure problem, where a bidder fails to secure additional licenses to complete a package and therefore prefers to not to win a license it did win at the end of the auction. Cramton (2006) argues that the price discovery advantages of and the withdrawal options in the FCC's simultaneous ascending auction design mitigate any exposure problem. Pairwise stability in matches will still hold under an exposure problem if total structural profit would not be increased by swapping licenses. Given the exposure problem, pairwise stability holds if the bidders are exposed on the "best of a bad menu" of licenses.

Fox (2007) introduces a semiparametric maximum score estimator for many-to-many matching games with transferable utility.[35] The estimator is semiparametric as no parametric distribution for the unobservables ε_{aj} is imposed. This paper is the first empirical application of the estimator introduced in Fox. The estimator is based on forming the empirical analog of the inequalities in Definition 2. When using only matches data, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$Q^{\text{match}}(\beta) = \frac{2}{L(L-1)} \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} 1[a(i) \neq a(j)] \cdot$$
$$1\left[\bar{\pi}_\beta\left(a(i), J_{a(i)}\right) + \bar{\pi}_\beta\left(a(j), J_{a(j)}\right) \geq \bar{\pi}_\beta\left(a(i), \left(J_{a(i)} \setminus \{i\}\right) \cup \{j\}\right) + \bar{\pi}_\beta\left(a(j), \left(J_{a(j)} \setminus \{j\}\right) \cup \{i\}\right)\right]. \tag{5}$$

The notation $a(i)$ refers to the bidder, a, that won territory i. The objective function $Q^{\text{match}}(\beta)$ considers all combinations of two licenses won by different bidders, $a(i) \neq a(j)$. The inequality is satisfied whenever, for any pair of licenses i and j and bidders $a(i)$ and $a(j)$, the deterministic part of utility is not increased by an exchange of licenses. The license specific unobservables ξ_j enter into both sides of (2) and difference out. Therefore, we do not need to directly estimate these parameters as fixed effects. If an inequality is satisfied, the count or score of correct predictions increases by 1.[36] The estimate $\hat{\beta}$ maximizes the score of correct predictions. The estimator is relatively simple to compute.[37]

One advantage of the maximum score estimator is that it involves only a small number of counterfactual packages. Computing a likelihood would require evaluating all possible packages.[38] Several papers in the collection Cramton, Shoham and Steinberg (2006) explore how even computing a winning bid in an alternative combinatorial auction is an active area of research in computer science. Cramton (2006) argues that a major motivation for using the simultaneous ascending auction over a package-bidding combinatorial auction is the computational challenge in evaluating all packages. Evaluating all possible packages is not a tractable estimation strategy in the C block environment that has more packages than the atoms in the universe.

[35] This estimator may be more appropriately labeled a "maximum rank correlation" estimator (Han, 1987), because of the asymptotic argument discussed below.

[36] The estimator's inequalities include only the deterministic portion of structural profits, $\bar{\pi}_\beta(w_a, x_J)$. Many inequalities will remain unsatisfied, even at the true parameter vector, because of the unobserved realizations of private values ε_{aj}, which also affect matches. Because not all inequalities can be satisfied, changing the score objective to squaring the deviations from deterministic pairwise stability makes the estimator inconsistent. See Sections 3.6–3.9 in Fox (2007) for a discussion of the properties of the private values ε_{aj} (and other forms of disturbances) needed for identification and consistency. Also, Monte Carlo studies in Fox show the estimator works relatively well when the private values are i.i.d. and independent of w_a and x_J.

[37] $Q^{\text{match}}(\beta)$ is a step function and as a result, in a finite sample there will be multiple parameters that maximize it. Any maximizer is a consistent estimator. In practice, reporting a 95% confidence region for each element of β removes this ambiguity. We use the global optimization routine differential evolution to maximize $Q^{\text{match}}(\beta)$ (Storn and Price, 1997). We find that differential evolution is more likely to find the global optimum than simulated annealing.

[38] Presumably, evaluating a likelihood would require computing assuming a particular dynamic game and computing equilibria to it. As private value realizations for all bidders enter the dynamic game, the likelihood would be an integral over one such game for each realization of error terms.

5.2 Structural profit functions

In our application, we let $w_a = \{\text{elig}_a\}$ be the initial (before the auction begins) eligibility of bidder a. Also, let

$$x_J = \left\{ \{\text{pop}_j\}_{j=1}^{J}, \text{complem.}_J \right\}$$

be equal to the population of all licenses in the package J as well as a vector complem.$_J$ of constructable empirical proxies for the complementarities in the package. Our choice of $\bar{\pi}_\beta (w_a, x_J)$ is

$$\bar{\pi}_\beta (w_a, x_J) = \pm 1 \cdot \text{elig}_a \cdot \left(\sum_{j \in J} \text{pop}_j \right) + \beta' \text{complem.}_J$$

The interaction $\text{elig}_a \cdot \left(\sum_{j \in J} \text{pop}_j \right)$ captures the fact in Table 2 that bidders with more initial eligibility won more licenses. The scalar $w_a = \{\text{elig}_a\}$ is our main measure of bidder characteristics, given that Table 2 shows financial measures were uncorrelated with winning a license. The coefficient on $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$ has been normalized to ± 1 because dividing both sides of the inequality in (3) by a positive constant will not change the inequality. The term β'complem.$_J$ provides the total contribution of the several complementarity measures in the vector complem.$_J$. Each element of complem.$_J$ is a nonlinear construction from the characteristics of the underlying licenses in the package J, so complem.$_J$ does not cancel in (3). The parameters β describe the relative importance of each complementarity measure in terms the units of $\text{elig}_a \cdot \left(\sum_{j \in J} \text{pop}_j \right)$.[39] Recall that (4) has license specific unobservables, ξ_j. These unobservables capture the common element to the valuation of licenses, such as the base contribution of population, the fact that spectrum is more scarce in more densely populated territories and the fact that competition from incumbent carriers may be stronger in some territories than others. The ξ_j's have no impact on whether the assignment is pairwise stable, and drop out of the maximum score estimation inequalities.

We choose a simple functional form for $\bar{\pi}_\beta (w_a, x_J)$ in order to demonstrate that a parsimonious model is able to fit the data quite well. A more complicated functional form would have little benefit in terms of the overall fit of the model and would obscure the interpretation of the parameters. We discuss nonparametric identification of the model below.

5.3 Three proxies for potential complementarities

We construct proxies for geographic economies of scope and use them as our primary measure of complementarities. Three alternative measures are used in order to examine the robustness of our results.

[39] Eligibility is the initial eligibility of a bidder, as seen in Table 2. Population is just the number of residents (in the 1990 census) of the license. To aid interpretation, we divide both measures by the population of the continental United States, so that an eligibility or population of 1 corresponds to a true value of 253 million people. By the fractional normalization, the mean population $\sum_{j \in J} \text{pop}_j$ among the 85 winning packages is 0.012 (standard deviation of 0.044), and the mean $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j \right)$ is 0.004 (standard deviation 0.029).

5.3.1 Geographic distance

Our first proxy for geographic scope is based on the geographic distance between pairs of licenses within a package.[40] For a package J in the set L of all licenses, potential complementarities are

$$\text{geocomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\left(\sum_{j \in J, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{ij}^{\delta}} \right)}{\left(\sum_{j \in L, j \neq i} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{ij}^{\delta}} \right)}, \tag{6}$$

where population is measured in fractions of the US total population and distance is measured in kilometers.[41] The distance, dist_{ij}, between licenses i and j is raised to a power $\delta = 4$ to make this measure overweight nearby territories.[42] The measure geocomplem._J proxies for short-distance travel and cost and marketing synergies across nearby territories. Also, geocomplem._J is similar to the well-known gravity equation in international trade. The measure geocomplem._J has the desirable feature that any firm's complementarities cannot decrease by adding licenses to a package.

5.3.2 Two travel measures

Geographic measures of distance may not capture the returns to scope that carriers are concerned about. Mobile phone customers may travel by means other than ground transportation. For example, many business users travel by air between Los Angeles and New York. In fact, the C block bidder NextWave won both the New York and Los Angeles licenses. We have two complementarity proxies based upon travel between two licenses. The first measure, from the 1995 American Travel Survey (ATS), is proportionate to the number of trips longer than 100 km between major cities. All forms of transportation are covered. The downside of this measure is that for privacy reasons the ATS does not provide enough information about rural origin and destinations to tie rural areas to particular mobile phone licenses. Our second measure, from the Airline Origin and Destination Survey for the calendar year 1994, is the projected number of passengers flying between two mobile phone license areas.[43] The drawback of the air travel measure is that it assumes all passengers stay in the mobile phone license

[40] We measure distance between two licenses using the population-weighted centroid of each license. The population-weighted centroid is calculated using a rasterized smoothing procedure using county-level population data from the US Census Bureau.

[41] This geographic complementarity proxy can be motivated as follows. Consider a mobile phone user in a home market i. That mobile phone user potentially wants to use his phone in all other markets. He is more likely to use his phone if there are more people to visit, so his visit rate is increasing in the population of the other license, j. The user is less likely to visit j if j is far from his home market i, so we divide by the distance between i and j. We care about all users equally, so we multiply the representative user in i's travel experience by the population of i.

[42] The choice of $\delta = 4$ is somewhat arbitrary and was chosen to make the clusters of licenses seen in Figure 1 have non-trivial levels of complementarities.

[43] Intermediate stops are not counted for either dataset. For both datasets, geographic information software (GIS) was used to match origins and destinations with mobile phone licenses. For airports, the origin and destination license areas are easy to calculate. For the MSAs (Metropolitan Statistical Areas) used in the ATS, the equivalent C block license area was found using the centroid of the origin or destination MSA. The C block license boundaries for urban areas roughly follow MSAs.

area where their destination airport is located.[44] Both travel measures for a package S are population-weighted means across licenses, and take the form

$$\text{travelcomplem.}_J = \sum_{i \in J} \text{pop}_i \frac{\sum_{j \in J, j \neq i} \text{trips (origin is } i, \text{destination is } j)}{\sum_{j \in L, j \neq i} \text{trips (origin is } i, \text{destination is } j)}. \tag{7}$$

where our ATS measure uses the count of raw trips in the survey, and the air travel count is inflated to approximate the total number of trips during 1994.[45] As with geographic distance, if $J = L$, $\text{travelcomplem.}_J = \sum_{i \in L} \text{pop}_i = 1$. Here again, adding a license to a package cannot take away complementarities between other licenses, so travelcomplem._J only increases as licenses are added to J.[46]

5.4 Consistency and inference

Fox (2007, Theorem 9) proves consistency of the maximum score estimator as $L \to \infty$.[47] Also, he proves (Theorem 10) the estimator converges at the $\sqrt{L}$ rate and is asymptotically normal, although the variance-covariance matrix is cumbersome to estimate.[48] Therefore, we use a resampling procedure, known as subsampling, for inference.[49]

Section 7 of Fox (2007) considers asymptotics as $L \to \infty$.[50] The argument is subtle: if the true number of bidders N expands, then the winning packages of the real 85 winners might change as new competitors enter. Asymptotics where the dependent variable changes in this manner are outside the scope of Fox (2007). Fox presents an alternative asymptotic argument: the observed C block is simply a subset of a very large, aggregately deterministic spectrum auction with many licenses and many bidders. The number of bidders (or winning bidders) N reflects the number of winning packages with recorded data. As N (or L) increases, we collect more data on the outcome of the same spectrum auction.[51]

[44] We effectively code that there are zero potential complementarities between rural licenses for both travel measures.

[45] Our airline passenger measure does not distinguish between origins and destinations, so we simply divide the formula for the complementarity proxy by 2. If all airline travel is round-trips during the same calendar year, this measure should be exactly correct.

[46] For all geographic complementarity proxies, some fraction of the winning packages has a value of 0. For example, 26 out of the 85 winning packages contain only one license in the continental United States.

[47] Han (1987) first proved the consistency in L of maximizers U-statistic (maximum rank correlation) objective functions and Sherman (1993) first derived the asymptotic distribution.

[48] The consistency and asymptotic distribution theorems rely on an intermediate property about the stochastic structure of a matching game. Sections 3.6–3.9 in Fox (2007) discuss this property in more detail and present simulations of its validity.

[49] Subsampling is consistent if the limiting distribution has a continuous $(1-\alpha)$ quantile (Politis, Romano and Wolf, 1999). Fox shows the limiting distribution is normal, which has continuous quantiles.

[50] In auctions of a single item, asymptotics are typically expressed as limits as the number of distinct auctions goes to infinity (Guerre, Perrigne and Vuong, 2000). The economic logic is that the number of statistically independent but otherwise similar economic events becomes large. Section 2.1 discusses why we prefer the C block auction to the AB and DEF blocks auctions. At best, using all three original PCS / mobile phone spectrum auctions would provide only three distinct auctions. Three auctions is not enough for an asymptotic approximation in the number of auctions to provide a good approximation to the true sampling distribution, so some other asymptotic argument must be used.

[51] Any asymptotic argument is just a way of approximating the finite sample distribution of estimators for inference. This is not actually a model of the counterfactual experiment of adding new bidders to a real-life spectrum auction, just as asymptotics

5.5 Nonparametric identification

Fox (2007) proves a sequence of theorems about the nonparametric identification of $\bar{\pi}(w_a, x_J)$. Theorems 4–7 consider the cardinal identification of features of $\bar{\pi}(w_a, x_J)$. The units of profit (dollars) are not identifiable from data on matches. However, because of the transferable utility structure, cardinal features of profits can be identified from qualitative match data.[52] Let $y = \{w_a, x_J\}$. Two variables are said to be complements when $\frac{\partial \bar{\pi}(y)}{\partial y_1 \partial y_2} > 0$, where y_1 and y_2 are two components of y. One theorem shows that the sign of $\frac{\partial \bar{\pi}(y)}{\partial y_1 \partial y_2}$ is identified.[53] Let y_1 and y_2 be characteristics from two different licenses, or one bidder and one license characteristic. Let y_3 and y_4 be two characteristics from different entities as well.[54] Another theorem shows the ratio $\frac{\partial \bar{\pi}(y)}{\partial y_1 \partial y_2} / \frac{\partial \bar{\pi}(y)}{\partial y_3 \partial y_4}$ is identified.[55] Therefore, we can cardinally identify the relative importance of sorting on different types of characteristics, such as bidder eligibility, license population and geographic complementarities.[56]

6 Estimates of profit functions

Table 5 lists estimates of β in structural profit function, (4), from the matching maximum score estimator.[57] The numbers in parentheses are 95% confidence intervals from subsampling.[58]

6.1 Main estimates

Columns 1 and 2 report estimates using the pairwise stability inequalities in the objective function with matches only (5). As in (4), because matches are qualitative outcomes, we normalize the coefficient on

in the number of US states is not modeled as an annexation of Canada.

[52]To express the identifiable features as derivatives, these theorems work with only continuous characteristics.

[53]Two very different functions may have the same $\frac{\partial \bar{\pi}(y)}{\partial y_1 \partial y_2}$. For both $-(y_1 - y_2)^2$ and $2y_1y_2$, $\frac{\partial \bar{\pi}(y)}{\partial y_1 \partial y_2} = 2$. Theorems 1–3 of Fox (2007) prove that $\bar{\pi}(w_a, x_J)$ is nonparametrically ordinally identified. We can identify whether structural profit $\bar{\pi}(w_a, x_J)$ is higher or lower at any two combinations of bidder and package characteristics. We can distinguish between $-(y_1 - y_2)^2$ and $2y_1y_2$.

[54]The identities of the entities in the two pairs can be the same or not.

[55]The relevant Theorem 7 alone requires data on bidders that win no licenses to provide a baseline of zero profit. We have data on losers, but do not need to use it because our parametric functional form allows us to compare various matches without a reference point.

[56]Our measures of geographic complementarities are actually package and not license characteristics, and fall under Theorems 4 and 5 in Fox (2007).

[57]The objective function was numerically maximized using the global optimization algorithm known as differential evolution (Storn and Price, 1997). More than ten runs were performed for all specifications. The reported point estimates are the best found maxima, although care was taken to ensure that runner up computed maxima were qualitatively the same as the best found values.

[58]We use 150 replications. For each replication, we use subsets equal to a random sample (without replacement) of 25 of the 85 packages. As a winning package is a dependent variable, it may be statistically cleaner to sample by the number of licenses. In unreported results, we take subsets of the data by using only the inequalities corresponding to 120 out of the 480 licenses in the United States. For each license, we evaluate the structural profit functions using the full winning package, whether all of the package's licenses are among the subset of 120 or not, to avoid using economically false restrictions in estimation. The confidence regions from drawing random licenses are similar to the regions found by drawing packages. Subsampling has not been extended to allow for spatial autocorrelation, so we do not adjust for such correlation, although see Politis and Romano (1993) for related results on the bootstrap.

$\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ to be ±1. Computationally, we estimate the other parameters β separately for the $+1$ and -1 normalizations and pick the vector with the highest number of satisfied inequalities. The results in columns 1 and 2 of Table 5 show that $+1$ is the correct point estimate. This fits the fact in Figure 2 that bidders with more initial eligibility win packages with more total population. Bidders with higher values win more.

Column 1 includes only one proxy for geographic complementarities: geographic distance, (6). The coefficient of $\beta_{\text{geo.}} = 0.69$ means, at the furthest extrapolation, that if one bidder with the maximum eligibility of 1 were to win the entire United States (population of 1), then the also maximized complementarities (value of $1\cdot\beta_{\text{geo.}}$) would give a total package value of $1\cdot1+0.69\cdot1 = 1.69$. The value from complementarities corresponds to $0.69/1.69 = 41\%$ of the total package value. This extrapolation is not representative of the variation in the data. Across the 85 winning packages, the standard deviation of $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ is 0.03 and the standard deviation of geocomplem._J is 0.024. The means of the two explanatory variables, $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ and geocomplem._J , are also nearly identical. Given the similar means and standard deviations, the coefficient estimate $\beta_{\text{geo.}} = 0.69$ implies that variation in the geographic location of licenses, geocomplem._J, is roughly $0.69/1 = 69\%$ as important in explaining the sorting pattern we see as variation in the match between bidders with more eligibility and packages with more population, $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$.[59]

Column 2 adds the two travel based complementarity measures to the specification. Now, not only do we measure the relative importance of $\text{elig}_a\left(\sum_{j\in J}\text{pop}_j\right)$ and complementarities in sorting, we see which measure of complementarities is most important. Total trips using all forms of travel has a coefficient of 0.32, while the coefficient on geocomplem._J, 0.65, remains roughly the same as in column 1. One interpretation is that the geographic pattern of clustering reflects more than just customers wishing to make calls while traveling. Other forms of complementarities include marketing and cost-of-service synergies. The second travel measure, air travel, has a positive coefficient of 0.234, and is statistically insignificant. The point estimate of 0.234 does show a important role for air travel synergies.[60] The standard deviation of air travel complementarities 0.017, which is only a little smaller than say geographic distance complementarities, at 0.024. Given the similar standard deviations, the point estimate shows air travel is important but not as important as geographic distance or the composite measure of travel.

Column 2 is our preferred, final set of estimates. The estimates use the pairwise stability in matches

[59] Any deterministic measure such as complementarities can be explained away by bidders having correlated private values for nearby licenses. However, the evidence suggests that the largest winners were not local businessmen with special attachments to particular, large regions. Many of the the largest winners, such as NextWave, Omnipoint and GWI/MetroPCS, won small clusters in many regions of the country. MetroPCS has its headquarters in Dallas, but won licenses only near Atlanta, Miami and San Francisco. DCR/Pocket won licenses stretching from Detroit to Dallas, an oddly-shaped region to be a regional specialist in. PCS2000 won mainly a cluster of licenses in the West, but had its headquarters far away in Puerto Rico. Further, Table 1 shows that the typical winning bidder bid on 39 license and won 5, for a ratio of around 8. The largest winners bid on many more licenses than that.

[60] The point estimate on air travel is a lower bound on the complementarities from air travel, as air travel also appears in the ATS survey and is being double counted. Roughly 75% of trips in the ATS are by car, but the fraction by air increases with distance.

inequalities. As previously argued, the estimator is consistent under forms of intimidation and demand reduction.[61]

6.2 Estimators not consistent under intimidation and demand reduction

This section explores two sets of alternative estimators that are inconsistent under intimidation and demand reduction. We show that alternative estimators that abstract from these possibilities generate bizarre estimates of bidder valuations.

6.2.1 Estimates with forced transfers of licenses

Columns 3 and 4 of Table 5 consider a variant of the maximum score estimator where bidder a adds a license j to its package J without swapping the license for another. License j was won in the data by bidder $a(j)$, so the inequality involves an increase in the number of a's licenses by 1 and a decrease in the number of $a(j)$'s licenses by 1. Let B be the set of 85 winning bidders. The estimator is any parameter value that maximizes

$$Q^{\text{addmatch}}(\beta) = \sum_{a=1}^{B} \sum_{j=1}^{L} 1[a \neq a(j)] \cdot 1\left[\bar{\pi}_\beta(a, J_a) + \bar{\pi}_\beta\left(a(j), J_{a(j)}\right) \geq \bar{\pi}_\beta(a, J_a \cup \{j\}) + \bar{\pi}_\beta\left(a(j), J_{a(j)} \setminus \{j\}\right)\right],$$

where $J_{a(j)}$ is the complete package won by the bidder that won license j. The estimator imposes the condition that a did not increase its package by one license because the total structural profit of a and b would go down from doing so: it would be less efficient. If a has reduced its demand, it may not compete for an extra license to reduce its payment to the seller rather than because of efficiency reasons. Therefore, maximizing $Q^{\text{addmatch}}(\beta)$ produces an inconsistent estimator under demand reduction. A similar argument applies to the intimidatory equilibria in Brusco and Lopomo (2002).

Columns 3 and 4 report a priori unreasonable estimates. First, the coefficient on $\text{elig}_a\left(\sum_{j \in J} \text{pop}_j\right)$ is the wrong sign: negative. Figure 2 shows that bidders with higher values (proxied by initial eligibility) win packages with greater populations which is inconsistent with this conclusion. Also, the coefficients on complementarities are implausibly large. For example, in column 3 the point estimate of 44.9 shows the structural profit from complementarities is 45 times the (negative) profit from winning an equivalent amount of population (times eligibility).

6.2.2 Estimates with prices

Columns 5 and 6 of Table 5 report estimates using both matches and prices data. The maximum score objective function is based on Definition 1, pairwise stability with matches and prices. When using

[61] Table 5 lists the percentage of satisfied inequalities at the point estimates, which is a measure of statistical fit. 95% of the inequalities are satisfied. Simply, vertical differences in bidder valuations for licenses and complementarities can explain most of the sorting patterns at the pair of licenses level.

price data, the estimator $\hat{\beta}$ is any vector that maximizes the objective function

$$Q^{\text{price}}(\beta) = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L} 1\left[a(i) \neq a(j)\right] \cdot 1\left[\bar{\pi}_\beta\left(a(i), J_{a(i)}\right) - \bar{\pi}_\beta\left(a(i), \left(J_{a(i)} \setminus \{i\}\right) \cup \{j\}\right) \geq p_i - p_j\right], \tag{8}$$

where p_i is the final, closing price of license i. Akkus and Hortacsu (2006) were the first to use the estimator with prices and perform a Monte Carlo study. In all of our Monte Carlo experiments (see the appendix for some) with i.i.d. private value terms ε_{aj}, the estimator performs extremely well.[62]

Section 4.2 discusses how pairwise stability in both prices and matches does not necessarily hold under intimidatory equilibria. Therefore, if prices are determined by an intimidatory equilibria, then the estimator with prices will likely be inconsistent. In columns 5 and 6, we have included price, measured in *trillions* of dollars. The coefficient on price is normalized to -1.[63] Taken literally, the coefficient on 0.372 on $\text{elig}_a\left(\sum_{j \in J} \text{pop}_j\right)$ in column 5 says that the value of a bidder with eligibility equal to the entire US's population winning the entire US is \$372 billion (although it is not statistically distinct from zero). Likewise, the value of complementarities from a nationwide license is \$418 billion. These estimates are absurdly high, given that the bids for the C block totalled \$10.1 billion.[64]

How is the model fitting the outcome data? Only the ratio of two parameters that enter structural payoffs linearly, say $\beta_{\text{geo}}/\beta_{\text{price}}$, is identified from an inequality. A high dollar value for non-price package and bidder characteristics is equivalent to saying the estimated coefficient on license price β_{price} would be economically quite small in magnitude if some other characteristic's coefficient was normalized to ± 1. A small coefficient on price is consistent with the finding in Section 2.4 that population and population density, characteristics mostly subsumed into ξ_j, explain most price variation.[65] As we discussed in Section 2.5, Ausubel, Cramton, McAfee and McMillan (1997) included measures of the runner up bidder's potential complementarities in a license level price regression, and found a nonzero but economically small coefficient. Together, the estimates from (8) and the price regressions suggest that prices may not clear the market in the sense of sorting price taking bidders to different

[62] In the Monte Carlos, we draw the error terms and then compute the fake data prices using a linear program (Roth and Sotomayor, 1990). Empirically, there is no need to instrument for the endogenous prices because of the simultaneous determination of prices and matches in a tatonnement mechanism. The dependent variable in a maximum score inequality enters the inequality. In (8), the dependent variable in an inequality is the collection $\left\{J_{a(i)}, J_{a(j)}, p_i, p_j\right\}$, even though every element of $J_{a(j)}$ does not appear in the inequality. There is no analog to regressing a discrete choice on endogenous prices, unless one worries about omitted variable bias from ξ_j. See Berry, Levinsohn and Pakes (1995) for one approach to dealing with the correlation of p_j and ξ_j.

[63] We estimate the model for a coefficient of $+1$ and find the objective function is lower.

[64] The annual revenue for the wireless phone industry in 2006, with nine or more active licenses per territory (not just the C block), was \$113 billion. It is unlikely that bidders in 1996 felt the C block had 7–8 times the structural *profit* potential as the *revenue* from all blocks combined 10 years later.

[65] In Definition 1 and the objective function (8), ξ_j does not difference out of the inequality, like it does in Definition 2. While the specifications in columns 5 and 6 in Table 5 emphasize comparisons with the columns that do not use price data, we have also estimated specifications including population and population density. The point estimates on the covariates that affect the efficiency of alternative assignments of licenses to bidders are then \$886 billion for winning the entire US's population for $\text{elig}_a\left(\sum_{j \in J} \text{pop}_j\right)$ and \$743 billion for the geographic complementarities geocomplem._J for winning the entire US. These estimates dramatically reinforce the finding that the coefficient β_{price} is to estimated to be economically small.

packages in a competitive market. Pairwise stability in prices and matches, Definition 1, may not be satisfied.

We have suggested intimidation as a non-competitive behavior that is possible in the simultaneous ascending auction. Previous descriptive empirical evidence and our own anecdotes about jump bidding and actual retaliation suggest, at least, that intimidation through signalling is empirically common (Cramton and Schwartz, 2000). Our results suggest that the link between prices and valuations have been contaminated by jump bids and retaliation and that an estimator, as in column 2, which is robust to these possibilities is preferable.

7 Policy implications

In this section, we discuss the policy implications of our estimates, including efficiency and the design of future auctions.

7.1 Actual and counterfactual surplus

In this section, we compare the surplus from the observed assignment of licenses to several counterfactual license assignments. The results from the previous section suggest that various measures of complementarities are important determinants of bidder valuations. However, the auction allocated licenses to 85 different bidders, which suggests that an efficiency improvement is possible by grouping licenses into larger winning packages. Furthermore, our earlier results suggest that demand reduction and intimidatory collusion may be present in the auction, which exacerbates this inefficiency. For a given assignment of licences, Table 6 reports the value of $\sum_{a \in B} \bar{\pi}_{\beta}(a, J_a)$. We ignore the license specific unobservables ξ_j since these terms do not interact with bidder characteristics. Hence, the assignment of licenses to different bidders does not influence the surplus from ξ_j. We also do not consider the private value ε_{aj} terms.[66] Identifying the distribution of private values is difficult because imposing a specific model of collusion or intimidation would be needed to know what area of the tail winning valuations (order statistics) are drawn from. When intimidatory collusion under symmetric private values and demand reduction take place, winners focus on only the licenses with high private value realizations.[67]

It is easiest to look at the last row of Table 6 first. The last row considers the largest winner (and bidder with the highest initial eligibility), NextWave, winning all 480 licenses in the continental United States. NextWave was initially eligible for 176 million people, or 0.697 of the 1990 population.[68]

[66] If all bidders have symmetric bidder and license specific private value distributions, auctioning smaller licenses would maximize the values of the private values. In Section 2.3, we argued that we believe these private values have a relatively small distribution as most winning bidders were national bidders.

[67] Computationally, in a qualitative-outcome (like a match) model, estimating the distribution of error terms usually involves a likelihood. A likelihood involves comparing all possible winning packages, and there are more potential packages than atoms in the universe. Also, the distribution of private values is not identified under the minimal assumptions needed for the consistency of maximum score estimators.

[68] Under the auction rules, NextWave would have been ineligible to bid on a national license unless it raised its initial

Therefore, the contribution to total value from NextWave's differential use for licenses is 0.697, or around 0.7. For the three geographic complementarity proxies, NextWave winning all licenses would maximize these, at a value of 1. So the total differential value (excluding the ξ_j's) of a nationwide license is $1 \cdot 0.697 + \beta_1 \cdot 1 + \beta_2 \cdot 1 + \beta_3 \cdot 1$, where the three β's are the complementarity parameters estimated in column 2 of Table 5. The total value of a nationwide license is then 1.91.[69]

Now consider the other four efficiency evaluations. The first row considers the actual assignment of bidders to licenses in the C block auction. The total surplus generated by the C block is 0.784, quite a bit less than the 1.91 from the nationwide license. The terms in all four columns are considerably smaller than in the bottom column, suggesting that the C block failed to maximize the potential benefits from complementarities. The lost surplus from complementarities due to air travel and ATS trips is particularly large compared to a single nationwide license.

The second row considers an extreme where all 480 licenses are won by separate bidders. There can be no across-license complementarities. We impose that BTA licenses (those auctioned in the C block) are the lowest level of disaggregation possible. There are 255 C block bidders (losers and winners). We assortatively match bidders to licenses by initial eligibility for bidders and population for bidders, so that NextWave wins New York, for example. For the $480 - 255 = 225$ licenses with the smallest populations, we say they are won by bidders with the lowest (255th) level of initial eligibility. The results show that the contribution from the $\text{elig}_a \left(\sum_{j \in J} \text{pop}_j\right)$ term is 0.171, smaller than the actual assignment's value of 0.345 by about half. This reflects bidders with low valuations winning licenses.[70]

The third row considers grouping the 480 BTA licenses into 47 packages reflecting the 47 Major Trading Areas (MTAs) in the continental United States used for the 1995 AB spectrum auction. No BTA belongs to more than one MTA. The MTAs are natural groupings centered around large metropolitan areas, but including lots of rural territory as well. MTAs do not correspond exactly to American states. Again, we assortatively match winning bidders to licenses based on initial eligibility and population, so again NextWave wins New York. However, in the C block auction NextWave won New York and a lot more, so here the contribution from differential bidder values is quite low, at 0.182. However, the design of the MTA boundaries ensures that most local, geographic distance complementarities are captured. The measure of geographic distance complementarities rises from 0.47 to 0.72. On the other hand, the MTAs are only local areas, and so a great deal of travel between regions occurs across MTAs. The values of the travel geographic complementarity measures are very small under the MTA scenario. The total value of this assignment is 0.72, not too much lower than the outcome of the actual C block.

The fourth row considers splitting the United States into four large regions: the northeast, midwest,

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eligibility. In our empirical work, eligibility is not used to define a budget set, as it is in the auction rules. Rather, we use initial eligibility as an observed proxy for bidder heterogeneity in valuations. Even if NextWave changed its initial eligibility, its true structural profit function would remain the same.

[69] Our functional forms are used to measure the efficiency of the counterfactuals. However, much of the increase in geocomplem._J, (6), arises from combining nearby or even adjacent licenses in the same package. This is especially true as we set $\delta = 4$ in (6) to emphasize nearby licenses in geocomplem._J. Nearby licenses are observed in the winning packages, so estimating the value of a nationwide package uses less out-of-sample extrapolation than may be apparent at first glance.

[70] Many small licenses may maximize the benefits from i.i.d. bidder and license specific private values.

south and west. We assign each of the 47 MTAs to one of these groupings. The midwest is roughly from Pittsburgh to Wichita, and Washington, DC is in the north. We take the four largest winners by initial eligibility and assortatively match them to the four regions by population. NextWave's package is the midwest; it is still slightly smaller in population than the package NextWave won in the C block. The fourth row shows that the contribution from differential bidder valuations is now higher, the measure of geographic distance complementarities is close to 1, and the two travel measures are about twice as high as the C block values. Thus, a system of four large regions doubles the value from complementarities compared to the C block, and significantly raises the amount of the US population won by high-value bidders. The United States is much bigger than a typical Western European nation; auctioning four licenses is workable plan that captures a large fraction of the maximum possible value, 1.42 out of 1.91.[71]

7.2 Policy implications for bidder anonymity

In 2006, the FCC requested comments on a proposed policy change to make the bidder identities of submitted bids anonymous. The intention of this rule change is to limit intimidation and signalling. A previous draft of this paper addressed one mechanism of signalling other bidders (jump bids) more explicitly. Here, our policy counterfactuals suggest that the simultaneous ascending auction produced inefficiently small winning packages. If bidder anonymity is one way of reducing the scope of intimidation, then it may make the final assignment of licenses to bidders more efficient.

7.3 Competitive scale-reducing economic forces

Intimidation and demand reduction reduce the size of winning packages and make the resulting mobile phone industry lack true national players. At least three other economic forces that are compatible with competitive bidding work in the same direction. First, bidders may have budget constraints, so that financial constraints from outside of the auction make the auction outcome inefficient.[72] Second, bidders may run down eligibility by focusing on a smaller license and be unable to switch to a license with a larger population once the price of the smaller license becomes too expensive. Path dependence may lock a bidder into considering only substitute licenses with relatively small populations.[73] Third, the FCC's rules prevented one bidder from winning more than 98 licenses in the C and F auctions. Only the largest bidder, NextWave, was anywhere close to bumping up against this constraint.

The previous descriptive literature and our bidding anecdotes in Section 2.6 show that bid signalling did go on during the C block auction (Cramton and Schwartz, 2000). However, measuring the extent

[71]By efficiency, we mean the total structural profit of bidders. Bajari, Fox and Ryan (2007) use demand estimation to measure the willingness of consumers to pay for larger coverage areas.

[72]Most auction estimators are inconsistent if agents have budget constraints. Likewise, our pairwise stability in matches condition may not hold if agents have budget constraints. However, to some degree the estimator reflects the spirit of budget constraints: identification uses data on what items each bidder won, rather than how many items each bidder won.

[73]Path dependence does not invalidate our estimator if the resulting assignment is pairwise stable.

or effectiveness of signalling seems difficult when these other factors operate in the same direction. We note all three of the competitive reasons for inefficiently small winning packages are consistent with larger licenses raising efficiency.

8 Conclusions

We measure the efficiency of the outcome of a FCC spectrum auction using a structural model of bidder valuations. A spectrum auction is a complex dynamic game, with many bidders and many items for sale. There is no one standard model of the equilibria to this dynamic game. In particular, the simultaneous ascending auction is potentially susceptible to demand reduction as well as intimidatory collusion. These phenomena may result in winning packages that are inefficiently small, as bidders split the market to coordinate on paying less to the seller.

Following the spirit of Haile and Tamer (2003), our approach to estimation uses necessary conditions that hold across a set of possible models. We use the condition known as pairwise stability in matches: the sum of structural profit functions from two winning bidders must not be increased by swapping licenses. Demand reduction takes place when bidders exploit the decreasing returns to scale of rivals to split the market. We show that pairwise stability holds under straightforward bidding after bidders have unilaterally reduced their demand. Intimidatory collusion under symmetric private values is sustained by bid signalling and threats of retaliation by reverting to straightforward bidding. We reinterpret existing theorems about collusion under private values to prove that pairwise stability hods under many perfect Bayesian equilibria, including those with complementarities.

We employ a matching maximum score estimator, which maximizes the number of inequalities that satisfy pairwise stability. This is the first empirical application of the estimator. While our application focuses on a spectrum auction, the pairwise stability condition behind our estimator is applicable to other multiple unit auctions.

There are more potential packages than the atoms in the universe in the C block. The estimator is computationally simple as it avoids evaluating all possible counterfactual packages, which would be needed for a likelihood based approach. Also, the estimator controls for additive license specific unobservables. To our knowledge, we are the first to estimate valuations using pairwise stability, which uses data on only the matches between bidders and licenses, not the closing prices. Indeed, we show that two alternative maximum score estimators, including one with prices, are not consistent under intimidation and demand reduction. These estimators produce bizarre estimates using the C block data.

Our estimates empirically validate the FCC's focus on complementarities when designing the mechanism for allocating radio spectrum. Also, the spectrum auction itself produces a much higher surplus than awarding licenses through the FCC's prior practices, such as lotteries. However, we find that the final allocation of licenses was inefficient. Total surplus is nearly doubled by awarding four large, regional licenses to the four highest-value bidders. A nationwide license would capture even more of the total surplus. To some degree, our findings validate the European approach of offering nationwide licenses and hence capturing all geographic complementarities.

A Monte Carlo for estimator with both matches and price data

Fox (2007) presents Monte Carlo studies showing that the performance of the matches-only maximum score estimator is pretty good. However, for a small number of bidders and licenses and a high variance of the error term, the estimator uses data only on matches can have high bias and root mean squared error (RMSE) in a finite sample, as random noise from the ε_{aj} terms dominates the matching, leaving little signal in the sorting pattern seen in the data. Like similar results in Akkus and Hortacsu (2006), Table 7 reports results from a Monte Carlo study from a one-to-one, two-sided matching market. Each bidder a matches to at most one license j, and the payoff of a bidder is $\bar{\pi}_\beta(aj) + \varepsilon_{aj} = x_{1,a}x_{1,j} + \beta x_{2,a}x_{2,j}$. There are two characteristics for bidders and two for licenses, with characteristics for each side distributed as a bivariate normal with means $(10,10)$, variances $(1,1)$ and covariance 0.5. The errors are i.i.d. normal with standard deviations listed in the table. For each auction we draw observable characteristics and unobservable error terms and compute an equilibrium assignment and vector of prices using the primal and dual linear programs for two-sided matching (Koopmans and Beckmann, 1957; Shapley and Shubik, 1972). The true β is 1.5. The example is chosen to make using only matches look bad: there is not much signal about $\bar{\pi}_\beta$ in the sorting patterns if the realized matches are visually plotted in characteristic space, especially in the second half of the table where the standard deviation of ε_{aj} is five times higher than in the upper part of the table. Note that for the C block the map in Figure 1 shows that there are clear sorting patterns; this Monte Carlo study makes using match data bad to show the potential advantages of using price data. The finite sample bias and RMSE are always much lower with continuous transfer data, even though the data on matches alone are uninformative. For all four cases the bias is small on an absolute scale for small samples, and for three of the four cases the RMSE is low compared to the true value of 1.5.

Table 7 shows a major advantage of using price data: the finite sample performance is much better if prices are generated from a tatonnement process. There are several advantages to using only match data, even if the prices are generated by a tatonnement process. This first is transparency: there is only one type of dependent variable, so inferring parameters from the US map of winning bidders is straightforward. With two types of dependent variables, it is not as clear where identifications arises from. The second is robustness. In this paper, we review models where prices are not generated by a tatonnement process, but the matches are still robust to pairwise swaps.

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Figure 1: Map of the licenses won by the top 12 winning bidders and bidders who won only one license

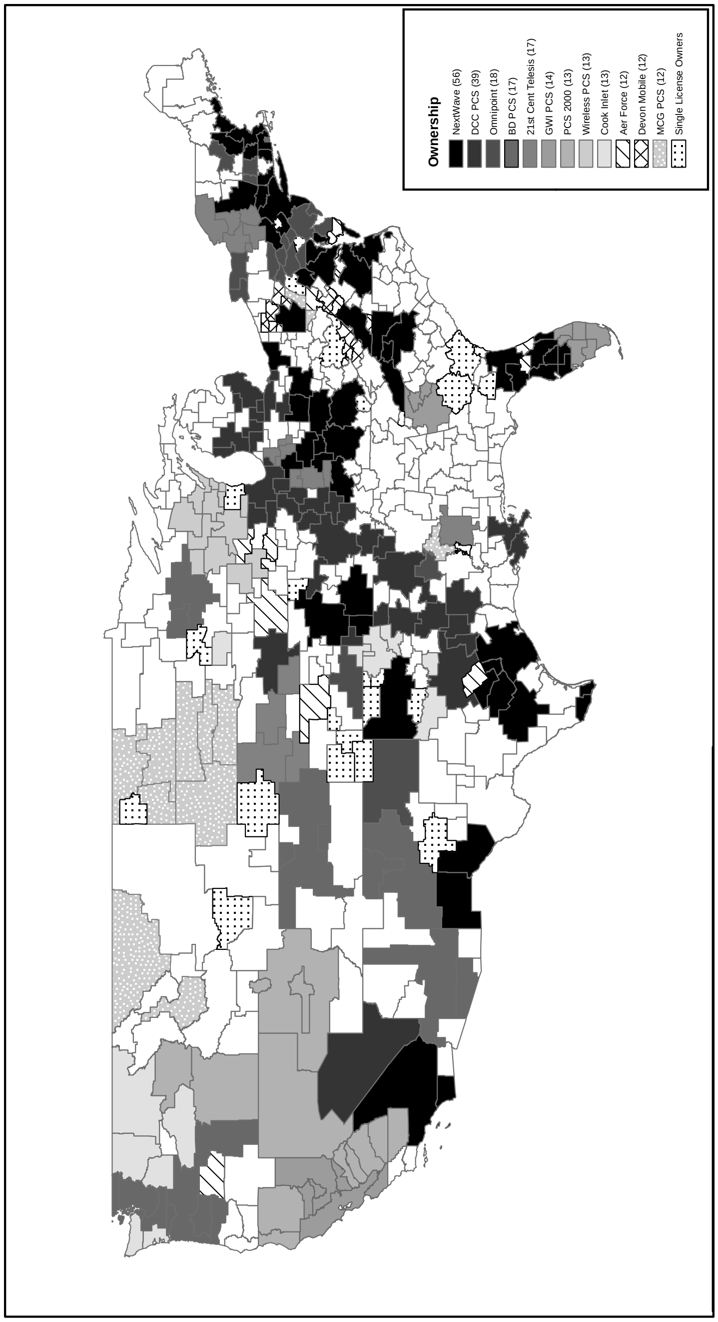

Figure 2: Log of a winning package's population by the log of the winning bidder's initial eligibility

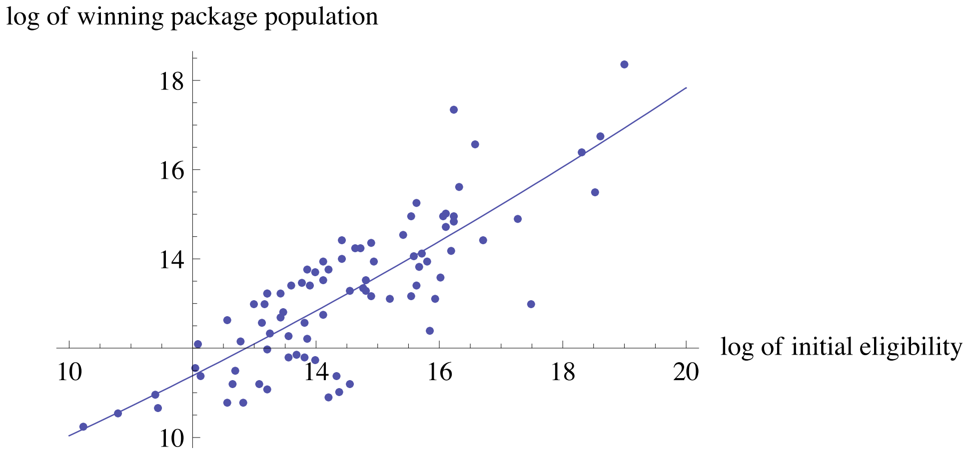

Figure 3: The Number of Jump Bids per Round

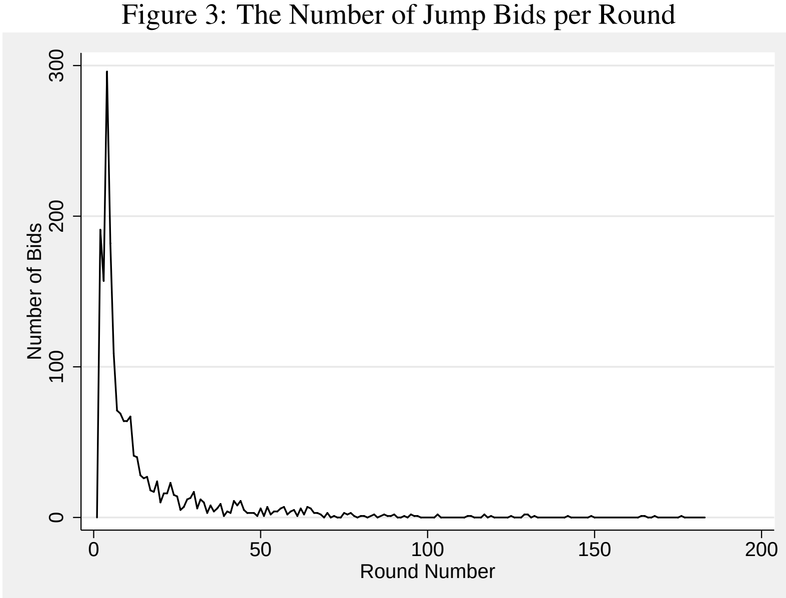

Table 1: Characteristics of winners and non-winners of packages in the continental United States

	Winners		non-Winners	
Characteristic	Mean	Stand. dev.	Mean	Stand. dev.
initial eligibility (millions of residents)	9.77	27.2	5.15	18.5
assets ($ millions)	13.1	21.8	12.3	18.8
revenues ($ millions)	40.7	67.8	39.9	72.3
# of licenses won	5.3	7.1	0	0
# of licenses ever bid on	38.5	70.6	14.8	44.2
# of bidders	85		170	

Table 2: Total closing prices and population of the 85 winning packages

Characteristic	Mean	Standard deviation	Min	Max
Total price ($millions)	116.2	496.1	0.102	4,201
Total population in 1994 (millions)	2.91	10.93	0.027	93.8

Table 3: Payoffs for two bidder examples of demand reduction

	Bidder a	Bidder b, case 1	Bidder b, case 2
License 1	$\pi_{a1} \geq \pi_{a2}$	$\pi_{b1} \leq \pi_{a1}, \pi_{b1} \leq \pi_{b2}$	$\pi_{b1} \leq \pi_{a1}, \pi_{b1} \geq \pi_{b2}$
License 2	$\pi_{a2} \leq \pi_{a1}$	$\pi_{b2} \leq \pi_{a2}, \pi_{b2} \geq \pi_{b1}$	$\pi_{b2} \leq \pi_{a2}, \pi_{b2} \leq \pi_{b1}$
Both 1 & 2	$\pi_{a12} = \pi_{a1} + \pi_{a2}$	$\pi_{b12} = \max\{\pi_{b1}, \pi_{b2}\}$	$\pi_{b12} = \max\{\pi_{b1}, \pi_{b2}\}$

Table 4: Winning package population weighted means of geographic complementarity proxies

Characteristic	Mean	Standard Deviation	Min	Max
Population / distance two markets in a package	0.0055	0.00232	0	0.198
Trips between markets in a package in the American Travel Survey	0.0032	0.0201	0	0.150
Total trips between airports in markets in a package (thousands)	0.0023	0.0166	0	0.182

The sample is the 85 winning packages in the continental United States. The formulas for these measures are equations (6) and (7).

Table 5: Maximum score estimates of profit function parameters

Characteristic	(1)	(2)	(3)	(4)	(5)	(6)
Type of inequalities	Swaps of 1 license each		Transfer of 1 license		Swaps of 1 license w/ prices	
Consistent under demand reduction & intimidation?	Y	Y	N	N	N	N
Population * bidder eligibility	+1	+1	-1	-1	0.372	0.357
	superconsistent				(-0.143,0.476)	(-0.156,0.446)
Population / distance two markets in a package	0.687 (0.511,0.917)	0.654 (0.464,0.826)	44.9	131 (166,193)	0.418 (0.133,0.569)	0.364 (0.047,0.465)
Trips between markets in a package in the American Travel Survey		0.321 (0.138,0.460)		-5.52 (-17.4,-8.25)		0.126 (-0.133,0.180)
Total trips between airports in markets in a package (thousands)		0.234 (-0.081,0.331)		-0.640 (-69.4,-13.8)		0.201 (0.069,0.353)
Price (in *trillions*)					-1	-1
					superconsistent	
# possible inequalities	111,192		40,320		222,384	
% inequalities correct	0.944	0.950	0.930	0.935	0.911	0.918

Eligibility, population and all three complementarity proxies range from 0 to 1. These estimates include licenses only from the continental United States. The maximum score / maximum rank correlation estimators are described in the text. The parentheses are 95% confidence intervals computed using subsampling. Subsampling uses 150 replications, 25 packages per replication and a convergence rate of $\sqrt{\#\text{packages}}$, as found by Sherman (1993). For each 25 packages, we use only the inequalities where all licenses are from the sampled packages. Subsampled confidence regions are not necessarily symmetric around the point estimate. Parameters that can take on only a finite number of values (here ± 1) converge at an arbitrarily fast rate; they are superconsistent.

Table 6: Counterfactual (differential) efficiency from five assignments

Assignment	$\text{elig}_a \left(\sum_{j \in J} \text{pop}_j\right)$	Geographic distance	Air travel	ATS trips	Total
C block: 85 winning packages	$1 \cdot 0.345 =$ 0.345	$0.645 \cdot 0.470 =$ 0.307	$0.234 \cdot 0.197 =$ 0.046	$0.321 \cdot 0.268 =$ 0.086	0.784
All 480 licenses won by different bidders	$1 \cdot 0.171 =$ 0.171	$0.645 \cdot 0 =$ 0	$0.234 \cdot 0 =$ 0	$0.321 \cdot 0 =$ 0	0.171
Each 47 MTAs separate package	$1 \cdot 0.182 =$ 0.182	$0.645 \cdot 0.722 =$ 0.472	$0.234 \cdot 0.037 =$ 0.009	$0.321 \cdot 0.168 =$ 0.054	0.717
Four large, regional licenses (top four of the 85 actual winners win)	$1 \cdot 0.513 =$ 0.513	$0.645 \cdot 0.964 =$ 0.631	$0.234 \cdot 0.379 =$ 0.088	$0.321 \cdot 0.586 =$ 0.188	1.42
Nationwide license for entire United States (NextWave wins)	$1 \cdot 0.697 =$ 0.697	$0.645 \cdot 1 =$ 0.645	$0.234 \cdot 1 =$ 0.234	$0.321 \cdot 1 =$ 0.321	1.91

Eligibility, population and all three complementarity proxies range from 0 to 1. These counterfactuals use the parameter estimates from column 2 of Table 5. Only licenses in the continental United States are considered. For the 47 MTAs in the continental United States as well as the four large regions, the top winners in the actual auction are assortatively matched to the counterfactual packages in order of population. For example, NextWave always wins the package with the highest population.

Table 7: Maximum score Monte Carlo: comparing using data on only matches to data on both matches and prices under tatonnement assumptions with noise dominating matches, true value is 1.5

				Matches		Matches + Prices	
# bidders	# licenses per auction	# spectrum auctions	error std. dev.	bias	RMSE	bias	RMSE
30	30	1	1	0.587	1.93	0.005	0.03
10	10	10	1	0.330	1.05	0.009	0.07
30	30	1	5	1.22	4.22	0.02	0.09
10	10	10	5	1.69	7.36	-0.02	0.446