Split-award Auctions with Investment*

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Abstract

This paper studies split-award procurement auctions where a buyer can either divide full production among multiple suppliers or award the entire production to a single supplier. The literature shows that single sourcing usually dominates multiple sourcing. This paper challenges the "winner-takes-all" argument. In a framework of generalized second-price auctions with pre-auction investment, we show that splitting the award improves the suppliers' investment incentives, intensifies competition at the bidding stage, and minimizes the buyer's procurement costs. Finally, in an N-supplier setting, using quadratic investment technology, we illustrate that it is optimal for a buyer to restrict the number of suppliers to two.

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1 Introduction

This paper examines the effect of using multiple suppliers as investment incentives. The issue is relevant to the purchase of generic drugs, defense procurement and other contracting environments.

One of the controversial issues hotly contested in the health care reform debate is the idea that the federal government should leverage its bargaining power to purchase prescription drugs on behalf of consumers in an attempt to reduce medicine expenditures, currently accounting for roughly 10% of the total health care costs.[1] Advocates of this school of thinking cite examples in Canada and Australia that see significantly lower prices vis-à-vis the U.S. for the same drugs.

Collective bargaining is actually already happening. The Department of Veteran Affairs (VA) and the Department of Defense (DoD) have carried out several joint programs over the years that engage in various drug purchase arrangements with pharmaceutical companies. Recent debates of the Obama health care reform cite cost savings as high as 40% for the VA through its huge bargaining power. Opponents argue that government's middleman role in drug sales amounts to price controls that will reduce biotechnology and pharmaceutical R&D efforts.[2]

The opponents' argument does not apply to generic drugs. Since compound formulas for generic drugs and the manufacturing process are well-known once patents of branded drugs expire, the remnant room for innovation seems to be only in areas of production efficiency and manufacturing cost reduction.

In the absence of innovation and R&D issues, it seems to be good public policy to extend the government purchase practices in VA and DoD programs to all of Medicare recipients.[3] However, while the idea of government collective bargaining in drug purchase has stirred much controversy in the health care reform debate, how to go about designing efficient mechanisms to purchase generic drugs has attracted scant attention. Currently,

[1]According to a Kaiser Family Foundation report, the average health care expenditure in the US in 2008 is about $7,400. See "Health Care Costs, a Primer," March, 2009.

[2]For example, it is reported that "the resulting government-induced loss of capitalized pharmaceutical R&D expenditures was $188 billion (in 2000 U.S. dollars) from 1960 to 2001. This "lost" R&D may be translated into human life years "lost", ··· approximately 140 million life years between 1960 and 2001." See Vernon, Santerre, and Giaccotto (2004).

[3]We will investigate the impact of various government purchase mechanisms on investment incentives to innovate in new drug development in a forthcoming paper.

VA and DoD conduct a competition for an exclusive contract with one manufacturer for supply of generic drugs. Due to the share size of the potential market and incentive concerns for the supplier to continuously invest in new technologies to streamline production and reduce costs, the single source solution may be less than optimal from the perspective of minimizing procurement costs.

The practice of employing an auction format that allows for a split-award outcome is widely observed. Burnett and Kovacic (1989) reported that in DoD's procurement program, guaranteeing a minimum share of production is particularly important when DoD wishes to induce a firm to bid against an established producer. In Japan's telecommunications industry, as reported in Fransman (1995), "Competition between the suppliers is not of the 'winner-takes-all' variety. Rather, it involves controlled competition in so far as, contingent on reasonable performance as judged and monitored by NTT, each supplier can expect to receive a sizable share of NTT's order." China Mobile, the world's largest mobile carrier, also conducts regularly several rounds of supplier tournaments in each year, where large chunks of equipment and mobile handset contracts are divided among a few vendors. The practice is also very common in the private sector. Wal-mart's vitamin business adopts a multiple-source model, relying on several of its vendors in China for supply contracts. Similarly, Tunca and Wu (2009) documented that companies such as Sun and HP that use online auctions to procure products worth hundreds of millions of dollars frequently opt for multiple sourcing.

The issue of single-source versus multiple-source contract has been studied extensively in the literature. The literature overall concludes that single sourcing usually dominates multiple sourcing in a variety of settings, as we will detail below. This paper challenges the "winner-takes-all" conclusion. In a framework of generalized second-price auctions (GSP) preceded by a stage where suppliers have an opportunity to invest to lower their marginal costs of production, we show that splitting the award improves the suppliers' investment incentives, intensifies competition at the bidding stage, and lowers the buyer's overall procurement costs in comparison to single sourcing.

In a two-supplier setup, two sets of pure strategy equilibrium may emerge: symmetric and asymmetric equilibrium. The dichotomy between the two depends critically on the percentage of the split α that goes to the winner, which, not surprisingly, should stay between 0.5 and 1. If α is too small, for example close to 0.5, the incentive to win is very low. The two suppliers would end up in a symmetric equilibrium with mediocre investments. If α is large, for example equal to 1, which implies that the winner would be awarded handsomely while the loser gets nothing, one of the two suppliers will choose to invest heavily to lower marginal cost while the best response of the other supplier is to invest little. This leads to an asymmetric equilibrium. The resulting procurement cost for the buyer is also high because the loser creates little competitive threat. By reducing α below 1, the incentive of winning at the bidding stage is lower, but the high cost supplier makes a positive investment, and becomes a stronger competitive threat. This chain reaction in improvement of investment incentives drives the suppliers to lower their bids in the bidding stage, and hence can offset the disincentive of winning when α is marginally below 1. This increased competition also results in a lower procurement cost for the buyer. Consequently the optimal split for the buyer would lie somewhere in the interval $(0.5, 1)$, determined by the point where the marginal disincentive of winning and the incentive of investment exactly offset each other.

Using a quadratic investment function as an example, we show that the optimal split is about 0.707 for the low bidder and 0.293 for the high bidder. The result of the example matches remarkably well with the practice of the U.S. government in its defense procurement activities, who typically commits that the high bidder will receive a fraction of the order (e.g. 30%) if the high bid is within "the competitive range". (See Gilbert and Klemperer (2000).) In this example, the savings can be more than 20% over the winner-takes-all mechanism.

We then extend our analysis to N-suppliers, where we first derive sufficient conditions on the fractional assignments to guarantee truthful bidding. The optimal design problem then reduces to a series of conditions that need to be imposed upon the fractional assignments in the buyer's cost minimization program. Assuming a quadratic investment technology, we characterize the optimal fractional assignments for the buyer. The result shows that the buyer can do no worse with only two suppliers than with more, a strong result reminiscent of Fullerton and McAfee (1999). In this special case, the 2-supplier analysis can be applied for policy implications without loss of generality.

Our results shed light on the implementation mechanisms for government drug purchase programs that are gaining increasing traction in the current health care reform debate. One policy implication is that introducing just one more supplier to the procurement process can adequately intensify the competition such that each party would be engaged in substantial investments to reduce costs, ultimately benefiting the government. The administrative cost increase of managing two suppliers versus one in the process seems to be modest, while the potential cost savings can be substantial. Our results can also be applied to other procurement contexts where the product at issue is fairly standardized and supplier investment incentives are sought after, such as the purchase of computers.

The paper is organized as follows. We start by relating our contribution to the literature on split-award auctions, auctions preceded with investments and generalized second-price auction. Section 2 introduces the setup of the model and the time structure of the procurement game. Section 3 establishes the equilibrium bids at the bidding stage, while Section 4 presents major results of the paper. Section 5 extends the analysis to many suppliers and shows the optimality of restricting the number of suppliers to two. Concluding remarks are contained in Section 6. All proofs and technical details can be found in the Appendix.

Related Literature

There is a sizable literature studying optimal procurement practices where the primary concern is a buyer's strategic choice between single sourcing and multiple sourcing.[4] One of the early seminal contributions to this literature is that by Anton and Yao (1989). In a framework where suppliers submit bids on each possible split of a contract, they show that split-award low-price auctions typically lead to higher price for the buyer. Indeed, they conclude that "(the) equilibria (of the split-award auctions) have the property that the price to the buyer is maximized$\cdots$ Thus, not only do split-award auctions fail to promote competition, they effectively present bidders with an invitation for implicit price collusion". Perry and Sákovics (2003) analyzes a sequential second-price auction where a larger primary contract and a smaller secondary contract are awarded, and show that if the number of suppliers is fixed, sole sourcing leads to a lower procurement cost. Inderst (2008) confirms the result that a monopolistic buyer conducting an auction strictly prefers single sourcing. A number of papers then go on to develop arguments for when and why split-award auctions could still be beneficial. For instance, Riordan and Sappington (1989) show how second sourcing reduces information rents in a dynamic setting. Anton and Yao (1992) argue that if suppliers have poor information on each other's cost, their ability of coordinating bids is limited and split-award may be beneficial. Anton, Br-

[4] The analysis of split-award auctions started with Wilson (1979), which analyzes share auctions where bidders receive fractional shares of the item at a price that equates demand and supply of shares and he shows that share auctions generally decrease seller revenue in comparison to unit auctions where the item is awarded to the highest bidder. Bernheim and Whinston (1986) analyse first-price menu auctions and show that they always achieve efficient allocation.

usco, and Lopomo (2009) argue that uncertainty regarding scale economies may provide a rationale for the use of split-award. Using a dataset on the missile system by the U.S. Defense Department, Lyon (2006) empirically supports that dual sourcing indeed lowers government procurement costs significantly. In our paper, the driving force of the optimality of split-award auctions is the provision of investment incentives to the suppliers so that they invest to lower their marginal costs of production, which in turn lower their bids in the bidding stage and hence the buyer's procurement cost becomes lower as well.

The choice of auction formats affects bidders' investment incentives in a nontrivial way. King, Welling, and McAfee (1992) and Arozamena and Cantillon (2004) show that while a second-price auction typically provides efficient investment incentives, suppliers tend to underinvest under first-price auctions. Bag (1997) shows that if the buyer can charge discriminatory entry fees, a second-price auction is efficient and is optimal for the buyer. Tan (1992) shows that first- and second-price auction are revenue equivalent if investment technologies exhibit diminishing and constant return to scales, and in Piccione and Tan (1996), it is shown that these auction formats are also efficient.[5] We depart from this literature by considering the optimal share splitting structure that minimizes a buyer's procurement cost, instead of focusing on sole-sourcing standard auctions.

Finally, our formulation of the bidding stage is related to the growing literature on generalized second-price auction and its wide application in the online market, eg. sponsored search auctions run by Overture (now part of Yahoo) and Google. Representative works include Edelman and Ostrovsky (2007), Edelman, Ostrovsky, and Schwarz (2007), Börgers, Cox, Pesendorfer, and Petricek (2007). These studies focus on bidders' strategic behaviors under this auction format. By putting the analysis in perspective, we show that provision of investment incentives has a great impact on the design of the allocation structure of the auction. Our work suggests that the GSP encourages more competition via enhanced ad design compared to placing a single advertisement.

2 The Model

A buyer must procure one unit of certain good which is fully divisible. There are two potential suppliers, $i = \{1, 2\}$. The suppliers have marginal costs $c_i \in [0, \omega]$ for producing

[5] King, McAfee, and Welling (1993) present a model where governments first invest in their infrastructure level, and then compete in auctions for a plant to be built in their region by a single firm. They show that the unique equilibrium of the game exhibits asymmetry in investment.

one unit of the good.

In allocating the contract, the buyer uses a generalized second-price split-award auction where each supplier submits a sealed price b_i for the contract. The supplier with lower bid is awarded an α fraction of the production contract at a price equal to the higher bid. The supplier who has submitted the higher bid is awarded the fraction $1-\alpha$ of the contract at a price equal to the maximum cost ω.

Before bidding takes place, each supplier has an opportunity to make an investment to reduce his marginal cost. In particular, marginal cost c requires an investment $g(c)$ which is three times continuously differentiable and has the following properties:

$$g'(c) < 0, \qquad g''(c) > 0, \tag{1}$$

$$g(\omega) = 0, \qquad \lim_{c \to \omega} g'(c) \uparrow 0, \qquad g'(0) < -1 \tag{2}$$

Condition (1) implies that the investment technology is strictly decreasing and convex. In Condition (2), $g(\omega) = 0$ implies that if a supplier invests nothing, his marginal cost of production is ω. $g'(\omega) = 0$ implies at $c = \omega$, it is nearly costless for a supplier to make some investment to improve his technology. Therefore, if there is any positive probability of winning a contract, it is worthwhile for a supplier to invest positively to reduce his marginal cost below ω. $g'(0) < -1$ implies that the cost of achieving a marginal cost of 0 is higher than its value.

The time structure of the game is:

1. The buyer chooses and announces α.

2. Suppliers simultaneously and independently choose investment levels g_i. Marginal costs c_i are realized and revealed.

3. Bids are submitted simultaneously, and restricted to lie in $[0, \omega]$. The lower bidder is awarded fraction α of the contract at a price equal to the higher bid. The higher bidder is awarded fraction $1-\alpha$ of the contract at a price equal to ω. In the event of a tie, each bidder is assigned an equal probability of being the lower bidder.

The buyer's sole objective is to design α in such a way that her procurement cost is minimized.

Note that for the buyer, any outcome with $\alpha \in [0, \frac{1}{2}]$ can be replicated by setting $\alpha = 1$. Given any $\alpha \in [0, \frac{1}{2}]$, the unique equilibrium in the bidding stage is $b_1 = b_2 = \omega$. Total procurement cost for the buyer is ω.

If the buyer sets $\alpha = 1$, the bidding stage then becomes a standard second-price procurement auction. In that case, bidders truthfully bid their costs, and the procurement cost is equal to the higher bid, which in turn is equal to the higher marginal cost. A supplier knowing that he will be the higher bidder expects to receive a zero fraction of the contract, which means zero payoff from the bidding stage, and will not invest anything to reduce his marginal cost. Hence, when $\alpha = 1$, the total procurement cost for the buyer is also ω. *In the following we can safely restrict our analysis to* $\alpha \in (\frac{1}{2}, 1]$.

3 The Bidding Stage

If $\alpha = 1$, the game at the bidding stage is a standard Vickrey auction and truthful bidding is an equilibrium in dominant strategies for the suppliers. When $\alpha \in (\frac{1}{2}, 1)$, bidders no longer have dominant strategies and truthful bidding may no longer be an equilibrium. Suppose supplier 2 bids truthfully and consider supplier 1's strategy. If supplier 1 has lower marginal cost than supplier 2, ie. $c_1 < c_2$, by bidding truthfully, supplier 1 gets a payoff equal to $\alpha(c_2 - c_1)$. If he places a bid above c_2, his payoff would be $(1-\alpha)(\omega - c_1)$. Truthful bidding forms an equilibrium if and only if $\alpha(c_2 - c_1) \geq (1-\alpha)(\omega - c_1)$. If $\alpha = 1$, this condition always holds. When $\alpha < 1$, truthful bidding is supported as an equilibrium only if c_2 is sufficiently larger than c_1 and α is sufficiently large.

In the next proposition, we characterize pure strategy equilibrium at the bidding stage. We denote the marginal costs realized from the investment stage by c_L and c_H with $c_L \leq c_H$, and denote (b_L, b_H) to be the bids of the low cost and high cost suppliers respectively.

Proposition 1. *The following strategies form an equilibrium of the bidding stage:*

$$b_L^* = c_L, \qquad b_H^* = \max\{c_H, \frac{1-\alpha}{\alpha}(\omega - c_L) + c_L\} \tag{3}$$

Proposition 1 implies that at the bidding stage, the low cost supplier bids less than the high cost supplier in equilibrium. As a result the low cost supplier is awarded α fraction of the contract and the high cost supplier is awarded $1-\alpha$ fraction of the contract. Also note that b_H^* is a decreasing function in α. A large value of α leads to more aggressive bid for the high cost supplier.

The equilibrium bid identified in Proposition 1 is not unique. Indeed, any b_L from the interval $[c_L, c_H)$ and b_H^* are mutually best responses and form an equilibrium of the bidding game, and all these equilibria achieve the same allocation. Among these equilibria, the bidding functions identified in Proposition 1 are the lowest equilibrium bids consistent with undominated strategies.

In the next proposition, we characterize the suppliers' payoffs and buyer's payment at the bidding stage.

Proposition 2. *At the bidding stage, the payoffs of the low cost and high cost suppliers are respectively*

$$\pi_L^* = \max\{\alpha(c_H - c_L), (1-\alpha)(\omega - c_L)\}; \qquad \pi_H^* = (1-\alpha)(\omega - c_H) \tag{4}$$

Total procurement cost of the buyer for given α is:

$$m(\alpha) = \max\{(1-\alpha)\omega + \alpha c_H, 2\omega(1-\alpha) + (2\alpha - 1)c_L\} \tag{5}$$

If $\alpha = 1$, the buyer awards the entire contract to the low cost supplier at a price equal to c_H, and the low cost supplier gets a payoff equal to $c_H - c_L$, which is exactly the outcome of a standard Vickrey auction.

4 The Investment Stage

At the investment stage, the two suppliers simultaneously choose their investment levels, which determine their marginal costs. Due to the monotonicity of the investment technology, the game at the investment stage is exactly the same as one where the two suppliers choose simultaneously their marginal costs of production. Using the results of Proposition 2, the suppliers' expected payoffs at the investment stage are respectively

$$\pi_L(c_H, c_L) = \max\{\alpha(c_H - c_L) - g(c_L), (1-\alpha)(\omega - c_L) - g(c_L)\}; \tag{6}$$

$$\pi_H(c_H, c_L) = (1-\alpha)(\omega - c_H) - g(c_H) \tag{7}$$

Note that for the supplier that turns out to be the high cost one, there is a unique optimal choice of marginal cost c_H^*, defined by

$$-(1-\alpha) = g'(c_H^*). \tag{8}$$

For the other supplier, there are two candidates for marginal costs, one the same as c_H^* and the other denoted c_L^* that solves

$$-\alpha = g'(c_L^*). \tag{9}$$

Therefore, there are two candidates for pure strategy equilibria at the investment stage:

1. Symmetric equilibrium: $(c_H^*(\alpha), c_H^*(\alpha))$;

2. Asymmetric equilibria: $(c_H^*(\alpha), c_L^*(\alpha))$ and its mirror case $(c_L^*(\alpha), c_H^*(\alpha))$.

Given that the other supplier has chosen c_H^* as his marginal cost, a given supplier compares the magnitude of $\alpha(c_H^* - c_L^*) - g(c_L^*)$ and $(1-\alpha)(\omega - c_H^*) - g(c_H^*)$ to determine his optimal choice. If the former is bigger, he prefers the lower marginal cost c_L^*. Otherwise he prefers the higher marginal cost c_H^*. Which scenario occurs depends on the buyer's procurement policy α.

Lemma 1. *There exists a unique $\tilde{\alpha}$ such that $\alpha \gtreqless \tilde{\alpha}$ if and only if,*

$$LHS := \alpha\left(\omega - c_L^*\right) \gtreqless \omega - c_H^* - \left(g(c_H^*) - g(c_L^*)\right) := RHS \tag{10}$$

When LHS is bigger than RHS in condition (10), $\alpha(c_H^* - c_L^*) - g(c_L^*) \geq (1-\alpha)(\omega - c_H^*) - g(c_H^*)$ holds. It is then optimal for a supplier to choose the marginal cost c_L^* over c_H^*, given the other's choice of c_H^*. In the next proposition, we state the equilibria of the investment stage.

Proposition 3. *1. If $\alpha \geq \tilde{\alpha}$, (c_L^*, c_H^*) and (c_H^*, c_L^*) are equilibria at the investment stage;*

2. If $\alpha < \tilde{\alpha}$, (c_H^*, c_H^*) forms an equilibrium at the investment stage.

From Proposition 3, we learn that by choosing different α, the buyer can elicit different equilibrium investment behavior from the suppliers. Now we consider the question which equilibrium is more desirable to the buyer, that is, which value of α minimizes her procurement cost. We first establish that choosing a sufficiently big α to elicit the asymmetric equilibria is always better than a small α where the symmetric equilibrium occurs, and then analyze the buyer's optimal procurement policy.

Lemma 2. *Choosing $\alpha \geq \tilde{\alpha}$ dominates $\alpha < \tilde{\alpha}$ for the buyer.*

When the splitting rule decreases from some arbitrary $\alpha_1 > \tilde{\alpha}$ to arbitrary $\alpha_2 < \tilde{\alpha}$, the buyer's expected payment to the high bidder increases by $(\alpha_1 - \alpha_2)\omega$, and the payment to the low bidder decreases from $\alpha_1 b_H^*(\alpha_1)$ to $\alpha_2 b_H^*(\alpha_2)$. The cost difference is at least equal to $(2\alpha_1 - 1)(\omega - c_L^*(\alpha_1)) - (2\alpha_2 - 1)(\omega - c_H^*(\alpha_2))$. Since

$$c_H^*(\alpha_1) > c_H^*(\alpha_2) > c_L^*(\alpha_2) > c_L^*(\alpha_1)$$

by convexity of $g(\cdot)$ and $\alpha_1 > \alpha_2$, the cost difference is positive and the procurement cost under $\alpha < \tilde{\alpha}$ is higher.

The buyer's design problem is now reduced to finding the optimal $\alpha^* \geq \tilde{\alpha}$ that minimizes the following objective:

$$m(\alpha \mid \alpha \geq \tilde{\alpha}) = \max\{(1-\alpha)\omega + \alpha c_H^*, 2\omega(1-\alpha) + (2\alpha - 1)c_L^*\} \tag{11}$$

Rewriting (11) gives:

$$\begin{aligned} m(\alpha \mid \alpha \geq \tilde{\alpha}) =& (1-\alpha)\omega + \alpha c_L^* + g(c_L^*) \\ &+ \max\{\alpha(c_H^* - c_L^*) - g(c_L^*), (1-\alpha)(\omega - c_L^*) - g(c_L^*)\} \end{aligned}$$

For $\alpha \geq \tilde{\alpha}$, we have

$$\alpha(c_H^* - c_L^*) - g(c_L^*) \geq (1-\alpha)(\omega - c_H^*) - g(c_H^*) > (1-\alpha)(\omega - c_L^*) - g(c_L^*)$$

where the last inequality obtains due to the definition of c_H^*. Therefore,

$$\begin{aligned} m(\alpha \mid \alpha \geq \tilde{\alpha}) &= (1-\alpha)\omega + \alpha c_L^* + g(c_L^*) + \alpha(c_H^* - c_L^*) - g(c_L^*) \\ &= (1-\alpha)\omega + \alpha c_H^* \end{aligned} \tag{12}$$

In the next proposition, we establish that the buyer's optimal choice of α is strictly smaller than 1, ie. splitting the contract is strictly better than awarding the whole production to a single supplier.

Proposition 4. *Splitting the contract between the two suppliers by choosing $\alpha < 1$ is optimal for the buyer.*

This proposition is demonstrated by showing $\frac{\partial m}{\partial \alpha} |_{\alpha=1} > 0$, and thus a slight reduction in α reduces buyer cost, starting at $\alpha = 1$. A reduction in α has a zero first order effect on cost but a positive first order effect on the price paid to the low cost supplier. More technically, the objective of the buyer can be decomposed into two parts: the payment made to the high bidder $(1-\alpha)\omega$ and the payment made to the low bidder αc_H^*. When the suppliers' marginal costs are exogenous, it is optimal for the buyer to set $\alpha = 1$ and award the entire contract to the low bidder. However, when marginal cost is endogenous, the choice of α affects the suppliers' payoffs from the bidding stage, in turn their investment incentives, and finally the buyer's procurement cost. At $\alpha = 1$, though the payment to the high bidder is zero, the payment to the low bidder is maximized since for $\alpha = 1$

the equilibrium outcome at the investment stage is such that one of the suppliers does not invest at all and hence the resulting higher marginal cost takes its maximal value ω. Therefore, the buyer's procurement cost at $\alpha = 1$ is ω, the highest possible cost. By decreasing α marginally, the payment to the high bidder increases at a rate equal to ω. The high cost supplier makes positive investment to get strict positive payoff and that reduces his marginal cost below ω, which in turn reduces the price of the share of the contract awarded to the low cost supplier. As a result, the buyer's expected payment to the low bidder will decrease, at a rate equal to $c_H^* + \frac{\alpha}{g''(c_H^*)} = \omega + \frac{\alpha}{g''(c_H^*)}$. Since $g''(\cdot) > 0$, the payment decrease dominates the payment increase and hence it is optimal for the buyer to reduce α marginally below 1.

We close this section with an example assuming a particular investment technology.

Example 1. Suppose the investment technology can be represented by a quadratic function

$$g(c) = \delta(\omega - c)^2 \tag{13}$$

where $\delta > 0$ and $\delta\omega > \frac{1}{2}$ to insure costs are positive.

The ex ante expected payoffs of the two suppliers are respectively:

$$\pi_L(c_L, c_H) = \max\{\alpha(c_H - c_L), (1-\alpha)(\omega - c_L)\} - \delta(\omega - c_L)^2; \tag{14}$$

$$\pi_H(c_L, c_H) = (1-\alpha)(\omega - c_H) - \delta(\omega - c_H)^2 \tag{15}$$

$\pi_H(c_L, c_H)$ is uniquely maximized at $c_H^* = \omega - \frac{1-\alpha}{2\delta}$ and $\pi_L(c_L, c_H)$ can be maximized at either $c_L^* = \omega - \frac{\alpha}{2\delta}$ or c_H^*, depending on the parameters. The candidates for pure strategy equilibria at the investment stage are: 1) the symmetric candidate: (c_H^*, c_H^*); and 2) the asymmetric candidates: (c_L^*, c_H^*) and (c_H^*, c_L^*).

When $\alpha < \frac{\sqrt{2}}{2}$, the symmetric candidate forms an equilibrium at the investment stage. When $\alpha \geq \frac{\sqrt{2}}{2}$, the asymmetric candidates form equilibria at the investment stage.

Using Proposition 2, the buyer's expected payment is:

$$m(\alpha \mid \alpha \geq \frac{\sqrt{2}}{2}) = \omega - \frac{\alpha(1-\alpha)}{2\delta} \tag{16}$$

which takes its minimum value at $\alpha = \frac{\sqrt{2}}{2}$, and

$$m(\alpha \mid \alpha < \frac{\sqrt{2}}{2}) = \omega - \frac{(1-\alpha)(2\alpha - 1)}{2\delta} \tag{17}$$

which takes its minimum value at $\alpha = \frac{\sqrt{2}}{2} - \epsilon$, where ϵ is an infinitely small positive number. Comparing the buyer's expected payment at different α, we conclude that

Remark 1. *If the investment technology is quadratic, the optimal choice of* α *that minimizes buyer's procurement cost is equal to* $\frac{\sqrt{2}}{2} \approx 0.71$*. It is a curious fact that the optimal share with quadratic investment costs is independent of the level of costs parameter* δ *and the maximum marginal cost of production* ω*.*

Under winner-takes-all mechanism with $\alpha = 1$, the buyer's procurement cost is equal to ω. Splitting the award by $\alpha = \frac{\sqrt{2}}{2}$ leads a percentage cost saving of $\frac{\sqrt{2}-1}{4\delta\omega}$, which can be more than 20% depending on the magnitude of δ and ω.

5 Extension: N-Suppliers

When there are many suppliers ($N \geq 3$), the buyer chooses a vector α which specifies the shares to be awarded to each supplier. We index the suppliers by $n = \{1, \cdots, N\}$ according to the ranks of their realized marginal costs after the investment stage, with $c_1 \leq c_2 \leq \cdots \leq c_N$. Denote $\alpha := (\alpha_1, \cdots, \alpha_N)$, with α_i the share of production to be awarded to the i-th lowest bidder.

The allocation rule at the bidding stage is such that the lowest bidder obtains an α_1 fraction of the contract at a price equal to the second lowest bid, the second lowest bidder obtains an α_2 fraction of the contract at a price equal to the third lowest bid, and so on. The highest bidder obtains an α_N fraction of the contract at a price equal to the maximal possible cost ω.

We make a working hypothesis that α is a decreasing series, ie. $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N$, which will be confirmed when the optimal procurement policy of the buyer is characterized. In the following lemmas and propositions, we first state the equilibrium strategies of the suppliers at the bidding stage, and then step by step we characterize the equilibrium marginal costs at the investment stage, and finally the α-series that are optimal to the buyer. At the bidding stage, we focus on a pure strategy equilibrium where suppliers have no incentive to make upward or downward deviations locally.

Proposition 5. *The following strategies form an equilibrium at the bidding stage:*

$$b_1 = c_1; \tag{18}$$

$$b_i = \max\{c_i, \frac{\alpha_i}{\alpha_{i-1}}(b_{i+1} - c_{i-1}) + c_{i-1}\}, \quad \textit{for } i \in \{2, \cdots, N-1\}; \tag{19}$$

$$b_N = \max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}(\omega - c_{N-1}) + c_{N-1}\} \tag{20}$$

So the equilibrium bids of the suppliers as functions of marginal costs are determined by a system of difference equations which can be solved backward starting from the supplier with the highest cost.

Corollary 1. *For $i \in \{2, \cdots, N\}$, b_i is an increasing function in $\{c_{i-1}, c_i, \cdots, c_N\}$.*

Corollary 1 implies that when a supplier reduces his marginal cost, that never increases the bid of any supplier at the bidding stage. This suggests that by providing proper incentives of investment to the suppliers at the investment stage, the buyer may be able to lower the procurement cost at the bidding stage. Using the equilibrium strategies at the bidding stage from Proposition 5, we formulate the expected payoffs of the suppliers at the investment stage in the next lemma.

Lemma 3. *The suppliers' expected payoffs at the investment stage are:*

$$\pi_i = \{\alpha_i(c_{i+1} - c_i) - g(c_i), \alpha_{i+1}(b_{i+2} - c_i) - g(c_i)\}, \quad \textit{for } i \in \{1, \cdots, N-2\} \tag{21}$$

$$\pi_{N-1} = \{\alpha_{N-1}(c_N - c_{N-1}) - g(c_{N-1}), \alpha_N(\omega - c_{N-1}) - g(c_{N-1})\}, \tag{22}$$

$$\pi_N = \alpha_N(\omega - c_N) - g(c_N) \tag{23}$$

From now on, we impose the assumption that the investment technology is given by the quadratic form $g(c) = \delta(\omega - c)^2$. Maximization of π_N in equation (23) gives the marginal cost of the least efficient supplier: $c_N = \omega - \frac{\alpha_N}{2\delta}$. For an arbitrary supplier $i \in \{1, \cdots, N-1\}$, there are always two choices of marginal costs, one relatively low and one relatively high, given respectively by $c_{iL} := \omega - \frac{\alpha_i}{2\delta}$ and $c_{iH} := \omega - \frac{\alpha_{i+1}}{2\delta}$. Since b_i is increasing in c_i and c_{i-1}, as shown in Corollary 1, while the buyer's procurement cost

$$m(\alpha_1, \cdots, \alpha_N) = \alpha_N\omega + \sum_{i=2}^{N} \alpha_{i-1} b_i \tag{24}$$

increases with b_i, whenever possible, it is always optimal for the buyer to design α-series such that each supplier chooses the lower marginal cost.

Lemma 4. *Among all equilibrium candidates at the investment stage, the following candidate $\mathbf{c}^*$ minimizes the buyer's procurement cost*

$$c_i = \omega - \frac{\alpha_i}{2\delta}, \ \textit{for all } i. \tag{25}$$

A necessary condition for the existence of this equilibrium is $\alpha_i \geq 2\alpha_{i+1}$, for $i \in \{1, \cdots, N-1\}$.

When $\alpha_i \geq 2\alpha_{i+1}$, the equilibrium bids at the bidding stage turn out to be truthfully bidding and the whole analysis is greatly simplified. In effect, the optimal α-series indeed satisfy this condition.

Lemma 5. *When $\alpha_i \geq 2\alpha_{i+1}$, for $i \in \{1, \cdots, N-1\}$, holds, truthful bidding constitutes an equilibrium at the bidding stage.*

In the next Lemma, we state the restrictions one must impose on the α-series for every supplier to choose the lower marginal cost at the investment stage.

Lemma 6. $\mathbf{c}^* = (c_1^*, \cdots, c_N^*) = (c_{1L}, \cdots, c_{(N-1)L}, c_N)$ *forms the equilibrium of the investment stage if the α-series satisfy*

$$\alpha_i^2 - 2\alpha_i\alpha_{i+1} \geq \alpha_{i+1}^2 - 2\alpha_{i+1}\alpha_{i+2}, \; for\ i \in \{1, \cdots, N-2\} \tag{26}$$

$$\alpha_{N-1}^2 - 2\alpha_N\alpha_{N-1} \geq \alpha_N^2 \tag{27}$$

$$\alpha_N > 0 \tag{28}$$

These conditions imply that α-series are indeed decreasing and $\alpha_i \geq 2\alpha_{i+1}$.

Lemma 6 transforms the suppliers' incentive problems into a sequence of requirements on α-series. So the buyer's optimization problem is to choose the α-series to minimize her procurement cost:

$$\begin{aligned} m(\alpha_1, \cdots, \alpha_N) &= \alpha_N\omega + \sum_{i=2}^{N} \alpha_{i-1}b_i \\ &= \alpha_N\omega + \sum_{i=2}^{N} \alpha_{i-1}(\omega - \frac{\alpha_i}{2\delta}) \\ &= \omega - \frac{1}{2\delta}\sum_{i=2}^{N} \alpha_{i-1}\alpha_i \end{aligned}$$

subject to constraints (26)-(28). Note that the minimization problem is equivalent to choosing α-series to maximize $\bar{m} = \sum_{i=2}^{N} \alpha_{i-1}\alpha_i$ subject to the same set of constraints.

Proposition 6. *The optimal α-series have the following properties:*

1. *α_N is as small as possible.*

2. *α_i for $i \in \{3, \cdots, N-2\}$ is as small as possible, subject to the constraints*

$$\alpha_i^2 - 2\alpha_i\alpha_{i+1} = \alpha_{i+1}^2 - 2\alpha_{i+1}\alpha_{i+2}, \; for\ i \in \{3, \cdots, N-2\}, \; and$$

$$\alpha_{N-1}^2 - 2\alpha_N\alpha_{N-1} = \alpha_N^2$$

3. α_2 is as big as possible, subject to the constraint that $\alpha_1^2 - 2\alpha_1\alpha_2 = \alpha_2^2 - 2\alpha_2\alpha_3$.

Proposition 6 implies that when there are many suppliers in the game, each supplier obtains a strictly positive share, even though only the first two shares are significant and the shares going to the remaining suppliers should be marginal. However, if the buyer can restrict the number of suppliers, the optimal number that minimizes her procurement cost is two, as shown in the next proposition.

Proposition 7. *For the buyer, restricting the number of suppliers to $N = 2$ is an optimal strategy.*

The result in Proposition 7 can be illustrated with an example of three suppliers. Denoting the share going to the highest bidder by ϵ. The optimal shares for the buyer are $(\alpha_1, \alpha_2, \alpha_3) = (\frac{\sqrt{2}-\epsilon}{2}, \frac{2-\sqrt{2}-\epsilon}{2}, \epsilon)$, which gives $\bar{m}(N=3) = \frac{\sqrt{2}-1}{2} - \frac{(\sqrt{2}-1)\epsilon}{2} - \frac{\epsilon^2}{4}$. Obviously, this is a decreasing function in ϵ and approaches $\bar{m}(N=2) = \frac{\sqrt{2}-1}{2}$ when ϵ approaches 0.

6 Concluding Remarks

This paper studies split-award procurement contract where a buyer can either divide full production among multiple suppliers or award the entire production to a single supplier. The literature shows that single sourcing usually dominates multiple sourcing. This paper challenges the "winner-takes-all" argument. In a framework of generalized second-price auctions with pre-auction investment, we show that splitting the award not only improves the suppliers' investment incentives, but also intensifies competition at the bidding stage.

The superiority of the split-award mechanism in such a setting lies in the fact that the buyer is essentially balancing the investment incentives for the low cost supplier to continue to lower costs with a credible threat, with the existence of a higher cost supplier in the process, versus the additional burden of having to accommodate the latter. By reducing the share for a sole winner marginally below 1, though the incentive of winning at the bidding stage is lower, the buyer can induce the high cost supplier to make positive investment, so for the low cost supplier to win the auction, he has to make even larger investment to achieve an even lower marginal cost. The chain reaction in improvement of the investment incentives drives the suppliers to lower their bids at the bidding stage, and hence may offset the disincentive of winning when the contract split is marginally below 1.

We then look at the N-supplier scenario to see if it pays for the buyer to enlarge the supplier pool. While the model explodes to unmanageable complexity as one would expect, we did derive a strong result when assuming a specific functional form for the cost reduction technology, namely the quadratic investment technology. There we illustrate that it is optimal for the buyer to restrict the number of suppliers to only two. That result is important as it confirms the conclusion of Fullerton and McAfee (1999) that two is the optimal number (in a very different setting), and enables bulk of the results in the simple 2-supplier case to be interpreted without loss of generality.

Our result can be directly applied to the sourcing mechanism for government drug purchase programs, which is a policy element that is gaining increasing traction in the current health care reform initiative championed by the Obama administration. Generic drugs, which account for 68 percent of all prescriptions, but comprise only 16 percent of pharmaceutical expenditures according to the Generic Pharmaceuticals Association, are unique in that the product is essentially homogeneous and the manufacturing process is highly standardized. In the absence of R&D issues, the procurement process can then be stylized by a reverse auction model with a pre-auction investment option that offers an opportunity for suppliers to reduce costs. Our results suggest that it offers a unique opportunity for the VA and DoD drug purchase programs to potentially save costs by adopting a multiple sourcing solution.

Our results can be certainly applied in many procurement areas beyond that in the health care reform. It needs to be noted however, that our model is based on the assumption that the procurement item at issue is homogenous and the buyer objective is to minimize procurement cost, directly during the auction stage, and indirectly via suppliers' investment to reduce costs prior to the auction. The contracts awarded to both suppliers are the same, except the price that follows a second-price auction rule and the quantity that is pre-determined by the split-award rule. There is no product quality issue in our analysis, limiting the scope of application. The quality dimension inevitably introduces the issue of R&D effort in the pre-auction stage. For example, to address government purchase of new patented prescription drugs as a possible element in the health care reform would require a model that incorporates contract differentiation among suppliers. Quality represents the most valuable potential extension.

Appendix

Proof of Proposition 1. The plan is to show that b_L^* and b_H^* are mutually best responses.

First, given the high cost supplier's strategy b_H^*, by placing a bid $b_L = c_L$, the low cost supplier places a bid lower than that of the high cost supplier. He is thus awarded fraction α of the contract at price b_H^*. His payoff at the bidding stage in that case is:

$$\begin{aligned}\pi_L &= \alpha \left(\max\{c_H, \frac{1-\alpha}{\alpha}(\omega - c_L) + c_L\} - c_L \right) \\ &= \max\{\alpha(c_H - c_L), (1-\alpha)(\omega - c_L)\}\end{aligned}$$

When contemplating a deviating strategy, the low cost supplier can only change his status of ranking by placing a bid above b_H^*. In that case, he is awarded fraction $1-\alpha$ of the contract at price ω and his payoff is

$$\bar{\pi}_L = (1-\alpha)(\omega - c_L) \leq \pi_L$$

Therefore, there is no improvement by deviation and b_L^* is indeed a best response to b_H^*.

Second, we show that b_H^* is also a best response to b_L^*. Given b_L^*, by following b_H^*, the high cost supplier places a bid higher than that of the low cost supplier. Thus he is awarded a fraction $1-\alpha$ of the contract at price ω. His payoff is

$$\pi_H = (1-\alpha)(\omega - c_H) \geq 0$$

By deviating, he only changes the outcome of bidding by placing a bid below b_L^*. In that case, his payoff is

$$\bar{\pi}_H = \alpha(c_L - c_H) < 0 \leq \pi_H$$

Hence, deviation makes the high cost supplier worse off and b_H^* is indeed a best response to b_L^*. □

Proof of Proposition 2. Suppliers' expected payoffs (4) can be obtained directly from the proof of Proposition 1. For the buyer, for α fraction of the contract, she pays the equilibrium higher bid b_H^* and for $1-\alpha$ fraction of the contract, she pays the maximum cost ω. Therefore, the buyer's equilibrium payment is:

$$\begin{aligned}m(\alpha) &= \alpha b_H^* + (1-\alpha)\omega \\ &= \alpha \max\{c_H, \frac{1-\alpha}{\alpha}(\omega - c_L) + c_L\} + (1-\alpha)\omega \\ &= \max\{(1-\alpha)\omega + \alpha c_H, 2\omega(1-\alpha) + (2\alpha - 1)c_L\}\end{aligned}$$

□

Proof of Lemma 1. First, both RHS and LHS, as functions of α, are continuous and monotonic. From (8) and (9), one has

$$\frac{dc_L^*}{d\alpha} = -\frac{1}{g''(c_L^*)} < 0, \quad \frac{dc_H^*}{d\alpha} = \frac{1}{g''(c_H^*)} > 0$$

The LHS of (10) is a monotonically increasing function in α since:

$$\frac{\partial LHS}{\partial \alpha} = \omega - c_L^* - \alpha \frac{dc_L^*}{d\alpha} > 0$$

The derivative of RHS with respect to α is

$$\begin{aligned}\frac{\partial RHS}{\partial \alpha} &= -\frac{dc_H^*}{d\alpha} - g'(c_H^*)\frac{dc_H^*}{d\alpha} + g'(c_L^*)\frac{dc_L^*}{d\alpha} \\ &= -\frac{1}{g''(c_H^*)} + (1-\alpha)\frac{1}{g''(c_H^*)} + \alpha\frac{1}{g''(c_L^*)} \\ &= \alpha\frac{g''(c_H^*) - g''(c_L^*)}{g''(c_H^*)g''(c_L^*)}\end{aligned}$$

Therefore, RHS is monotonically increasing in α if $g'''(\cdot) \geq 0$ and decreasing otherwise.

Second, note that for α approaches $\frac{1}{2}$, $c_H^* = c_L^*$ in the limit and $LHS = \alpha(\omega - c_L^*) < RHS = \omega - c_H^*$. When $\alpha = 1$, $c_H^* = \omega$, and we have $LHS = \omega - c_L^* > RHS = g(c_L^*)$. This implies that the LHS must intersects RHS from below at some unique interior point $\alpha = \tilde{\alpha}$. Hence, LHS is bigger than RHS if and only if $\alpha \geq \tilde{\alpha}$. □

Proof of Proposition 3. The plan is to show that the two suppliers' strategies are mutually best responses.

1. First, suppose supplier 2 follows strategy $c_2 = c_H^*$ and consider supplier 1's strategy. By choosing $c_1 = c_L^*$, supplier 1 is the low cost supplier and his payoff is given by:

   $$\pi_1(c_L^*, c_H^*) = \max\{\alpha(c_H^* - c_L^*), (1-\alpha)(\omega - c_L^*)\} - g(c_L^*)$$

   Recall that $\alpha \geq \tilde{\alpha}$ implies that $\alpha(\omega - c_L^*) \geq \omega - c_H^* - (g(c_H^*) - g(c_L^*))$. Applying this, we have:

   $$\alpha(c_H^* - c_L^*) - g(c_L^*) \geq (1-\alpha)(\omega - c_H^*) - g(c_H^*) > (1-\alpha)(\omega - c_L^*) - g(c_L^*)$$

   where the second inequality obtains by the definition of c_H^*. Therefore,

   $$\pi_1(c_L^*, c_H^*) = \alpha(c_H^* - c_L^*) - g(c_L^*)$$

When contemplating marginal cost other than c_L^*, if supplier 1 chooses some $c_1 < c_H^*$, he is still the low cost supplier and his payoff can not improve over $c_1 = c_L^*$ since c_L^* is the unique maximizer of $\pi_1(c_1, c_H^* \mid c_1 < c_H^*)$.

By choosing $c_1 \geq c_H^*$, supplier 1 becomes the high cost supplier. His payoff in that case is:

$$\begin{aligned}\pi_1(c_1, c_H^* \mid c_1 \geq c_H^*) &= (1-\alpha)(\omega - c_1) - g(c_1) \\ &\leq (1-\alpha)(\omega - c_H^*) - g(c_H^*) \\ &\leq \pi_1(c_L^*, c_H^*)\end{aligned}$$

which implies that it is not worthwhile for supplier 1 to choose a marginal cost other than c_L^* given $c_2 = c_H^*$.

It remains to be shown that $c_2 = c_H^*$ is also a best response to $c_1 = c_L^*$. By choosing $c_2 = c_H^*$, supplier 2 is the high cost supplier and his payoff is:

$$\pi_2(c_L^*, c_H^*) = (1-\alpha)(\omega - c_H^*) - g(c_H^*)$$

Deviating to any $c_2 \geq c_L^*$ does not change supplier 2's status of ranking at the bidding stage and decreases his payoff instead since $\pi_2(c_L^*, c_2 \mid c_2 \geq c_L^*)$ is uniquely maximized at $c_2 = c_H^*$. If supplier 2 contemplates $c_2 < c_L^*$, he becomes the low cost supplier and his payoff is given by:

$$\begin{aligned}\pi_2(c_L^*, c_2 \mid c_2 < c_L^*) &= \max\{\alpha(c_L^* - c_2), (1-\alpha)(\omega - c_2)\} - g(c_2) \\ &= \max\{\alpha(c_L^* - c_2) - g(c_2), (1-\alpha)(\omega - c_2) - g(c_2)\}\end{aligned}$$

The first term in the max operator takes its maximum value $-g(c_L^*)$ when c_2 goes infinitely close to c_L^*. The second term is strictly increasing in c_2 for $c_2 < c_L^* < c_H^*$. As c_2 approaches c_L^*, it takes its maximal value $(1-\alpha)(\omega - c_L^*) - g(c_L^*)$ which is smaller than $\pi_2(c_L^*, c_H^*)$. This concludes the proof that $c_2 = c_H^*$ is also a best response to $c_1 = c_L^*$.

2. Suppose $\alpha < \tilde{\alpha}$. We show that given $c_2 = c_H^*$ it is a best response for supplier 1 to choose $c_1 = c_H^*$.

   By following the prescribed strategy $c_1 = c_H^*$, supplier 1's expected payoff is:

$$\pi_1(c_H^*, c_H^*) = (1-\alpha)(\omega - c_H^*) - g(c_H^*)$$

When supplier 1 contemplates deviating from c_H^*, he has two choices: $c_1 > c_H^*$ or $c_1 < c_H^*$. In the following, we show that neither direction of deviation is profitable for supplier 1.

(a) Suppose supplier 1 contemplates $c_1 > c_H^*$. Then supplier 1 becomes the high cost supplier. His payoff is given by

$$\pi_1(c_1, c_H^* \mid c_1 > c_H^*) = (1-\alpha)(\omega - c_1) - g(c_1)$$

Since the $(1-\alpha)(\omega - c) - g(c)$ is uniquely maximized at c_H^*, $\pi_1(c_1, c_H^* \mid c_1 > c_H^*) < \pi_1(c_H^*, c_H^*)$. Therefore, deviating to $c_1 > c_H^*$ is not worthwhile.

(b) Suppose supplier 1 contemplates $c_1 < c_H^*$. Supplier 1 is the low cost supplier. His payoff is given by:

$$\pi_1(c_1, c_H^* \mid c_1 < c_H^*) = \max\{\alpha(c_H^* - c_1) - g(c_1), (1-\alpha)(\omega - c_1) - g(c_1)\}$$

For $c_1 < c_H^*$, the second term in the max operator is a monotonic increasing function, and is maximized as c_1 goes infinitely close to c_H^*. Therefore, the second term is always smaller than $\pi_1(c_H^*, c_H^*)$.

The first term in the maximum operator is maximized at an interior point $c_1 = c_L^*$ since $c_L^* < c_H^*$. So its maximum value is given by $\alpha(c_H^* - c_L^*) - g(c_L^*)$. Since

$$\begin{aligned}
&\alpha(c_H^* - c_L^*) - g(c_L^*) - ((1-\alpha)(\omega - c_H^*) - g(c_H^*)) \\
&= \alpha(\omega - c_L^*) - (\omega - c_H^*) + (g(c_H^*) - g(c_L^*)) \\
&< \frac{\omega - c_H^* - (g(c_H^*) - g(c_L^*))}{\omega - c_L^*}(\omega - c_L^*) - (\omega - c_H^*) + (g(c_H^*) - g(c_L^*)) \\
&= 0
\end{aligned}$$

where the inequality holds due to relation (10). Hence, $\alpha(c_H^* - c_L^*) - g(c_L^*) < \pi_1(c_H^*, c_H^*)$ holds as well. Therefore,

$$\pi_1(c_1, c_H^* \mid c_1 < c_H^*) < \pi_1(c_H^*, c_H^*)$$

Hence, we conclude that choosing $c_1 = c_H^*$ is a best response to $c_2 = c_H^*$. This establishes the symmetric equilibrium at the investment stage when $\alpha < \tilde{\alpha}$.

□

Proof of Lemma 2. For an arbitrary $\alpha_1 \geq \tilde{\alpha}$ and $\alpha_2 < \tilde{\alpha}$, since $g''(\cdot) > 0$, we have

$$c_H^*(\alpha_1) > c_H^*(\alpha_2) > c_L^*(\alpha_2) > c_L^*(\alpha_1) \tag{29}$$

The difference of the buyer's procurement costs given the split rule α_1 and α_2 is

$$\begin{aligned} m(\alpha_2) - m(\alpha_1) =& \alpha_2 b_H^*(\alpha_2) + (1-\alpha_2)\omega - \alpha_1 b_H^*(\alpha_1) - (1-\alpha_1)\omega \\ =& \alpha_2 \max\{c_H^*(\alpha_2), \frac{1-\alpha_2}{\alpha_2}(\omega - c_H^*(\alpha_2)) + c_H^*(\alpha_2)\} \\ & - \alpha_1 \max\{c_H^*(\alpha_1), \frac{1-\alpha_1}{\alpha_1}(\omega - c_L^*(\alpha_1)) + c_L^*(\alpha_1)\} + (\alpha_1 - \alpha_2)\omega \end{aligned}$$

where the last two lines obtain because when $\alpha_1 \geq \tilde{\alpha}$, the equilibrium marginal costs are $(c_H^*(\alpha_1), c_L^*(\alpha_1))$ and when $\alpha_2 < \tilde{\alpha}$, the equilibrium marginal costs are $(c_H^*(\alpha_2), c_H^*(\alpha_2))$. Rewriting the last two lines, we get

$$\begin{aligned} m(\alpha_2) - m(\alpha_1) =& \alpha_1\omega - (2\alpha_2 - 1)(\omega - c_H^*(\alpha_2)) \\ & - \max\{\alpha_1 c_H^*(\alpha_1), (1-\alpha_1)(\omega - c_L^*(\alpha_1)) + \alpha_1 c_L^*(\alpha_1)\} \\ =& \max\{\alpha_1(\omega - c_H^*(\alpha_1)) - (2\alpha_2 - 1)(\omega - c_H^*(\alpha_2)), \\ & (2\alpha_1 - 1)(\omega - c_L^*(\alpha_1)) - (2\alpha_2 - 1)(\omega - c_H^*(\alpha_2))\} \\ >& 0 \end{aligned}$$

where the inequality holds because $\alpha_1 > \alpha_2$ and $c_H^*(\alpha_2) > c_L^*(\alpha_1)$ and the second term in the max operator is positive. Hence for the buyer, choosing a low α and eliciting the symmetric equilibrium at the investment stage is dominated by choosing a high α and eliciting the asymmetric equilibrium.

□

Proof of Proposition 4. Taking the derivative of objective (12) with respect to α, one obtains

$$\frac{\partial m}{\partial \alpha}(\alpha \mid \alpha \geq \tilde{\alpha}) = -\omega + c_H^* + \alpha \frac{dc_H^*}{d\alpha}$$

Recall $c_H^* = \omega$ when $\alpha = 1$ and $\frac{dc_H^*}{d\alpha} = \frac{1}{g''(c_H^*)}$. Evaluating $\frac{\partial m}{\partial \alpha}(\alpha \mid \alpha \geq \tilde{\alpha})$ at $\alpha = 1$ gives

$$\frac{\partial m}{\partial \alpha}(\alpha \mid \alpha = 1) = \frac{1}{g''(\omega)} > 0$$

This means that marginally reducing α below 1 reduces the buyer's expected payment. Hence, splitting the contract between the two suppliers is optimal. □

Proof of Proposition 5. We first show that no one has any incentive to place a bid above the strategy prescribed for him. For an arbitrary supplier $i \in \{1, \cdots, N-2\}$, given other suppliers following the candidate strategies, by following the prescribed strategy, he is the ith lowest bidder. His payoff is

$$\begin{aligned}
\pi_i &= \alpha_i(b_{i+1} - c_i) \\
&= \alpha_i \left(\max\{c_{i+1}, \frac{\alpha_{i+1}}{\alpha_i}(b_{i+2} - c_i) + c_i\} - c_i \right) \\
&= \max\{\alpha_i(c_{i+1} - c_i), \alpha_{i+1}(b_{i+2} - c_i))\} \\
&\geq \alpha_{i+1}(b_{i+2} - c_i) = \bar{\pi}_i
\end{aligned}$$

where $\bar{\pi}_i$ is supplier i's payoff if he places a bid above b_{i+1} such that he becomes the $i+1$-th lowest bidder. Therefore, he has no incentive to bid above b_i. For $i = N-1$, there is no incentive to place a bid above b_{N-1} either since

$$\begin{aligned}
\pi_{N-1} &= \alpha_{N-1}(b_N - c_{N-i}) \\
&= \alpha_{N-1} \left(\max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}(\omega - c_{N-1}) + c_{N-1}\} - c_{N-1} \right) \\
&= \max\{\alpha_{N-1}(c_N - c_{N-1}), \alpha_N(\omega - c_{N-1}))\} \\
&\geq \alpha_N(\omega - c_{N-1}) = \bar{\pi}_{N-1}
\end{aligned}$$

where $\bar{\pi}_{N-1}$ is supplier $N-1$'s payoff if he raises his bid such that he becomes the highest bidder.

Finally, for $i = N$, raising his bid can not change his position since he is already the highest bidder. Therefore, bidding higher does not change his payoff either.

We now show that none of the suppliers has any incentive to bid below his prescribed strategy. For $i = 1$, bidding lower changes neither his position nor his payoff. For supplier $i = 2$, suppose he deviates from b_2 by placing a lower bid. He can only change his payoff if he bids low enough such that he becomes the lowest bidder. In that case, his payoff is

$$\bar{\pi}'_2 = \alpha_1(c_1 - c_2) \leq 0$$

Hence, the deviation is not profitable.

For any supplier $i \in \{3, \cdots, N-1\}$, by bidding below b_{i-1}, he then moves his ranking of bid from position i to position $i-1$. The bid of supplier $i-1$ now becomes the actual

i-th lowest bid. Supplier i's payoff from this deviation is

$$\begin{aligned}
\bar{\pi}'_i &= \alpha_{i-1}(b_{i-1} - c_i) \leq \alpha_{i-1}(b_i - c_i) \\
&= \alpha_{i-1}\left(\max\{c_i, \frac{\alpha_i}{\alpha_{i-1}}(b_{i+1} - c_{i-1}) + c_{i-1}\} - c_i\right) \\
&= \max\{0, \alpha_i(b_{i+1} - c_{i-1}) + \alpha_{i-1}(c_{i-1} - c_i)\} \\
&\leq \alpha_i(b_{i+1} - c_i) = \pi_i
\end{aligned}$$

where the last inequality holds because $b_{i+1} - c_i > 0$ and using the hypothesis that α is a decreasing series, one has $(\alpha_{i-1} - \alpha_i)(c_{i-1} - c_i) \leq 0$ which implies

$$\alpha_i(b_{i+1} - c_{i-1}) + \alpha_{i-1}(c_{i-1} - c_i) \leq \alpha_i(b_{i+1} - c_i)$$

Therefore, it is not worthwhile for any supplier $i \in \{3, \cdots, N-1\}$ to bid below the prescribed strategies.

For $i = N$, by bidding below b_{N-1}, he then moves his ranking from the highest to the second highest bidder. Supplier N's payoff from deviation is

$$\begin{aligned}
\bar{\pi}'_N &= \alpha_{N-1}(b_{N-1} - c_N) \leq \alpha_{N-1}(b_N - c_N) \\
&= \alpha_{N-1}\left(\max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}(\omega - c_{N-1}) + c_{N-1}\} - c_N\right) \\
&= \max\{0, \alpha_N(\omega - c_{N-1}) + \alpha_{N-1}(c_{N-1} - c_N)\} \\
&\leq \alpha_N(\omega - c_N) = \pi_N
\end{aligned}$$

where the last inequality holds because $\omega - c_N \geq 0$ and $(\alpha_{N-1} - \alpha_N)(c_{N-1} - c_N) \leq 0$ implies

$$\alpha_N(\omega - c_{N-1}) + \alpha_{N-1}(c_{N-1} - c_N) \leq \alpha_N(\omega - c_N)$$

Hence, it is not worthwhile for supplier N to bid below his prescribed strategy either. □

Proof of Corollary 1. The proof is done through induction.

1. For $i = N$, rewriting b_N gives

$$b_N = \max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}\omega + \frac{\alpha_{N-1} - \alpha_N}{\alpha_{N-1}}c_{N-1})\}$$

   Obviously, b_N is an increasing function in c_N and c_{N-1} as well due to the working hypothesis $\alpha_{N-1} \geq \alpha_N$.

2. For $i = N-1$, rewriting b_{N-1} gives:

$$b_{N-1} = \max\{c_{N-1}, \frac{\alpha_{N-1}}{\alpha_{N-2}} b_N + \frac{\alpha_{N-2} - \alpha_{N-1}}{\alpha_{N-1}} c_{N-2})\}$$

Therefore, b_{N-1} is an increasing function b_N, and c_{N-2} as well due to $\alpha_{N-2} \geq \alpha_{N-1}$. It has been shown that b_N is an increasing function in c_N and c_{N-1}, hence, b_{N-1} is also an increasing function in c_N and c_{N-1}.

3. For an arbitrary $i \in \{2, \cdots, N-2\}$, given that b_{i+1} is an increasing function in $\{c_i, \cdots, c_N\}$, rewriting b_i gives:

$$b_i = \max\{c_i, \frac{\alpha_i}{\alpha_{i-1}} b_{i+1} + \frac{\alpha_{i-1} - \alpha_i}{\alpha_i} c_{i-1}\}$$

Therefore, b_i is an increasing function in b_{i+1} and c_{i-1} due to $\alpha_{i-1} \geq \alpha_i$. Since b_{i+1} is an increasing function in $\{c_i, \cdots, c_N\}$, b_i is an increasing function in $\{c_{i-1}, c_i, \cdots, c_N\}$.

□

Proof of Lemma 3. 1. For supplier $i = N$, he expects to be awarded fraction α_N of the contract at price ω. Hence, his expected payoff at the investment stage is obviously given by (23).

2. Supplier $i = N-1$ expects to receive fraction α_{N-1} of the contract at price b_N. Hence:

$$\begin{aligned}
\pi_{N-1} &= \alpha_{N-1}\left(b_N - c_{N-1}\right) - g(c_{N-1}) \\
&= \alpha_{N-1}\left(\max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}(\omega - c_{N-1}) + c_{N-1}\} - c_{N-1}\right) - g(c_{N-1}) \\
&= \max\{\alpha_{N-1}(c_N - c_{N-1}) - g(c_{N-1}), \alpha_N(\omega - c_{N-1}) - g(c_{N-1})\}
\end{aligned}$$

3. Supplier $i \in \{1, \cdots, N-2\}$ expects to be awarded fraction α_i of the contract at price b_{i+1}. Hence:

$$\begin{aligned}
\pi_i &= \alpha_i\left(b_{i+1} - c_i\right) - g(c_i) \\
&= \alpha_i\left(\max\{c_{i+1}, \frac{\alpha_{i+1}}{\alpha_i}(b_{i+2} - c_i) + c_i\} - c_i\right) - g(c_i) \\
&= \max\{\alpha_i(c_{i+1} - c_i) - g(c_i), \alpha_{i+1}(b_{i+2} - c_i) - g(c_i)\}
\end{aligned}$$

□

Proof of Lemma 4. For each supplier except for $i = N$, there are two choices of marginal costs given by c_{iL} and c_{iH}. Altogether, there are 2^{N-1} equilibrium candidates for the investment stage. However, as shown in Corollary 1, bidding at the investment stage is increasing in the marginal costs and buyer's procurement cost increases with the bids. Hence, for the buyer to minimize her procurement cost, the optimal investment profile, if possible, must be such that all suppliers choose the lower investment level c_{iL}.

For the suppliers to choose the lower marginal costs in equilibrium, a necessary condition is that given every $j \neq i$ chooses c_{jL}, the expected payoff for i from choosing c_{iL} is nonnegative, which is equivalent to:

$$\alpha_i(c_{i+1} - c_i) - g(c_i) = \alpha_i \cdot \frac{\alpha_i - \alpha_{i+1}}{2\delta} - \frac{\alpha_i^2}{4\delta} \geq 0 \tag{30}$$

which implies $\alpha_i \geq 2\alpha_{i+1}$, for $i \in \{1, \cdots, N-1\}$. □

Proof of Lemma 5. Supplier $i = 1$ is bidding truthfully by Proposition 5. We still need to show that the same holds true for suppliers $i = \{2, \cdots, N\}$. The proof is done by induction.

1. For supplier $i = N$, given everyone else bidding truthfully, we have

$$\begin{aligned} b_N &= \max\{c_N, \frac{\alpha_N}{\alpha_{N-1}}(\omega - c_{N-1}) + c_{N-1}\} \\ &= \max\{\omega - \frac{\alpha_N}{2\delta}, \frac{\alpha_N}{2\delta} + \omega - \frac{\alpha_{N-1}}{2\delta}\} \end{aligned}$$

   Since

$$\omega - \frac{\alpha_N}{2\delta} - \left(\frac{\alpha_N}{2\delta} + \omega - \frac{\alpha_{N-1}}{2\delta}\right) = \frac{\alpha_{N-1} - 2\alpha_N}{2\delta} \geq 0,$$

   $b_N = c_N$ is optimal for supplier N.

2. For supplier $i = N - 1$, given that the other suppliers are bidding truthfully, we have

$$\begin{aligned} b_{N-1} &= \max\{c_{N-1}, \frac{\alpha_{N-1}}{\alpha_{N-2}}(b_N - c_{N-2}) + c_{N-2}\} \\ &= \max\{\omega - \frac{\alpha_{N-1}}{2\delta}, \omega - \frac{\alpha_{N-2}}{2\delta} + \frac{\alpha_{N-1}}{2\delta} - \frac{\alpha_N \alpha_{N-1}}{2\delta\alpha_{N-2}}\} \end{aligned}$$

   Since

$$\omega - \frac{\alpha_{N-1}}{2\delta} - \left(\omega - \frac{\alpha_{N-2}}{2\delta} + \frac{\alpha_{N-1}}{2\delta} - \frac{\alpha_N \alpha_{N-1}}{2\delta\alpha_{N-2}}\right) = \frac{\alpha_{N-2} - 2\alpha_{N-1}}{2\delta} + \frac{\alpha_N \alpha_{N-1}}{2\delta\alpha_{N-2}} > 0$$

   $b_{N-1} = c_{N-1}$ is optimal for supplier $N - 1$.

3. For supplier $i \in \{2, \cdots, N-2\}$, given $b_{i+1} = c_{i+1}$, we have

$$\begin{aligned} b_i &= \max\{c_i, \frac{\alpha_i}{\alpha_{i-1}}(b_{i+1} - c_{i-1}) + c_{i-1}\} \\ &= \max\{\omega - \frac{\alpha_i}{2\delta\alpha_{i-1}}, \omega - \frac{\alpha_{i-1}}{2\delta} + \frac{\alpha_i}{2\delta} - \frac{\alpha_i\alpha_{i+1}}{2\delta}\} \end{aligned}$$

since

$$\omega - \frac{\alpha_i}{2\delta} - \left(\omega - \frac{\alpha_{i-1}}{2\delta} + \frac{\alpha_i}{2\delta} - \frac{\alpha_i\alpha_{i+1}}{2\delta\alpha_{i-1}}\right) = \frac{\alpha_{i-1} - 2\alpha_i}{2\delta} + \frac{\alpha_i\alpha_{i+1}}{2\delta\alpha_{i-1}} > 0$$

Hence, $b_i = c_i$ is also an optimal strategy for supplier i.

□

Proof of Lemma 6. 1. For supplier $i = N$, there is only one choice of marginal cost $c_N = \frac{\alpha_N}{2\delta}$. To ensure that supplier N participates in the game, α_N should be such that $\pi_N \geq 0$ which is equivalent to the requirement that $\alpha_N \geq 0$. To avoid division by 0 in bidder $N-1$'s bidding strategy, α_N has to be strictly positive.

2. Supplier $i = N-1$ chooses $c_{(N-1)L} = \omega - \frac{\alpha_{N-1}}{2\delta}$ over $c_{(N-1)H} = \omega - \frac{\alpha_N}{2\delta}$ if

$$\begin{aligned} \alpha_{N-1}(c_N^* - c_{(N-1)L}) - g(c_{(N-1)L}) &\geq \alpha_N(\omega - c_{(N-1)H}) - g(c_{(N-1)H}) \\ \Leftrightarrow \alpha_{N-1}\frac{\alpha_{N-1} - \alpha_N}{2\delta} - \frac{\alpha_{N-1}^2}{4\delta} &\geq \alpha_N\frac{\alpha_N}{2\delta} - \frac{\alpha_N^2}{4\delta} \\ \Leftrightarrow \alpha_{N-1}^2 - 2\alpha_N\alpha_{N-1} &\geq \alpha_N^2 \end{aligned}$$

Since $\alpha_N^2 > 0$, $\alpha_{N-1}^2 - 2\alpha_N\alpha_{N-1} > 0$. Hence, if condition (27) is satisfied, $\alpha_{N-1} \geq 2\alpha_N$ must hold.

3. Given everyone else choosing c_{jL}, $j \neq i$, and everyone bidding truthfully at the bidding stage, for supplier $i = \{1, \cdots, N-2\}$ to choose $c_{iL} = \omega - \frac{\alpha_i}{2\delta}$ over $c_{iH} = \omega - \frac{\alpha_{i+1}}{2\delta}$, one must have

$$\begin{aligned} \alpha_i(c_{i+1}^* - c_{iL}) - g(c_{iL}) &\geq \alpha_{i+1}(b_{i+2}^* - c_{iH}) - g(c_{iH}) \\ \Leftrightarrow \alpha_i\frac{\alpha_i - \alpha_{i+1}}{2\delta} - \frac{\alpha_i^2}{4\delta} &\geq \alpha_{i+1}\frac{\alpha_{i+1} - \alpha_{i+2}}{2\delta} - \frac{\alpha_{i+1}^2}{4\delta} \\ \Leftrightarrow \alpha_i^2 - 2\alpha_i\alpha_{i+1} &\geq \alpha_{i+1}^2 - 2\alpha_{i+1}\alpha_{i+2} \end{aligned}$$

This confirms requirement (26). Furthermore, for $i = N-2$, one has

$$\alpha_{N-2}^2 - 2\alpha_{N-2}\alpha_{N-1} \geq \alpha_{N-1}^2 - 2\alpha_{N-1}\alpha_N > 0$$

which means $\alpha_{N-2} \geq 2\alpha_{N-1}$ must hold. Applying the procedure to every $i \in \{1, \cdots, N-3\}$ shows that $\alpha_i \geq 2\alpha_{i+1}$ must hold as well.

□

Proof of Proposition 6. Taking the derivative of $\bar{m}$ with respect to each element of the α-series, we get

$$\begin{aligned}\bar{\mathbf{m}}_{\mathbf{i}} &= (\bar{m}_1, \bar{m}_2, \bar{m}_3, \bar{m}_4 \cdots, \bar{m}_{N-1}, \bar{m}_N) \\ &= (\alpha_2, \alpha_1 + \alpha_3, \alpha_2 + \alpha_4, \alpha_3 + \alpha_5, \cdots, \alpha_{N-2} + \alpha_N, \alpha_{N-1})\end{aligned}$$

Since $\bar{m}_2 > \bar{m}_3 > \bar{m}_1 > \bar{m}_4 > \cdots > \bar{m}_N$, and given the constraint that $\sum_i \alpha_i = 1$, to maximize $\bar{m}$, the maximal weight should be given to α_2 and the lowest weight should be given to the remaining αs, subject to constraints (26) to (28).

1. Since there is no lower bound constraint imposed on α_N other than $\alpha_N > 0$, α_N should be made as small as possible.

2. For α_{N-1}, constraint (27) imposes a lower bound. Hence, the optimal α_{N-1} is such that (27) is binding.

   For α_{N-2}, constraint (26) is

   $$\alpha_{N-2}^2 - 2\alpha_{N-2}\alpha_{N-1} \geq \alpha_{N-1}^2 - 2\alpha_{N-1}\alpha_N$$

   whose right-hand side is a constant with respect to α_{N-2} while the left-hand side is an increasing function in α_{N-2}. This, hence, implies a lower bound for α_{N-2}. In optimum, constraint (26) must be binding for $i = N-2$. Working iteratively shows that α_i, for $i \in \{3, \cdots, N-3\}$, are all constrained in a similar way.

3. Note that constraint (26) for $i = 1$ is

   $$\alpha_1^2 - 2\alpha_1\alpha_2 \geq \alpha_2^2 - 2\alpha_2\alpha_3$$

   Since the left-hand side is a decreasing function in α_2 while right-hand side is an increasing function in α_2, the constraint imposes an upper bound on α_2. To make α_2 as big as possible implies that (26) for $i = 1$ must be binding.

□

Proof of Proposition 7. Recall that the buyer's objective is equivalent to maximizing

$$\bar{m} = \sum_{i=2}^{N} \alpha_{i-1}\alpha_i \tag{31}$$

subject to constraints (26) to (28).

When $N = 2$, $\bar{m}(N = 2) = \alpha_1\alpha_2$ subject to the constraint $\alpha_1^2 - 2\alpha_1\alpha_2 \geq \alpha_2^2$. Applying $\alpha_1 + \alpha_2 = 1$, we get $\bar{m} = \frac{\sqrt{2}-1}{2}$.

When $N \geq 3$, as shown in Proposition 6, small positive shares ε_i are given to supplier i for $i \neq \{1, 2\}$. For $i = 1$, constraint (26) implies that

$$2\alpha_2^2 = (\alpha_1 - \alpha_2)^2 + 2\alpha_2\varepsilon_3 \tag{32}$$

To make α_2 as big as possible means setting $\alpha_2 \approx (\sqrt{2} - 1)\alpha_1$, while keeping ε_3 as small as possible. Applying $\sum_{i=1}^{N} \alpha_i = 1$ leads to $\alpha_1 = \frac{\sqrt{2}}{2}(1 - \sum_{i=3}^{N} \varepsilon_i)$. Therefore,

$$\begin{aligned}
\bar{m}(N \geq 3) &= \alpha_1\alpha_2 + \alpha_2\alpha_3 + \sum_{i=4}^{N} \alpha_{i-1}\alpha_i \\
&= (\sqrt{2} - 1)\left(\frac{\sqrt{2}}{2}(1 - \sum_{i=3}^{N} \varepsilon_i)\right)^2 + (\sqrt{2} - 1)\left(\frac{\sqrt{2}}{2}(1 - \sum_{i=3}^{N} \varepsilon_i)\right) \cdot \varepsilon_3 + \sum_{i=4}^{N} \varepsilon_{i-1}\varepsilon_i \\
&= \frac{\sqrt{2} - 1}{2} - \frac{3\sqrt{2} - 4}{2}\varepsilon_3 - (\sqrt{2} - 1)\sum_{i=4}^{N} \varepsilon_i + O(2)
\end{aligned}$$

where $O(2)$ represents the remaining second-order terms. Since

$$-\frac{3\sqrt{2} - 4}{2}\varepsilon_3 - (\sqrt{2} - 1)\sum_{i=4}^{N} \varepsilon_i < 0$$

We conclude that

$$\bar{m}(N \geq 3) < \frac{\sqrt{2} - 1}{2} = \bar{m}(N = 2)$$

□

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