Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders*

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First Version: November 2008
This Version: March 2009

Abstract

In this paper we study the timber sales auctions in Oregon. We estimate a structual model of entry and bidding within the affiliated private value paradigm and with heterogeneous bidders. We find that the hauling distance plays a significant role in bidders' entry and bidding decisions, and we quantify the extent to which the potential bidders' private values and entry costs are affiliated. The structural estimates are then used to conduct counterfactual analyses to address policy related issues. In particular, we quantify the effects of reserve price, affiliation, and merger on the end auction outcomes.

*We thank Luke Froeb for helpful discussions on the merger issues, and seminar participants at Vanderbilt for comments and discussions.

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1 Introduction

Auctions have long been used as a means for price determination under a competitive setting and an incomplete information environment. Auction theory developed within the game-theoretic framework with incomplete information (Harsanyi (1967/1968)) not only helps us understand how auctions work, but also offers insight in analyzing many other economic problems. A celebrated result in auction theory is Vickrey's (1961) revenue equivalence theorem, which postulates that all the four auction formats (first-price sealed-bid, second-price sealed bid, English, and Dutch auctions) generate the same average revenue for the seller with symmetric, independent, and risk-neutral bidders.

The revenue equivalence theorem is a powerful result that offers insight into how auction mechanisms work, and also raises important questions as to how this powerful result can be affected when the standard assumptions are relaxed. A large part of the auction theory has focused on answering these questions. Milgrom and Weber (1982) give revenue ranking with symmetric and affiliated bidders in which the English auction generates highest revenue among the four formats and the second-price auction ranks next; they also establish that with symmetric, affiliated, and risk-averse bidders who have constant absolute risk aversion, the English auction can generate at least as high revenue as the second-price auction. Myerson (1981) derives the optimal auctions with asymmetric bidders, and Maskin and Riley (1984) consider the case with risk-averse bidders. Levin and Smith (1994) extend the revenue equivalence and ranking results from Vickrey (1961) and Milgrom and Weber (1982) to the case with symmetric bidders (independent or affiliated) using mixed entry strategies.

Using timber sale auctions organized by the Oregon Department of Forestry (ODF), this paper attempts to address a set of questions that include with heterogeneous bidders and when entry is taken into account, how the seller's revenue could change with the extent to which the bidders' private values are affiliated, and whether the reserve price currently set by the ODF is optimal with respect to maximizing the seller's revenue/profit. Moreover, merger and bidder coalition have been an important issue to economists interested in competition policy, yet no empirical work has studied this issue taking into account endogenous participation from potential bidders. That we consider heterogeneous bidders is motivated by the evidence from the previous work studying the timber auctions in Oregon (e.g. Brannman and Froeb (2000) using data consisting of oral auctions, and Li and Zhang (2008) using the same data used in this paper comprising first-price sealed-bid auctions), that hauling distance plays an important role in bidders' bidding (Brannman and Froeb (2000)) and entry (Li and Zhang (2008)) decisions. This means that bidders are asymmetric and heterogeneous. Furthermore, Li and Zhang (2008) find a small but strongly significant level of affiliation among potential bidders' private information (either private values or entry costs). Lastly, recent empirical work in auctions in general and in timber auctions in particular (e.g. Athey, Levin and Seira (2004), Bajari and Hortacsu (2003), Kransnokutskaya and Seim (2006), Li and Zheng (2005, 2007)) has demonstrated that bidders' participation and entry decision is an integrated part of the decision making process that has to be taken into account when studying auctions. In view of these, in this paper we attempt to study the timber auctions organized by the ODF within a general framework in which potential bidders are affiliated and heterogeneous, and they make endogenous entry decisions before submitting bids.

Auction theory offers little guidance in answering these questions for auctions with entry and asymmetric potential bidders with affiliated private values. On the other hand, to gain insight on these questions from an empirical perspective, one needs to observe two states of world, such as pre and post the change of the affiliation level, or pre and post merger. Usually in auction data, as is the case in our data, however, one cannot observe these two states of world. Therefore we adopt a structural approach in our empirical analysis. In particular, we estimate a game-theoretic auction model within the affiliated private value (APV) paradigm with asymmetric bidders and with entry. We then use the estimated structural parameters to conduct counterfactual analyses of our interest. We find that for a representative auction, the optimal reserve price should be much larger than the current one. In evaluating the merger effects we find that the merged bidder is very likely to participate in the auction regardless of the merging bidders' entry behaviors and that the merger has little impact on other bidders' entry behaviors. While the overall merger effect on the seller's revenue is not theoretically clear in our model, we find that at the current reserve prices and dependence levels, merger is beneficial to the seller, but it could mean a loss for the seller for some values of reserve price and dependence levels and that pro-competitive merger is better than anti-competitive merger with respect to the change in the seller's revenue.

We develop an entry and bidding model for asymmetric bidders within the APV paradigm. Because of the general framework we adopt, the answers to the aforementioned questions of our interest depend on the interactions of affiliation, entry, and asymmetry, as well as competition. As is well known, the optimal reserve price in a symmetric independent private value (IPV) model without entry does not depend on the number of potential bidders. This result can change if entry is introduced (see, e.g., Levin and Smith (1994), Samuelson (1985), Li and Zheng (2007)), or if bidders have affiliated private values (Levin and Smith (1996), Li, Perrigne, and Vuong (2003)). In our case, on the other hand, assessing the optimal reserve price is complicated further by the APV framework with entry and asymmetric bidders. Therefore we can only address this issue through a counterfactual analysis using the structural estimates. Furthermore, while the effect of the number of potential bidders on winning bids and seller's revenue is clear in an IPV model with symmetric bidders and without entry, it becomes less clear in a more general setting, such as the IPV model with entry and symmetric bidders (Li and Zheng (2005, 2007)), and the APV model without entry (Pinkse and Tan (2005)). In particular, Li and Zheng (2005, 2007) show that in terms of the relationship between the number of potential bidders and the expected seller's revenue, in addition to the usual "competition effect," there is an opposite effect due to the entry which they term as the "entry effect." On the other hand, Pinkse and Tan (2005) postulate that in a conditionally independent private value model, a special case of the APV paradigm, in addition to the "competition effect," there is an opposite effect caused by affiliation they term as the "affiliation effect." Zhang (2008) shows that in the APV model with entry and symmetric bidders, these three effects, namely, the "competition effect," the "entry effect," and the "affiliation effect" are at work. While we expect these three effects to remain in the APV framework with entry and asymmetric bidders, it becomes challenging to pinpoint them with asymmetric bidders. Since the effect of merger is closely related to how the seller's revenue changes with the set of potential bidders, i.e, not only the number of potential bidders, but also the identity of potential bidders when they are heterogeneous, and at the same time, theory does not yield good predictions, we rely on the structural analysis to gain insight on this issue.

Asymmetry is an indispensable element of the model given the asymmetric feature of the data. The analysis of the model, however, is complicated from both theoretical and econometric viewpoints due to the introduction of asymmetry. Because of the complexity of the model, and in particular, because that there is no closed form solution for the bidding function, we have to rely on some numerical approximation procedure. Moreover, while the structural analysis of auctions with asymmetric bidders has focused on the case with two types of bidders (Athey, Levin, and Seira (2004), Campo, Perrigne and Vuong (2003), and Kransnokutskaya and Seim (2006)), our model allows for all potential bidders to be different from each other, motivated by the fact that in our data, asymmetry is driven by the difference among bidders' hauling distances.

This paper makes contribution to the growing literature of the structural analysis of auction data since Paarsch (1992). While the structural approach has been extended to the APV paradigm by Li, Perrigne and Vuong (2000, 2002), Campo, Perrigne and Vuong (2003), and Li, Paarsch and Hubbard (2007), this paper is the first one in estimating a structural model within the APV paradigm and taking into account entry. On the other hand, while the recent work has started to pay attention to the problem of endogenous participation and entry, all the work has focused on the IPV framework with Bajari and Hortaçsu (2003) being an exception as they consider a common value (CV) model. In contrast, this paper considers the entry problem within the APV paradigm, a more general framework.

Our empirical analysis of the timber auctions and the resulting findings offer new insight on timber sale auctions and policy related issues. While most of the empirical analysis of timber sale auctions is based on the IPV model without entry (e.g. Paarsch (1997), Baldwin, Marshall and Richard (1997), Haile (2001), Haile and Tamer (2003), Li and Perrigne (2003)) or the IPV model with entry (Athey, Levin and Seira (2004), Li and Zheng (2007)), ours is based on the APV model with entry and heterogeneous bidders. As a result, our findings can be more robust, and also can be more useful for addressing the policy-related issues as our analysis takes into account the affiliation effect, the entry effect, and the asymmetry effect. Moreover and probably more interestingly, we study the merger effect within the asymmetric APV framework with entry, and offer new insight into how merger as well as other issues related to competition policy can be affected by complications arising from affiliation, entry, and asymmetry, and how they can be addressed within a unified framework as adopted in this paper.[1]

[1] It is worth noting that to the best of our knowledge, Brannman and Froeb (2000), considering oral timber auctions within an IPV paradigm without entry, is the only paper assessing the merger effect in auctions using the structural approach.

This paper is organized as follows. Section 2 describes the data we analyze in the paper. In Section 3 we propose the asymmetric APV model with entry. Section 4 is devoted to the structural analysis of the data, and Section 5 conducts a set of counterfactual analyses studying the effects of reserve prices, affiliation levels, and mergers. Section 6 concludes.

2 Data

The data we study in this paper are from the timber auctions organized by the ODF between January 2002 and June 2007. Before an auction is advertised, the ODF "cruises" the selected tract of timber and obtains information of the tract, such as the composition of the species, the quality grade of the timber and so on. Based on the information it obtains, the ODF sets its appraised price for the tract, which serves also as the reserve price. After the "cruise," a detailed bid notice is usually released 4-6 weeks prior to the sale date, which provides information about the auction, including the date and location of the sale, species volume, quality grade of the timber, appraised price as well as other related information. Potential bidders acquire their own information or private values through different ways and decide whether and how much to bid. Bids are submitted in sealed envelopes that are opened at a bid opening session at the ODF district office offering the sale. The sale is awarded to the bidder with the highest bid. All the sales are therefore first price sealed bid scale auctions.

The original data contain 415 sales in total. Among them, some sales have more than one bid species, which are deleted from our sample because of the "skewed bidding" issue discussed in Athey and Levin (2001). We focus on the sales in which Douglas-fir is the only bid species and drop the sales with other than Douglas-fir as bid species, because Douglas-fir is a majority species. Considering the time that our estimation program takes, we focus on the auctions with at most 5 potential bidders. The resulting final sample has 81 sales and 245 observed bids.

For each sale, we directly observe some sale-specific variables including the location and the region of the sale, appraised price, appraised volume, length of the contract, and diameter at breast height (DBH) as well. Noting that the bid species is often a combination of a mixture of several grades of quality, we use number 1, 2, $\cdots$, up to 18 to denote the letter-grades used by ODF so that the final grade of a sale is the weighted average of grades with volumes of grades as the weight. In addition to sale-specific variables, as shown in Brannman and Froeb (2000) and Li and Zhang (2008), respectively, hauling distance is an important bidder-specific variable that affects bidders' bidding and entry decisions. However, hauling distance is not observed directly. We use the hauling distance variable constructed in Li and Zhang (2008) who transfer the location of a tract into latitude and longitude through the Oregon Latitude and Longitude Locator[2] and find the distances between the tract and the mills of firms by using Google Map.

The key information related to endogenous entry is the identities of potential bidders, which are not observed. Unlike some procurement auctions, where information on bidders who have

[2] It is available at http://salemgis.odf.state.or.us/scripts/esrimap.dll?name=locate&cmd=start

requested bidding proposal is available and can be used as a proxy for potential bidders (Li and Zheng (2005)), we do not have such information in our case, as is usual for timber sale auctions. Therefore we follow Athey, Levin, and Seira (2004) and Li and Zheng (2007) to construct potential bidders. Specifically, we first divide all sales in the original data set into 146 groups each of which contains all sales held in the same district in the same quarter of the same year. The potential bidders of a sale are then all bidders who submit at least one bid in the sales of the group that the sale belongs to. In other words, all auctions in the same group have the same set of potential bidders. Note that in constructing the potential bidders we use the original data set including all auctions removed from our final sample. Summary statistics of the data are given in Table 1. Notably, the entry proportion, which is calculated as the ratio of the number of actual bidders and the number of potential bidders, is about 0.7 on average, meaning that while there is strong evidence of entry pattern from the potential bidders, on average more than half of the potential bidders would participate in the auction.

3 The Model

In this section we propose a theoretical two-stage model to characterize the timber sales, extending the models in Athey, Levin, and Seira (2004) and Krasnokutskaya and Seim (2006) with two groups of bidders within an IPV paradigm to the APV paradigm that allows potential bidders to be different from each other. Specifically, motivated by the finding of Brannman and Froeb (2000) that the hauling distance plays a significant role in bidders' bidding decision in oral timber auctions in Oregon, and the finding of Li and Zhang (2008) using the same data studied in this paper that the hauling distance is important in potential bidders' entry decision and potential bidders are affiliated through their private information (either private values or entry costs), we consider a first-price sealed-bid auction within the APV paradigm with a public reserve price, endogenous entry, and asymmetric bidders.

In the model, a single object is auctioned off to N heterogenous and risk-neutral potential bidders, who are affiliated in their private information. Bidder i has a private entry cost k_i, including the cost of obtaining private information and bid preparation, and does not obtain his private value v_i until he participates in the auction. We allow both private values and entry costs to be affiliated across bidders, that is $V_1, \ldots, V_N$ and $K_1, \ldots, K_N$ jointly follow a distribution $F(\cdot, \ldots, \cdot)$ with support $[\underline{v} = r, \overline{v}]^N$, and a distribution $G(\cdot, \ldots, \cdot)$ with support $\left[\underline{k}, \overline{k}\right]^N$, respectively, where r is the public reserve price of the auction. Affiliation is a terminology describing the positive dependence among random variables, which was first introduced into the study of auctions by Milgrom and Weber (1982). It is equivalent to the concept called multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature. Following Milgrom and Weber (1982), affiliation has the following formal definition.

Definition. *Let z and z' be any two values of a vector of random variables $Z \subseteq \mathbb{R}^n$ with a density $f(\cdot)$. It is said that all elements of Z are affiliated if $f(z \vee z') f(z \wedge z') \geq f(z) f(z')$, where $z \vee z'$*

denotes the component-wise maximum of z and z', and $z \wedge z'$ denotes the component-wise minimum of z and z'.

Intuitively, affiliation means that large values for some of the components in Z make other components more likely to be large than small. We also denote the marginal distribution and density of bidder i's private value by $F_i(\cdot)$ and $f_i(\cdot)$ and marginal distribution and density of bidder i's entry cost by $G_i(\cdot)$ and $g_i(\cdot)$, respectively, and assume that $f_i(\cdot)$ is continuously differentiable and bounded away from zero on $[\underline{v} = r, \overline{v}]$. The subscript of distribution function implies that all potential bidders are of different types. This assumption is motivated by the fact that heterogeneity among bidders arises from different hauling distances in our data.

3.1 Bidding Strategy

Because the entry decision is based on the pre-entry expected profit, which depends on the bidding strategy of bidder i, we first describe the bidding strategy of bidder i. We assume that bidder i knows the number of the actual competitors in the bidding stage,[3] and thus bidder i's bidding strategy is determined by the first order condition of the following maximization problem,

$$\max\left(v_i - b_i\right) \Pr\left(B_j < b_i | v_i; a_{-i}\right),$$

where B_j denotes the maximum bid among other actual bidders and

$$a_{-i} \in A_{-i} = \left\{\left(a_1, \ldots, a_N\right) | a_j = 0 \text{ or } 1, j = 1, \ldots N, j \neq i\right\}$$

is one possibility of the 2^{N-1} combinations of entry behaviors of $N-1$ other potential bidders. Denote the number of actual bidders of the combination a_{-i} by $n_{a_{-i}}$. As usual we consider a continuously differentiable and strictly increasing bidding strategy, $b_i = s_i(v_i)$, therefore the first order condition is

$$-F_{V_{-i}|v_i}\left(s_j^{-1}\left(b_i\right), j \neq i | v_i\right) + \left(v_i - b_i\right) \sum_{j \neq i}^{n_{a_{-i}}} \frac{\partial F_{V_{-i}|v_i}\left(s_j^{-1}\left(b_i\right), j \neq i | v_i\right)}{\partial v_j} \frac{\partial s_j^{-1}\left(b_i\right)}{\partial b_i} = 0, \quad (1)$$

where $F_{V_{-i}|v_i}$ denotes the joint distribution of $V_j, j \neq i$ conditional on $V_i = v_i$ and $s_i^{-1}(\cdot)$ is the inverse function of the bidding function of bidder i. A set of equation (1) for $i = 1, \ldots, n$ form a system of differential equations characterizing the equilibrium bids for all n actual bidders. We denote the post-entry profit of bidder i by $\pi_i\left(v_i | a_{-i}\right)$.

[3] When the lower support of private value is below the reserve price, bidder i only knows the active bidders who participate in the auction but not actual bidders who submit bids. In our case, the number of active bidders is equal to the number of actual bidders, since the lower support of private value is assumed to be just the reserve price.

3.2 Entry Decision

In the initial participation stage, each potential bidder i only knows his own entry cost, joint distributions of entry costs and private values. Therefore the entry decision of bidder i is determined by his pre-entry expected profit from participation, Π_i. Specifically, he participates in the auction only if his entry cost is less then Π_i. Let p_i denote the entry probability of bidder i, respectively. The ex ante expected profit Π_i is given by

$$\Pi_i = \sum_{a_{-i} \in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i \left(v_i | a_{-i}\right) dF_i \left(v_i\right) \Pr \left(a_{-i} | a_i = 1\right), \tag{2}$$

where $\Pr\left(a_{-i}|a_i = 1\right)$ is a function of $p_i, i = 1, \ldots, N$, which can be denoted by $\Pr\left(a_{-i}; p_1, \ldots, p_N | a_i = 1\right).$ As a result, the pre-entry expected profit is the sum of 2^{N-1} products of the post-entry profits and corresponding probabilities with the unknown private value integrated out. On the other hand, the ex ante probability of entry is given by $p_i = \Pr\left(K_i < \Pi_i\right) = G_i\left(\Pi_i\right).$

Note that although the number of potential bidders does not affect the bidding strategy in the bidding stage, it affects the number and the identities of actual bidders, which in turn have impact on the bidding strategy.

3.3 Characterization of the Equilibrium

Existence and uniqueness of the Bayesian Nash equilibrium with asymmetric bidders has been a challenging problem studied in the recent auction theory literature. See, e.g. Lebrun (1999, 2006) and Maskin and Riley (2000, 2003) within the IPV framework, Lizzeri and Persico (2000) within the APV framework and two types of bidders. The analysis of our model is further complicated by the introduction of affiliation and entry, as well as that we allow all potential bidders to be different from each other. To address the issue of existence and uniqueness in our case, we look at the case where the joint distribution of bidders' private values is characterized by the family of Archimedean copulas. For the copula concept and the characterization of the Archimedean copulas, see Nelsen (1999). Copula can provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions.

Specifically, by Sklar's theorem (Sklar (1973)), for a joint distribution $F\left(x_1, \ldots, x_N\right)$, there is a unique copula C, such that $C\left(F_1\left(x_1\right), \ldots, F_N\left(x_N\right)\right) = F\left(x_1, \ldots, x_N\right)$. For the Archimedean copulas, the copula C can be expressed as $C\left(u_1, \ldots, u_n\right) = \phi^{[-1]}\left(\phi\left(u_1\right) + \cdots + \phi\left(u_n\right)\right)$, where ϕ is a generator of the copula and is a decreasing and convex function, and $\phi^{[-1]}$ denotes the pseudo-inverse of ϕ[4]. The family of Archimedean copulas include a wide range of copulas. For example, the generators $\phi\left(u\right) = \frac{1}{q}\left(u^{-q} - 1\right)$, $\phi\left(u\right) = \left(-\ln\left(u\right)\right)^q$, and $\phi\left(u\right) = \ln\left(\frac{\exp(qu)-1}{\exp(q)-1}\right)$ correspond to the

[4] ϕ is a decreasing convex function from $[0, 1]$ to $(0, \infty]$ with $\phi(1) = 0$. $\phi^{[-1]}$ is defined as

$$\phi^{[-1]}\left(u\right) = \begin{cases} \phi^{-1}\left(u\right), & 0 \le u \le \phi\left(0\right), \\ 0, & \phi\left(0\right) \le u \le \infty. \end{cases}$$

widely used Clayton copula, Gumbel copula, and Frank copula, respectively. Since we consider a differentiable bidding strategy, we have to confine ourself to the strict generator, that is $\phi^{[-1]} = \phi^{-1}$. Since $C_i\left(F_1\left(x_1\right), \ldots, F_N\left(x_N\right)\right) = F_{X_{-i}|x_i}\left(x_1, \ldots, x_N\right)$ (e.g. Li, Paarsch, and Hubbard (2007)), the first order condition (1) determining the equilibrium bids can be written as follows

$$\frac{ds_i^{-1}(b)}{db} = \frac{\phi^{-1\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}{\left(n_{a_{-i}}-1\right)\phi'\left(F_i\left(s_i^{-1}(b)\right)\right) f_i\left(s_i^{-1}(b)\right)\phi^{-1\prime\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}\left[\sum_{k\neq i}\frac{1}{s_k^{-1}(b)-b} - \frac{n_{a_{-i}}-2}{s_i^{-1}(b)-b}\right] \tag{3}$$

and $\Pr\left(a_{-i}; p_1, \ldots, p_N | a_i = 1\right)$, for example, for the case that given the participation of bidder i, bidder 1 up to bidder $i-1$ participate in the auction while bidder $i+1$ up to bidder N do not, can be expressed as

$$\begin{aligned} & \Pr\left(a_1 = \cdots a_{i-1} = 1, a_{i+1} = \cdots a_N = 0 | a_i = 1\right) \\ = & \frac{\Pr\left(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0\right)}{\Pr\left(a_i = 1\right)} \end{aligned}$$

where

$$\begin{aligned} & \Pr\left(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0\right) \\ = & C\left(p_1, \ldots, p_i, 1, \ldots, 1; q_k\right) - \sum_{i+1\leq j\leq N} C\left(p_1, \ldots, p_i, p_j, 1, \ldots, 1; q_k\right) \\ & \cdots + (-1)^{N-i} C\left(p_1, \ldots, p_N; q_k\right), \end{aligned}$$

and $\Pr\left(a_i = 1\right) = C\left(1, \ldots, 1, p_i, 1, \ldots, 1; q_k\right)$.

Equilibrium of the model consists of two parts, entry equilibrium and bidding equilibrium. Based on the choice of Archimedean copulas for the joint distribution of private values, the existence of the equilibrium is guaranteed. Moreover, with some additional conditions, the bidding equilibrium is unique. The next paragraph describes the equilibrium formally.

Proposition (Characterization of Equilibrium)**.** *Assume* (a) *the marginal distribution of entry cost of bidder* i*,* G_i *is continuous over* $\left[\underline{k}, \overline{k}\right]$ *for all* i*;* (b) *marginal distribution of private value of bidder* i *is differentiable over* $(\underline{v}, \overline{v}]$ *with a derivative* f_i *locally bounded away from zero over this interval for all* i*;* (c) *joint distribution of private values follows an Archimedean Copula.*

i. Bidding Equilibrium *In the bidding equilibrium, bidder* i *adopts a continuously differentiable and strictly increasing bidding function* $b_i = s_i(v)$ *over* $(\underline{v}, \overline{v}]$*. The inverse functions of* s_i *for all* i*,* $s_1^{-1}, \ldots, s_n^{-1}$ *are the solution of the system of differential equations* (3) *with boundary conditions* (4) *and* (5) *:*

$$s_i^{-1}(\underline{v}) = \underline{v} \tag{4}$$

$$s_i^{-1}(\eta) = \overline{v}. \tag{5}$$

for some η.

ii. Uniqueness of Bidding Equilibrium *Moreover, if $F_i(\underline{v}) > 0$ and $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u, then the bidding equilibrium is unique.*

iii. Entry Equilibrium *In the entry equilibrium, bidder i chooses to participate in the auction if his entry cost is less than the threshold $\Pi_i(p)$ and stay out otherwise, where $p = (p_1, \ldots p_N)$ and p_i is the entry probability of bidder i and is determined by*

$$p_i = G_i(\Pi_i(p)). \tag{6}$$

As is seen here, the existence of the entry equilibrium is equivalent to the existence of the entry probability p_i, given by the equation (6). Since Π_i is continuous in p_i and thus G_i is continuous over $[0,1]$, there exists a solution p_i of equation (6), according to Kakutani's fixed point theorem (Kakutani (1941)). To show the uniqueness of the bidding equilibrium is to show that there is a unique η such that $s_i^{-1}(\eta) = \overline{v}$. Then starting from η, according to Lipschitz uniqueness theorem, s_i^{-1} is unique over $(\underline{v}, \eta]$. Note that a Clayton copula satisfies the condition for uniqueness that $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. The formal proofs are provided in the appendix.

4 The Structural Analysis

We estimate the model proposed in the last section using the timber sales data. Our objective is to recover the underlying joint distributions of private values and entry costs using observed bids and the number of actual bidders. The structural inference in our case is complicated because of the generality of our model that accounts for affiliation, asymmetry, and entry. Our approach circumvents the complications arising from the estimation of our model and makes the structural inference tractable. First, to model the affiliation in a flexible way, we adopt the copula approach in modeling the joint distribution of private values and the joint distribution of entry costs.[5] Second, since we allow bidders to be asymmetric, the system of differential equations consisting of equation (3) that characterizes bidders' Bayesian Nash equilibrium strategies does not yield closed-form solutions. To address this problem we adopt a numerical method based on Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004). Third, because of the various covariates we try to control for and the relatively small size of the data set, the nonparametric method does not work well here. Therefore, we adopt a fully parametric approach.

4.1 Specifications

We adopt the Clayton copula to model the joint distributions of both private values and entry costs. With the generator of Clayton copula given above, the joint distribution of private value is specified as $F(v_1, \ldots v_n) = \left(\sum_i F_i(v_1)^{-q_v} - n + 1\right)^{-1/q_v}$, and the joint distrition of entry costs is specified as

[5] Li, Paarsch, and Hubbard (2007) use the copula approach to model affiliation within the symmetric APV framework without entry and propose a semiparametric estimation method.

$G(k_1, \ldots, k_n) = \left(\sum_i G_i(v_1)^{-q_k} - n + 1\right)^{-1/q_k}$, where q_v and q_k are dependence parameters and F_i and G_i are the marginal distributions of private value and entry cost, which are specified as truncated exponential distributions given as follows, $F_{V_{\ell i}}(v|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}v\right)}{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\overline{v}\right)}$, $G_{K_{\ell i}}(k|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}k\right)}{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\overline{k}\right)}$ for bidder i of the ℓ-th auction, $\ell = 1, \ldots, L$, where L is the number of auctions, $\lambda_{v_{\ell i}}$ and $\lambda_{k_{\ell i}}$ are the private value and entry cost means and equal $\exp(\beta\mathbf{x}_{\ell i})$ and $\exp(\alpha\mathbf{x}_{\ell i})$, respectively, and $\mathbf{x}_{\ell i}$ is a vector of covariates that are auction specific or bidder specific, and in our case includes variables such as hauling distance, volume, duration, grade, and DBH.[6] In practice, $\underline{v}$ is equal to the reserve price of ℓ-th auction, $\overline{v}$ is equal to \$1500/MBF, the lower bound of entry cost is equal to zero and the upper bound $\overline{k}$ is \$940/MBF, an arbitrarily large number. We then model the joint distributions of private values and entry costs in auction ℓ as Clayton copula with different dependence parameters q_v and q_k. The use of the Clayton copula offers several advantages. First, it guarantees the existence and uniqueness of the equilibrium as discussed in Section 3.3. Second, it preserves the same dependence structure when the number of potential bidders changes. Third, it is relatively easy to draw dependent data from the Clayton copula, as it has a closed form that can be used to draw data recursively. Lastly, since q is the only parameter that measures the dependence, we can easily evaluate the impact of the dependence level on the end outcomes of an auction by changing the value of q.

Note that in these specifications, the asymmetry across potential bidders is captured by the inclusion of the hauling distance variable in $\mathbf{x}_{\ell i}$, while both α and β are kept constant across different bidders. This enables us to estimate a relatively parsimonious structural model and at the same time control for the asymmetry.

4.2 Estimation Method

Because of the complexity of our structural model, we employ the indirect inference method to estimate the model. Initially proposed in the nonlinear time series context by Smith (1993) and developed further by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996), the indirect inference method is simulation based and obtains the estimates of parameters by minimizing a measure of distance between the estimates for the auxiliary parameters of an auxiliary model using the original data and simulated data. More specifically, let θ denote the vector of parameters of interest, γ be the parameters of the auxiliary model, $\widehat{\gamma}_T$ and $\widehat{\gamma}_{ST}^{(p)}(\theta)$ be the estimates of the auxiliary model using the original data and the p-th simulated data out of P sets of simulated data from the

[6] Here we do not introduce unobserved auction heterogeneity into the model, as Li and Zhang (2008) show that it does not have a significant effect in bidders' entry behaviors.

model given a specific θ, respectively. Then the estimator of θ, denoted by $\widehat{\theta}_{ST}$, is defined as

$$\widehat{\theta}_{ST} = \arg\min_{\theta} \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{p=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right]' \Omega \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{P=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right], \tag{7}$$

where Ω is a symmetric semi-positive definite matrix.

The auxiliary model, which is usually simpler than the original model and easier to estimate as well, plays an important role in the indirect inference method. In this paper, following the idea in Li (2005) we employ a relatively simple and easy-to-estimate auxiliary model to make the implementation tractable and the inference feasible. Specifically, since we use both bids and the number of actual bidders in the estimation of entry and bidding model, our auxiliary model includes two separate regressions: a linear regression of the observed bids and a Poisson regression of the number of actual bidders, which are described as follows

$$\begin{aligned} b_\ell &= \gamma_{10} + \sum_{h=1}^{H} X_{h\ell}\gamma_{11h} + \sum_{h=1}^{H} X_{h\ell}^2\gamma_{12h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m\gamma_{1mh} + \varepsilon_1, \\ \Pr(n_\ell = k) &= \frac{\exp(-\lambda_\ell)\lambda_\ell^k}{k!}, \lambda_\ell = \gamma_{20} + \sum_{h=1}^{H} X_{h\ell}\gamma_{21h} + \sum_{h=1}^{H} X_{h\ell}^2\gamma_{22h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m\gamma_{2mh}, \end{aligned}$$

where b_ℓ is the average bid of auction ℓ, and $X_{h\ell}$, $h = 1, ...H$, denote the vector of auction-specific covariates of auction ℓ and the average of bidder-specific covariates, and H is the number of such covariates, which is 6 in our case. $m = 2$ makes our model over-identified.

An issue arising from the implementation of the indirect inference method is the discontinuity of the objective function of equation (7) because of the discrete dependent variable (the number of actual bidders) in the auxiliary model that makes gradient-based optimization algorithm invalid. We address this issue by using simplex, a nongradient-based algorithm. Alternatively, one can follow Keane and Smith (1993) to smooth the objective function using a logistic kernel.

4.3 Estimation Results

Table 2 reports the estimation results. For the (marginal) private value distribution, all the estimated parameters are significant at the 5% level, and also have the expected signs. Of particular interest is the parameter of the hauling distance variable, which is used to control for heterogeneity across bidders. Its estimate is negative, meaning that the longer the hauling distance is, the less is the private value mean. Furthermore, the average marginal effect of the hauling distance variable is about -1.79, meaning that one mile increase in the distance would reduce the private value mean by \$1.8/MBF while everything else is fixed. In other words, one mile increase in the distance could reduce the private value mean by 0.21%. Another parameter of particular interest is the dependence parameter q_v in private values, which turns out to be relatively small ($q_v = 0.1014$) but significant. To get some idea of how large the dependence is with $q_v = 0.1014$, we use a measure called Kendall's

τ (Nelsen (1999)), which is used to measure the concordance of two random variables. Concordance is not really the same concept as affiliation, but measures the positive dependence in a similar way. Kendall's τ is defined as the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = \Pr\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right] - \Pr\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right].$$

For the Clayton copula, $\tau = q/(q+2) = 0.047$. Therefore $q_v = 0.1014$ implies that the event of any two bidders' private values being concordant is about 4.7% more likely than the event of being discordant.

Two points in the estimates in the distribution of entry costs are worth noting. First, the hauling distance variable is significant and positive in the entry cost distribution and its marginal effect is 2.1736. Second, the dependence level among the entry costs is 0.5015, implying a Kendall's τ of 0.2. This indicates that the affiliation among the entry behaviors is mainly driven by the affiliation among the entry costs.

4.4 Model Fit

In this subsection, we assess the model fit from the structural estimation. Because our model yields two main outcomes, namely, the number of actual bidders and bids submitted by the actual bidders, we assess the model fit through these two outcomes. Specifically, we use the structural estimates to conduct 1000 simulations, each of which contains 81 auctions. We pool all simulated bids and the simulated number of actual bidders and compare two histograms of the simulated data with those of our sample. The histograms are provided in Figure 1, from which we can see that the distributions of the simulated data match the distributions of the original data quite well. The means of the simulated bids and number of actual bidders are \$378.40/MBF and 3.21, respectively, which are quite close to \$376.69/MBF and 3.02 of the sample means.[7]

5 Counterfactual Analyses

With the estimated structural parameters we can now answer the questions put forward in the introduction section empirically. We focus on both end outcomes, namely, the number of actual bidders, and winning bids (or seller's unit revenue). We conduct counterfactual analyses on the 99th auction of our data, which is not included in the estimation sample. We use this auction as a representative auction, as the values of covariates of this auction are close to the average values of all auctions in our data set. In particular, the number of potential bidders in this auction is 7, about the same as the average number of potential bidders in the data. In doing so we assume that the estimates derived from a subset of the data fit the whole data.

[7] Alternatively we could compare the estimated private value mean and the true private value mean. The latter, however, is not observed. Through our private communications with a staff at the ODF, the average private value is within the range of \$800/MBF and \$1000/MBF. The estimated average private value from our estimation is about \$833.49/MBF, well falling inside this range.

The seller's expected unit revenue is given as follows

$$\begin{aligned} E(w) &= E(w|w>0)\Pr(w>0) + E(w|w<0)\Pr(w<0) \\ &= E(w|w>0)\Pr(w>0) + v_0\Pr(w<0), \end{aligned}$$

where w denotes the winning bid and v_0 is the valuation of the timber to the seller, and the second equality follows the assumption that if the timber is not sold successfully then the seller gets his own value. In the following analyses we assume $v_0 = 0$, thus the expected revenue is equivalent to the expected profit.

5.1 Effects of Reserve Price and Dependence Level

Intuitively the effect of the reserve price can be seen from two aspects. On one hand, a higher reserve price is associated with a lower ex ante expected profit, i.e., a lower cut-off entry cost according to equation (2) as it narrows the integration range, and thus fewer participating bidders and lower probability of being sold, which may lower bids in our APV model with asymmetric bidders. On the other hand, a higher reserve price raises the lowest acceptable bids and of course makes bidders bid higher. Our counterfactuals shown in Figure 2 confirm such tradeoff. The three panels in Figure 2 show how the reserve price affects the number of actual bidders, the probability of being sold, and the seller's revenue. The number of actual bidders is decreasing in the reserve price as is shown in the first panel. The average number of participating bidders drops dramatically from 5.42 to 1.20 when the reserve price is raised from \$146.7/MBF to \$1320.4/MBF. The probability of being sold is negatively related to the reserve price when the reserve price is less than \$1000/MBF. The change in the winning bid is the final result of all effects associated with change in the reserve price. As is seen in the last panel, the optimal reserve price is around \$590/MBF, which is more than twice as large as the current reserve price. This implies that when the reserve price is below \$590/MBF the positive effect on the winning bid outweights the negative effect associated with the lower probability of being sold.

The APV model we estimated also enables us to quantify the effects of the dependence level among bidders. To this end, we change the values of the dependence parameters of both private values and entry costs while keeping other parameters fixed. We are able to conduct such analysis as the change of the dependence parameter does not affect other parameters, which appear only in the marginal distribution of private values or entry costs. As in the analysis of the effects of the reserve price, we are interested in three effects of the dependence parameters. Results are provided in Figure 3 and Figure 4. The probability of being sold remains at 1 as dependence levels change because the dependence levels do not affect the number of participating bidders as much as the reserve price does. The number of participating bidders is slightly negatively related with the dependence level of private values. The average number of participating bidders associated with a high dependence level, $q_v = 5$, is only about one less than the one in the almost independence case. On the other hand, the number of participating bidders is not monotone with respect to the dependence level of entry costs, as is seen from the first panel of Figure 4. One thing worth noting is that a relatively large increase in the number of participating bidders is associated with the change of q_k from almost zero to one. This is consistent with the intuition that affiliated bidders should have more similar entry behavior, which is also the idea of the affiliation test in Li and Zhang (2008). Moreover, two dependence parameters have opposite effects on the winning bid. The dependence of private values seems to be negatively related with the winning bid while the dependence of entry costs has a significant positive effect on the winning bid. As in the effects on entry, the change in the winning bid associated with the change from independence to affiliation is of interest. Note that the winning bid drops when q_v increases from alomost zero to a small positive number, meaning that the seller's profit could be higher when the bidders' private values are independent than when they are affiliated.

5.2 Effect of Bidding Coalition or Merger

Our asymmetric model is ideal for evaluating the merger effects on the end outcomes of auctions, because asymmetry is intrinsically involved in the merger. The merged bidder will be different from other bidders even if they are symmetric pre-merger. For the purpose of measuring the effects of bidding coalition or merger, we conduct two hypothetical mergers, pro-competitive and anti-competitive merger. In the pro-competitive merger two least competitive bidders are merged, which means that two bidders with the longest hauling distances are merged in our case. On the contrary, in the anti-competitive merger two bidders with the shortest hauling distances are merged into one entity. On the other hand, according to the pre-merger entry behaviors of the merging bidders, mergers can be divided into three groups: mergers between two participating bidders, mergers between two non-participating bidders, and mergers between one participating bidder and one non-participating bidder. It is obvious that most anti-competitive mergers belong to the first group while most pro-competitive mergers belong to the second group, because strong bidders are more likely to participate in the auction than weak bidders do. We therefore focus on the merger effects of the first group anti-competitive mergers and the second group pro-competitive mergers. These two polar cases should shed light on other mergers.

The private value V_m and the entry cost K_m of the merged bidder are defined as $V_m = \max(V_1, V_2)$ and $K_m = \min(K_1, K_2)$, respectively, assuming that bidder 1 and bidder 2 are merged without loss of generality where m denotes the merged bidder. Therefore the marginal distributions of private values and entry costs of the merged bidder are defined as $F_m(v_m) = C(F_1(v_m), F_2(v_m); q_v)$, and $G_m(k_m) = \widetilde{C}(1 - G_1(k_m), 1 - G_2(k_m); q_k)$ in terms of copula, where $\widetilde{C}$ is the survival copula associated with C. In practice we simulate 3000 auctions based on covariates of the representative auction and conduct pro-competitive and anti-competitive mergers for these 3000 auctions and compare the pre-merger and post-merger end outcomes.

When it comes to the merge effects on entry, we are not only interested in the merged bidder's entry behavior, but also cocerned with whether a merger induces non-participating bidders into the auction or crowds participating bidders out of the auction. The first panels of Figure 5 to Figure 10 demonstrate the entry behavior of the merged bidder in both anti-competitive and pro-competitive mergers. As is seen from Figure 5, Figure 6, and Figure 7, in the pro-competitive mergers the merged bidder always participates in the auction, while in the anti-competitive mergers, the merged bidders participates in the auction with a large probability of 0.80 as is shown in Figure 8, Figure 9, and 10, although the merging bidders did not participate in the auction pre-merger. This is not surprising because the merged bidder has a higher private value mean and lower entry cost mean. The effects of both mergers on other bidders' entry behaviors are very slight. Neither of the two types of mergers induces nonparticipating bidders into the auction or crowds participating bidders out of the auction, except that in the anti-competitive merger when the reserve price is above \$900/MBF or the dependence level of private values is more than 1, there is about 20% chance that a nonparticipating bidder will enter the auction due to the merger, as is shown in Figure 5 and Figure 6. Considering all effects on entry, it is obvious that the acution loses one participating bidder due to the anti-competitive merger and gains one participating bidder in the pro-competitive bidder.

The changes in the number and identities of participating bidders affect the final bids and thus the winning bids through several channels. The first channel is called the "competition effect." The increase or decrease in the number of participating bidders makes bidders more or less aggressive. To the merged bidder in anti-competitive merger, the competition between two merging bidders is removed due to the merger, which causes the merged bidder bid less. Second, the merger may affect the bids and winning bids through affiliation. Within the APV framework, a bidder would think that he may overestimate the common factor which affects all bidders' private values when he wins the auction. By taking this into account and trying to alleviate this effect, the bidder reduces his bid. This effect is called the "affiliation effect" in Pinkse and Tan (2005) and can make bidders bid more as the number of potential bidders decreases. Lastly, the merger yields a stronger bidder meaning a smaller marginal distribution, through which the merger affects bidders' bids and possibly the winning bids. Intuitively this should lower the winning bid in the anti-competitive merger and raise the winning bid in the pro-competitive merger, because a stronger winner is undesired while a stronger competitor is desired in terms of the degree of competition. Note that the first two channels are essentially through the change in the number of participating bidders and the last one is effective through the change in identities of participating bidders. How merger affects the seller's revenue depends on the interactions of these effects. Because of the complexity of the model we consider, however, analytically we cannot quantity the extent to which each effect impacts on the seller's revenue. Therefore we can only rely on the counterfactual analysis to quantity the overall effect of merger on the seller's revenue as we do here.

The effects on the revenue are presented in the second panels of Figure 5 to Figure 10. The first thing we note is that at the current levels of reserve price and dependence levels, both types of merges are beneficial to the seller, yielding gains of \$15/MBF and \$6/MBF in pro-competitive merger and anti-competitive merger respectively. This implies that the "competition effect" and the effect associated with a stronger competitor dominate the other effect in the pro-competitive merger, while in the anti-competitive merger the affiliation effect might be the dominant one. The effects on the revenue vary as the reserve price or dependence levels vary. In the anti-competitive merger, the merger could do harm to the seller for some values of the reserve price, for example, \$700/MBF, as is shown in Figure 5. From Figure 6 and Figure 7, it can be seen that high levels of dependece among bidders can make an anti-competitive merger undesirable. When the dependence level of private values is more than 0.2, the change in the revenue due to the merger is negative, and the change in the revenue is negatively related with the depence level of entry costs. This implies that high levels of dependence might alleviate the "affiliation effect" and strengthen the other two effects. On the other hand, in the pro-competitive merger, the interaction between the merger effect on revenue and the reserve price has almost the same pattern as in the anti-competitive merger. The merger could lower the revenue in some high level of dependence of private values, but the loss is becoming less as the dependence level increases from 1. In Figure 10, we can see that although the change in the revenue is decreasing in the dependence level of entry costs for the dependence larger than 0.2, it never becomes negative.

To summarize, we find the following results regarding the merger effects.

i. The merged bidder almost always participates in the auction.

ii. The merger almost has little impact on other bidders' entry behaviors.

iii. Both anti-competitive and pro-competitive mergers are beneficial to the seller at the current levels of reserve price and dependence levels.

iv. Pro-competitive merger is better than anti-competitive merger in terms of the change in the seller's revenue.

6 Conclusion

In this paper we study how affiliation and entry can affect bidders' bidding behavior and the seller's revenue using the timber sales auction data from the ODF. We develop an entry and bidding model with heterogeneous bidders within the APV framework, and establish the existence and uniqueness of the Bayesian-Nash equilibrium. We adopt a structural approach to obtain the estimates for the structural parameters in the bidders' private values distribution. We are able to quantify the extent to which the potential bidders' private values and entry costs are affiliated, respectively, and find that the affiliation among bidders' private information found in Li and Zhang (2008) is mainly driven by the affiliation among bidders' entry costs. We then use the structural estimates to conduct counterfactual analysis to address the policy-related issues. In particular, we quantify how the seller's revenue could change with the changes in the reserve price or the dependence level. Moreover, we quantify the merger effect and evaluate how it changes with the changes in the reserve price or the dependence level.

Since we allow bidders to be heterogeneous and have affiliated private values, and also take entry into account, our approach is general and closer to the real timber auction environment. On the other hand, the analysis of the end auction outcomes and welfare implications is complicated by the interactions of affiliation, asymmetry, and entry. The structural approach we propose offers a promising way to disentangle these effects through the counterfactual analysis in addressing policy-related issues such as the merger effect.

Appendix

The proof of Proposition adapts Lebrun (1999, 2006). We first need the following lemma.

Lemma. *Consider a continuously differentiable and strictly increasing bidding strategy. Assume* $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ *is decreasing in* u*. If* $\widetilde{\eta} > \eta$ *and* $\widetilde{s}_i^{-1}(b)$ *and* $s_i^{-1}(b)$ *for all* i *are two solutions of the system of differential equations* (3) *with boundary condition* (5) *over* $(\widetilde{\gamma}, \widetilde{\eta}]$ *and* $(\gamma, \eta]$*, respectively, then the inverse bidding functions satisfy the following condition:* $\widetilde{s}_i^{-1}(b) < s_i^{-1}(b)$ *for all* b *in* $(\max(\gamma, \widetilde{\gamma}), \eta]$*, where* $\gamma > \underline{v}$*.*

Proof. Since we know that s_i^{-1} is strictly increasing over $(\gamma, \eta]$, we have $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta) = \overline{v}$. Define g in $[\max(\gamma, \widetilde{\gamma}), \eta]$ as follows:

$$g = \inf\left\{b \in [\max(\gamma, \widetilde{\gamma}), \eta] \left| \widetilde{s}_i^{-1}(b') < s_i^{-1}(b'), \text{ for all } i \text{ and all } b' \in (b, \eta] \right.\right\}.$$

We want to prove that $g = \max(\gamma, \widetilde{\gamma})$. According to the definition of g, $\eta > g$. Suppose that $g > \max(\gamma, \widetilde{\gamma})$. By continuity, there exists i such that $\widetilde{s}_i^{-1}(g) = s_i^{-1}(g)$. From the definition of g, we also have $\widetilde{s}_j^{-1}(g) \leq s_j^{-1}(g)$ for all j. Moreover, there exists $j \neq i$ such that $\widetilde{s}_j^{-1}(g) < s_j^{-1}(g)$, because all the solutions conincide at the point g and therefore conincide in $(g, \eta]$ due to the fact that the right hand side of equation (3) is locally Lipschitz at $b = g$, which contradicts the fact that at point η $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta)$.

From equation (3), we know $ds_i^{-1}(b)/db$ is a strictly decreasing function of $s_j^{-1}(b)$, for all $j \neq i$, since $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. Consequently, $d\widetilde{s}_i^{-1}(b)/db > ds_i^{-1}(b)/db$. Therefore there exists $\delta > 0$ such that $\widetilde{s}_i^{-1}(b) > s_i^{-1}(b)$, for all b in $(g, g+\delta)$. This contradicts the definition of g. □

Proof of Proposition

Proof. First we prove the first part of the proposition by showing that there exist an η, such that $s_i^{-1}(\eta) = \overline{v}$.

(i) Bidding Equilibrium

Let $i, 1 \leq i \leq n$ denote bidders who have the highest bids, denoted by η', at the upper bound of private value $\overline{v}$ and $j, 1 \leq j \leq n$ denote bidders who has the second highest bid, denoted by η, at the upper bound of private value $\overline{v}$. So $\eta' \geq \eta$.

For bidder i, we know that

$$(\overline{v} - \eta') \Pr(B_{-i} < \eta' | \overline{v}) \geq (\overline{v} - \eta) \Pr(B_{-i} < \eta | \overline{v}).$$

It is obvious that $\Pr(B_{-i} < \eta' | \overline{v}) = 1$

$\Pr(B_{-i} < \eta | \overline{v}) = \Pr(b_j < \eta, b_k < \eta, k \neq i, j | v_i = \overline{v}) = \Pr(b_k < \eta, k \neq i, j | v_i = \overline{v})$, since b_j is not larger than η.

$\Pr(B_{-j} < \eta | \overline{v}) = \Pr(b_i < \eta, b_k < \eta, k \neq i, j | v_j = \overline{v})$.

Since the joint distribution of private values follows Archimedean copulas, we have

$$\begin{aligned}\Pr\left(B_{-i}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi\left(F_j\left(s_j^{-1}(\eta)\right)\right)+\phi\left(F_i\left(\overline{v}\right)\right)\right)\phi'\left(F_i\left(\overline{v}\right)\right)\\ &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi(1)+\phi(1)\right)\phi'(1)\end{aligned}$$

and

$$\begin{aligned}\Pr\left(B_{-j}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi\left(F_j\left(\overline{v}\right)\right)+\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)\right)\phi'\left(F_j\left(\overline{v}\right)\right)\\ &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi(1)+\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)\right)\phi'(1)\end{aligned}$$

If $F_i\left(s_i^{-1}(\eta)\right)<1$, then $\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)>\phi(1)$ and $\Pr\left(B_{-i}<\eta|\overline{v}\right)>\Pr\left(B_{-j}<\eta|\overline{v}\right)$ since $\phi'(1)<0$ and $\phi^{-1\prime}(x)$ is increasing in x. Therefore

$$\left(\overline{v}-\eta'\right)\Pr\left(B_{-j}<\eta'|\overline{v}\right)>\left(\overline{v}-\eta\right)\Pr\left(B_{-j}<\eta|\overline{v}\right)$$

since $\Pr\left(B_{-j}<\eta'|\overline{v}\right)=1$. But this is impossible because the optimal bid of bidder j at $\overline{v}$ is η, therefore we have $F_i\left(s_i^{-1}(\eta)\right)=1$ and $\eta'=\eta$.

(ii) Uniqueness of Bidding Equilibrium

Suppose that there exist two equilibria and thus two different values η and $\widetilde{\eta}$ such that the respective solutions $s_i^{-1}(b)$ and $\widetilde{s}_i^{-1}(b)$ are also solutions of the system of differential equations for all i. Without loss of generality, we can assume that $\eta<\widetilde{\eta}$. The value of $\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)$ at $b_i=\eta$ is strictly larger than the value of $\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}(b_i),j\neq i|v_i\right)\right)$ at the same point. We have shown that $\widetilde{s}_i^{-1}(b)<s_i^{-1}(b)$ for all b in $(\underline{v},\eta]$. When b converges to $\underline{v}$, $s_i^{-1}(\underline{v})=\underline{v}$.

On the other hand, the first order condition can be written as follows

$$\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)}{db}=\frac{1}{s_i^{-1}(b_i)-b_i}.$$

So $\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}(b),j\neq i|v_i\right)\right)}{db}<\frac{d\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}(b),j\neq i|v_i\right)\right)}{db}$. Therefore, the difference between these two logarithms increases as b decreases towards $\underline{v}$. On the other hand, $\ln\left(\Pr\left(v_j<\underline{v},j\neq i|v_i\right)\right)$ is a finite value since $F_j(\underline{v})>0$. Therefore for two solutions, $\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)$ cannot both coverge to the same finite value as b decreases towards $\underline{v}$. Therefore η and $\widetilde{\eta}$ conincide and the equilibrium is unique.

(iii) Entry Equilibrium

The entry probability p_i is determined by equation (6). Let $p=(p_1,\ldots,p_n)\in[0,1]^n$ and

$G_p = (G_1 \circ \Pi_1(p), \ldots, G_n \circ \Pi_n(p))$. Since $s_i(v)$ and G_i is continuous, the pre-entry expected proft Π_i and $G_i \circ \Pi_i$ is continuous in p. So $G_p : [0,1]^n \to [0,1]^n$ and is continuous in p. A fixed point of p follows Kakutani's fixed point theorem (Kakutani (1941)). $\square$

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Table 1: Summary Statistics of Bidder- and Auction-specific Covariates

	Observation	**Mean**	**Std. Dev.**
Bid	245	376.6915	96.5321
# of Potential Bidders	81	4.2099	0.8619
# of Actual Bidders	81	3.0247	1.3036
Entry Proportion	81	.7251	.2852
Appraised Price	81	329.7940	95.5464
Distance	341	75.9155	45.3968
Volume	81	3644.333	3085.016
Duration	81	803.4355	212.5968
Grade	81	10.3236	.4570
DBH	81	16.7655	4.8213

Table 2: Estimation Results

	Private Value distribution		**Entry Cost distribution**	
	Coefficient	**Std. Error**	**Coefficient**	**Std. Error**
Hauling Distance	-.1379	.006	1.3672	.5309
Volume	.0593	.0152	.0685	.024
Duration	-.0769	.0167	.0291	.0237
Grade	.9587	.0757	1.0283	.0057
DBH	.1751	.0661	-.0756	.071
Dependence Parameter	.1014	.0306	0.5015	.1363

Figure 1: Histograms of Original and Simulated Data

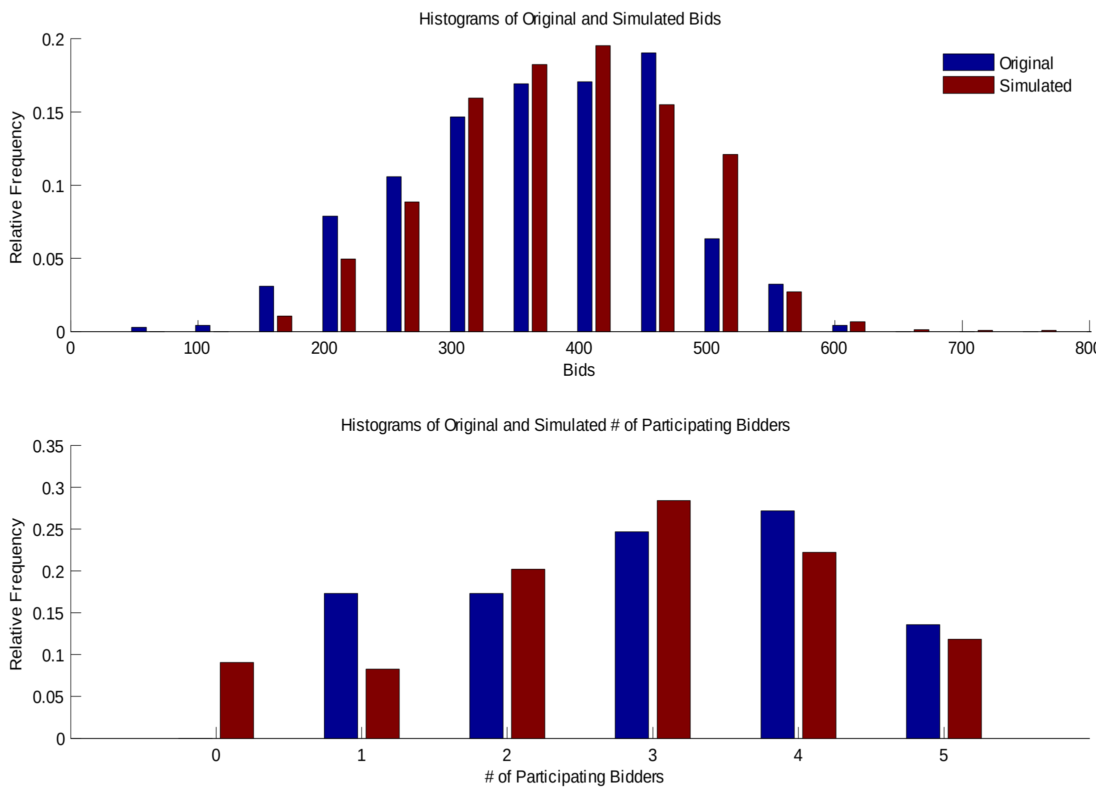

Figure 2: Effect of Reserve Price

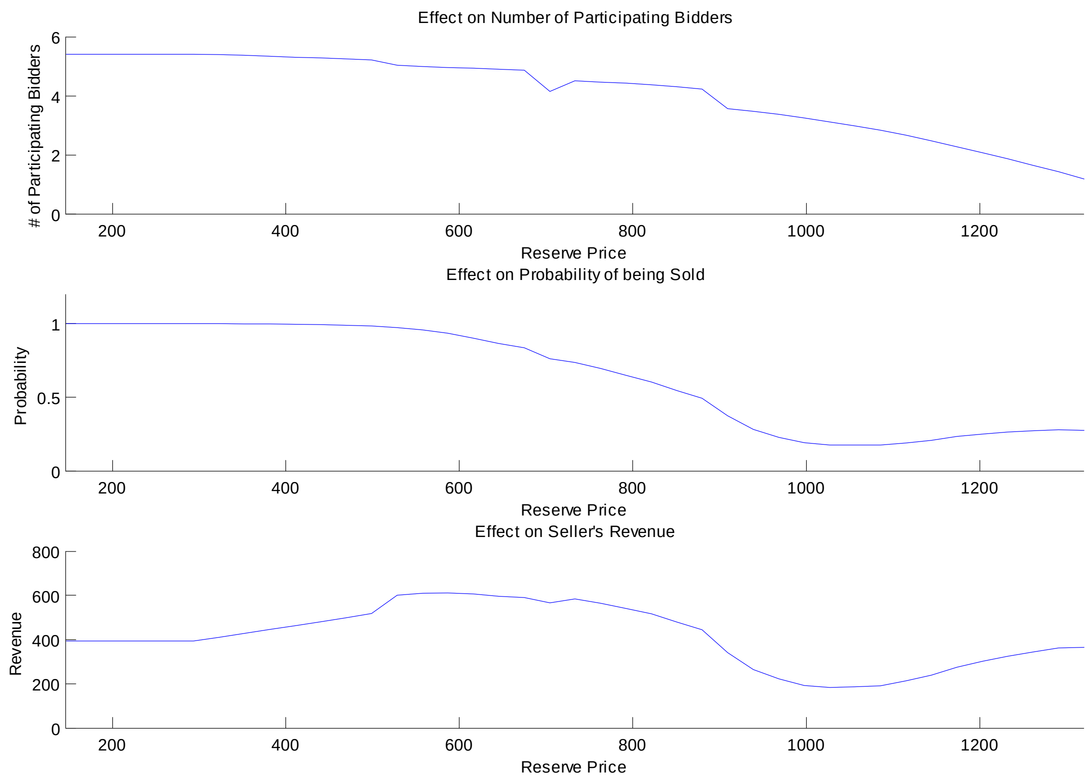

Figure 3: Effect of Dependence Level of Private Values

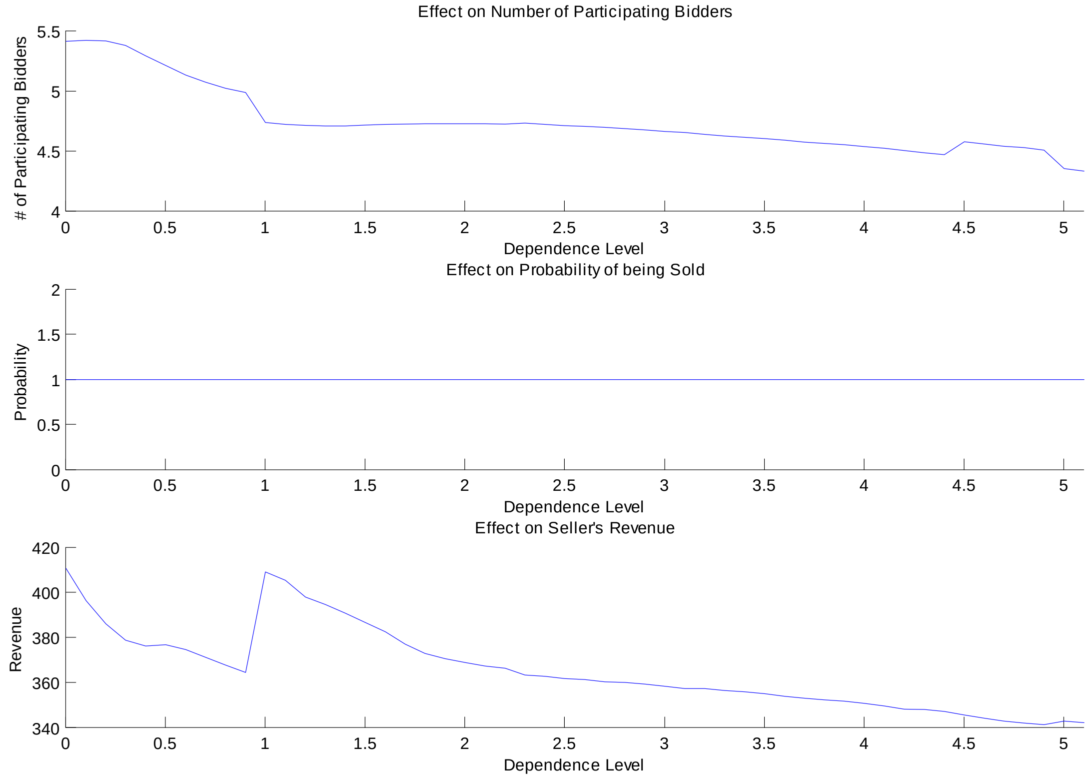

Figure 4: Effect of Dependence Level of Entry Costs

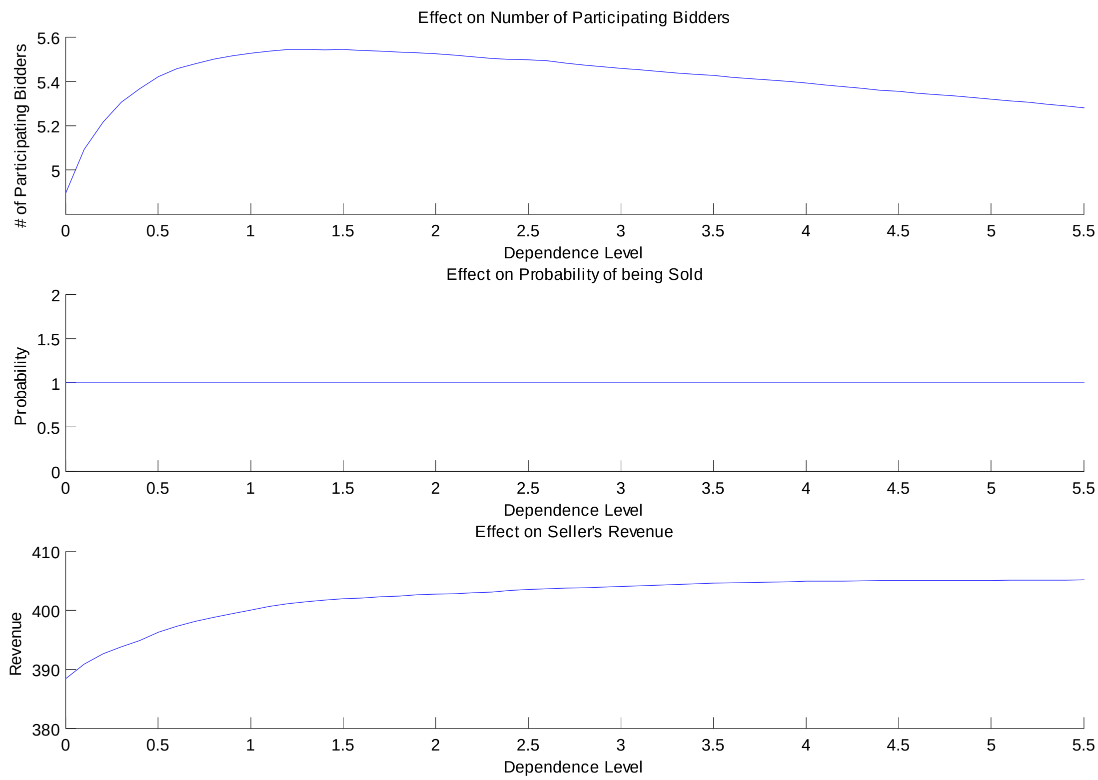

Figure 5: Interaction between Anti-competitive Merger and Reserve Price

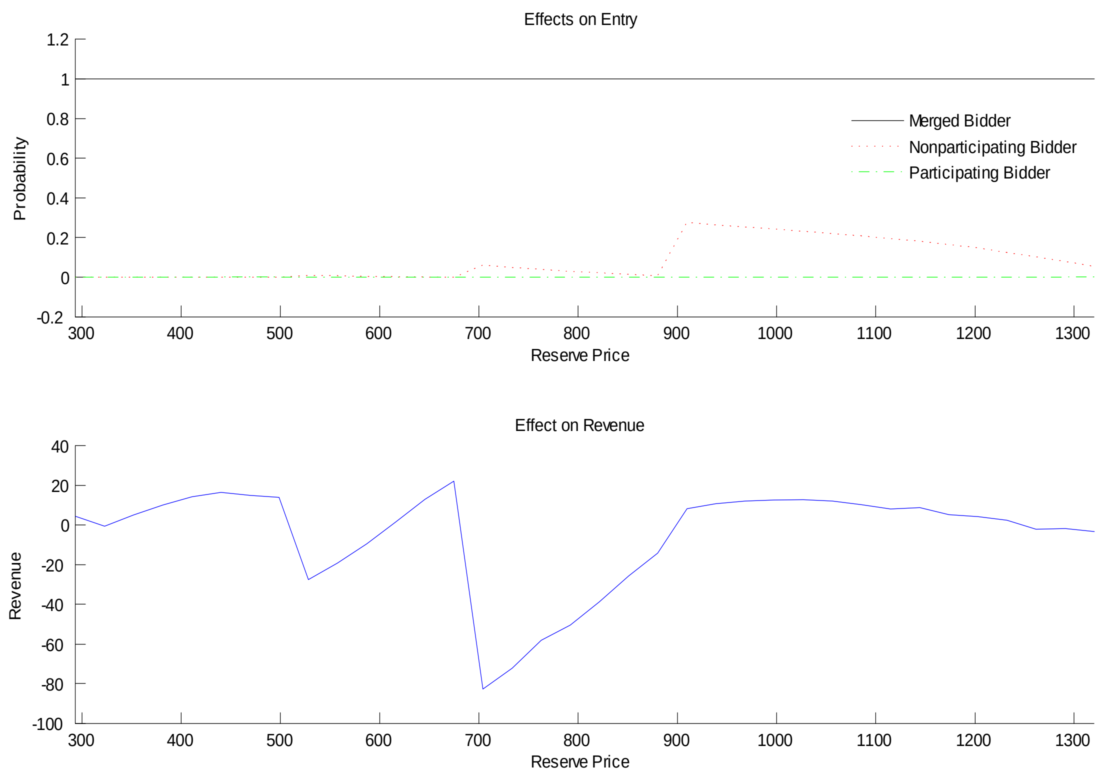

Figure 6: Interaction between Anti-competitive Merger and Dependence Level of Private Values

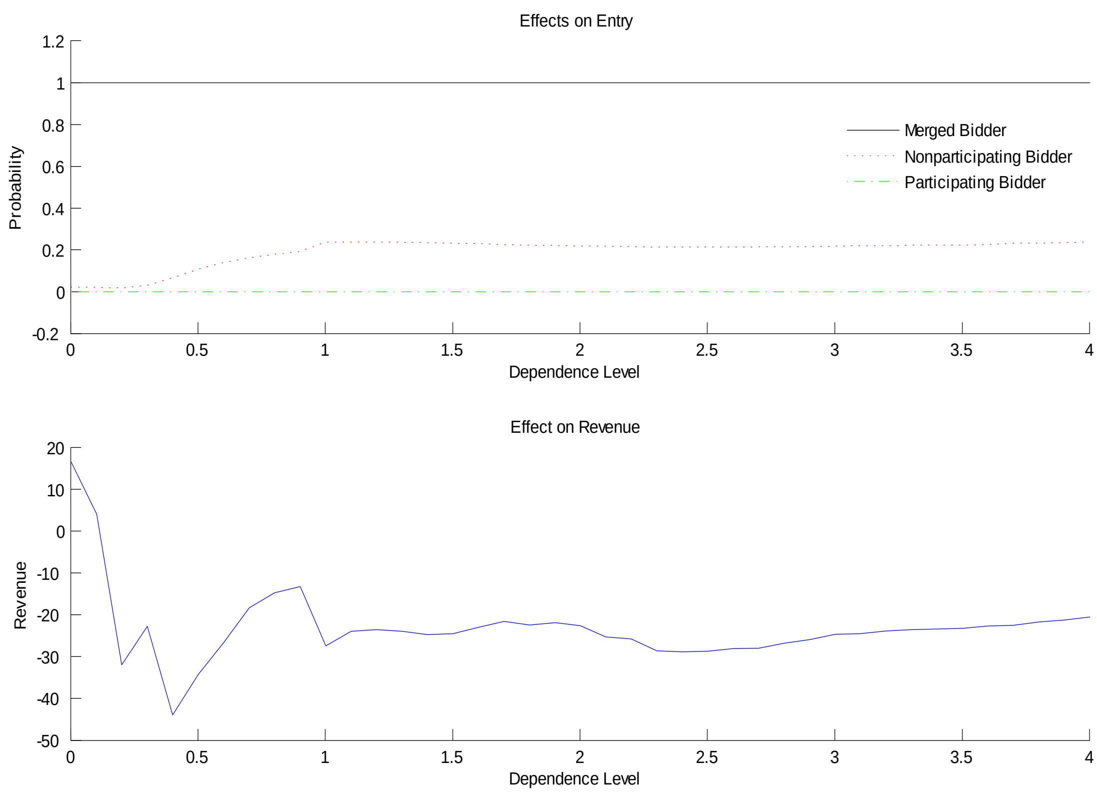

Figure 7: Interaction between Anti-competitive Merger and Dependence Level of Entry Costs

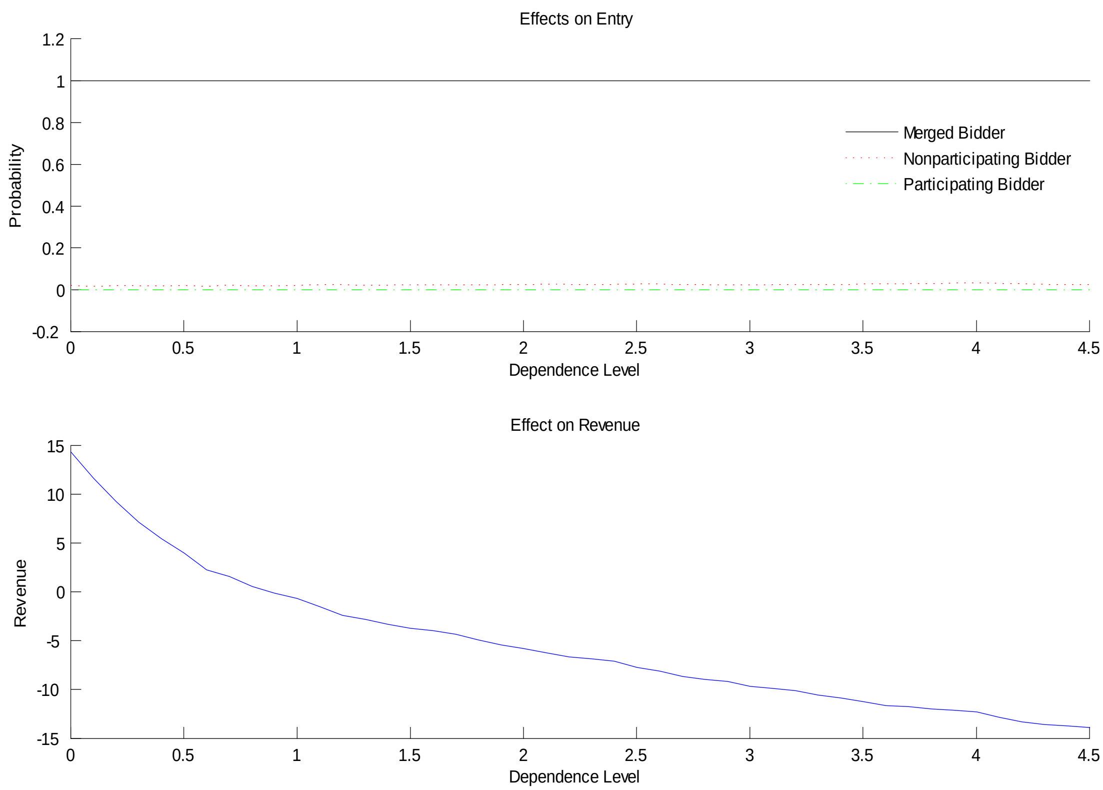

Figure 8: Interaction between Pro-competitive Merger and Reserve Price

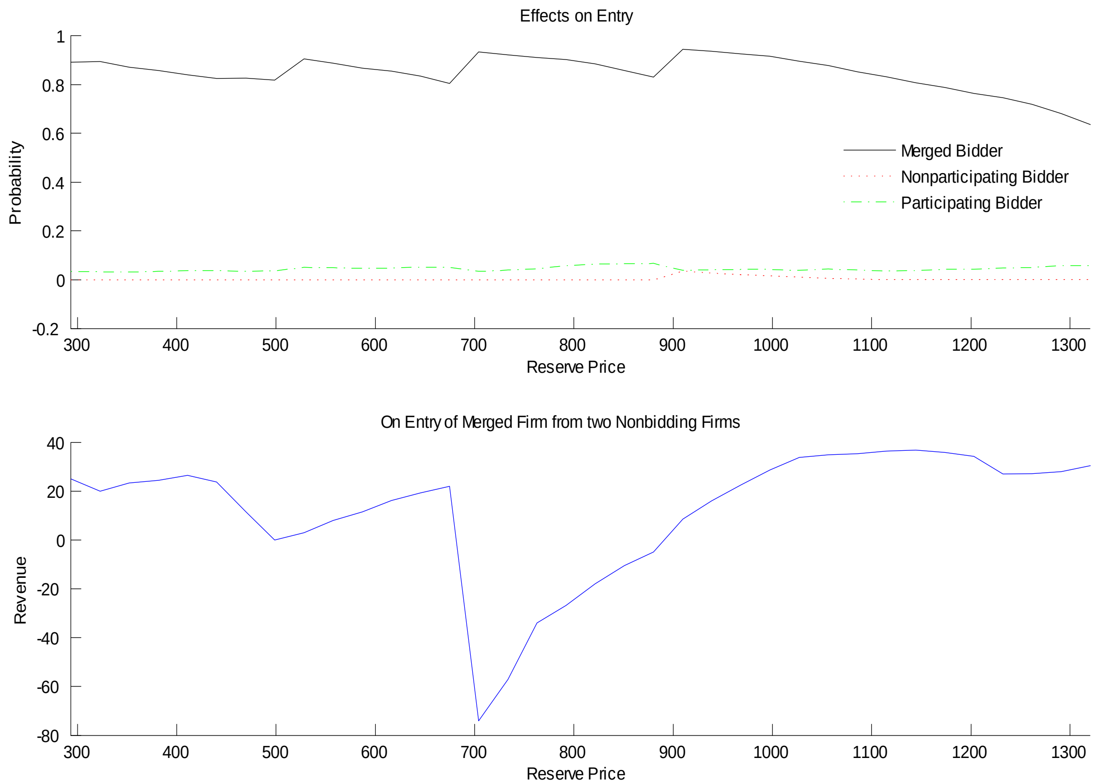

Figure 9: Interaction between Pro-competitive Merger and Dependence Level of Private Values

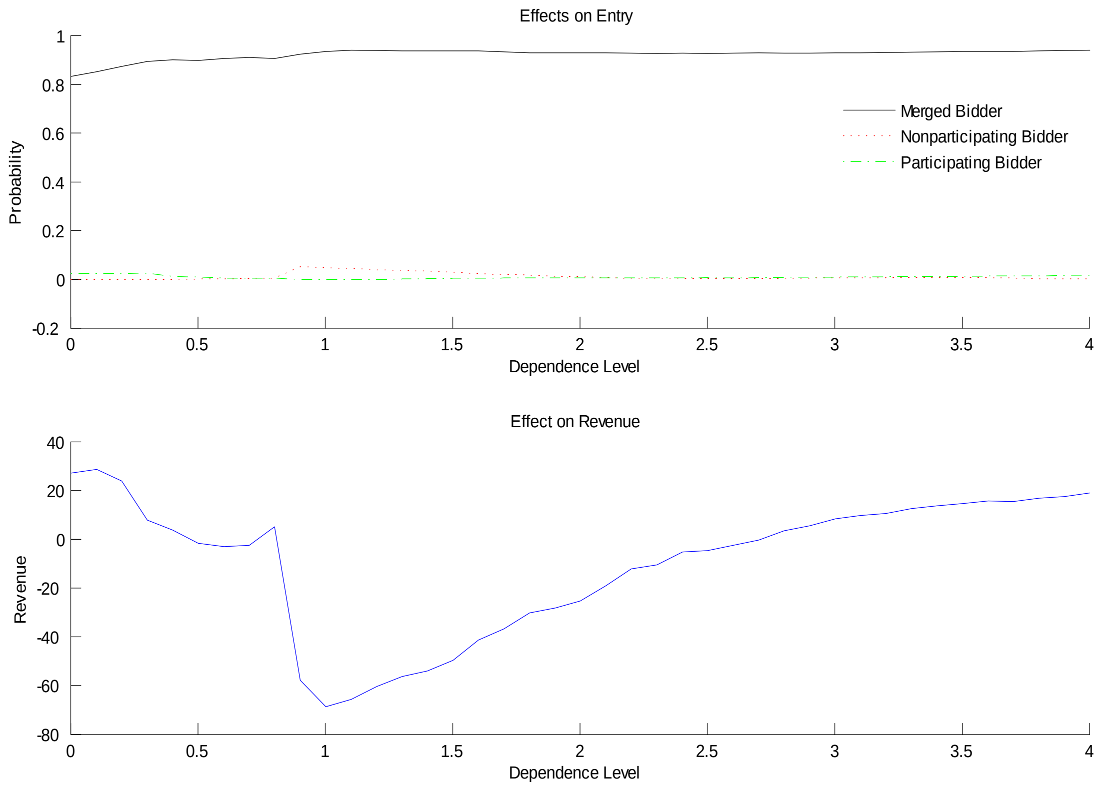

Figure 10: Interaction between Pro-competitive Merger and Dependence Level of Entry Costs