

Informed seller in a Hotelling market*

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Abstract

We consider the problem of a monopolist who is selling a good and is privately informed about some of its attributes. We focus on the case where goods with different attributes are horizontally differentiated: in other words, they appeal to different segments of the market. Can the monopolist profit from concealing her private information? Is it optimal for her to reveal all good's attributes upfront?

We show that in many circumstances, the monopolist maximizes her profit by not disclosing any information, which is in contrast to insights from auction theory and the informed-principal literature. We characterize the optimal selling mechanism for the informed monopolist. The optimal selling mechanism depends on the shape of the transportation cost function and on the base consumption value of agents. Still, if the base value is sufficiently high, then it is optimal not to disclose any information, if the base value is sufficiently low, then it is optimal to disclose the location. For intermediate base values, one can implement the optimal mechanism as the two-item menu: buy the good at the fixed price without disclosure, or buy the information about the location with the option of purchasing the good afterwards at a predetermined exercise price.

1 Introduction

It is a common characteristic of several business negotiations that the interaction between sellers and buyers originally begins in a particular condition of asymmetric information: the buyers are uncertain about their valuation for the good on sale, the sellers own private information that would help the buyers to resolve their uncertainty. For example, a seller may be better informed about attributes of the good that are not immediately visible to the buyer: the ingredients used to cook a dish, the internal components of a technological device, the details of a financial investment plan. In these situations each seller may try to use her information strategically: revealing or hiding what she knows in order to maximize her profits.

When is profitable for the seller to disclose her private information? How much to reveal? To whom?

We consider the problem of a profit maximizing seller who owns private information in an horizontally differentiated market. Such a market is represented via a standard Hotelling

*Preliminary and incomplete.

(1929) model. Each buyer's and seller's private information consists in locations on the Hotelling line. The seller's type which can be thought of as a location of the good corresponds to a particular characteristic of the good that is relevant to the buyer but is the private information of the seller. The buyer's type describes his preferences over the characteristic so that his utility from consuming the good is a base consumption value minus a cost depending on the distance between the buyer and the good.

We derive the optimal (revenue maximizing) mechanism in a variety of settings, first restricting the information disclosure options for the seller to full revelation or no revelation and then solving for the optimal mechanism without restrictions. We show that under restricted disclosure option, it is always better to disclose the information if the base consumption value is low, and it is always better not to disclose the information if the base consumption value is sufficiently high, no matter what is the cost function considered. The optimal unrestricted mechanism may provide partial disclosure so that different buyers receive different information and thus it entails price discrimination across buyers in terms of their valuation for information. We propose mechanisms to implement the optimal outcome. In one of them, the seller offers a menu: buy the good with no information attached or buy the information about the good and have an option to buy the good at a predetermined exercise price after learning the seller's private information.

Our analysis inherits the complexity of the Informed Principal problem (Myerson (1983), Maskin & Tirole (1990), Maskin & Tirole (1992)). After the seller learns her type, she designs the mechanism to sell the good. Any choice she makes affects the buyer's beliefs over her types, and, thus, the buyer's willingness to pay for the good. However, the seller cannot manipulate arbitrarily the buyer's beliefs. Any signal the seller sends to the buyer has to pass through a credibility test. Whenever the seller wants to induce some probability distribution over her types, these probabilities should be consistent with what is indeed in the interest of each of the seller's types from the perspective (i.e. given the information set) of the buyer. In this sense, the actual seller (i.e. the true type of the seller) needs to consider what the other seller's type would do, if she wants to credibly reveal or conceal her identity.

We consider an environment in which the good can be located only at the two extremes of the line and the buyers have ex-ante beliefs that the good is at each extreme with equal probabilities. The symmetry that we are imposing by construction simplifies our analysis and allows us to bring to light the interaction between the seller's types in a clearer way. Indeed, given symmetry, the perspective of one type is the mirror-version of the other's. This implies that the setting is one of pure horizontal differentiation: there exists no "better" or "high value" type for the seller who finds advantageous to always reveal her identity.¹

In our environment, each type of the seller would like to reveal her location to the buyers located close by, and be pooled with the other type in the beliefs of the buyers located further away. This strategy would maximize each buyer's expected valuation for the good, but it is infeasible. The strategy requires that the two types of the seller pool together in the beliefs of some buyers. However, the seller's types disagree in terms of which set of buyers to pool together for. Therefore, the incentives of the seller's types are misaligned in terms of how to

¹The informed principal literature (see Yilankaya (1999)) considers several environments where the presence of high and low types of the principal hinders any strategic use of private information by the principal and supports a complete unraveling of information (i.e. full disclosure) as unique equilibrium.

use the private information to manipulate the buyer’s expected valuation for the good.

We show that the opposite is true considering the buyer’s valuation for information. Under certain conditions, both seller’s types recognize the buyer’s types that are located closer to the extremes as the ones who are willing to pay the most for their information. Accordingly, in the optimal mechanism both types of the seller decide to pool together and offer the good with no information disclosed — a product we may refer to as an *opaque good*²— and an option-like contract that allows the buyer to learn information and then, if he wants, to buy the good.

The conditions that support such result are about the buyer’s types utility functions. The disutility they suffer from consuming the good they like the least should be sufficiently large. We characterize these conditions by considering different cost functions in the Hotelling model.

By selling the opaque good and the actual information separately the true type of the seller finds the way to sell something to everyone in the market. The buyers that are almost indifferent between the two types of the seller choose to buy the opaque good. The buyers who have strong biases in their preferences buy the option. Out of this latter group, the ones that learn “positive” information (i.e. the actual good is the one that they like the most) buy the good at the exercise price of the option. The ones who learn “negative” information do not exercise the option.

By buying the option and learning the true type of the seller, a buyer avoids to consume the least preferred good. The more variable is a buyer’s valuation for the good (depending on the seller’s type), the higher is the same buyer’s valuation for information. This implies that, for a given type of the seller, the buyers who are willing to pay the most for her information are also the ones who value the least her actual good.

We show that, given the optimal mechanism, the distribution of informational rents among the buyers differ depending on the shape of the cost function considered. If the cost function is concave, the buyers with the highest valuation for the actual good have the highest rents; if the cost function is convex, then the highest information rents are gained by the buyers who are indifferent between the types of the seller. Furthermore, the shape of the cost function affects which constraints determine the prices of the contracts offered by the seller. When the costs are convex the price of the option and the price of the opaque good are pinned down by the individual rationality constraints of two (symmetric) types of buyers who are left with no surplus by the seller. When the costs are concave the price of the opaque good is determined by the individual rationality of the buyer located in the middle of the Hotelling line. Instead, the price of the option is derived from the incentive compatibility constraint of the buyer who is indifferent between the two contracts offered by the seller.

If the seller’s types are asymmetric, then the characterization of the optimal mechanism is more complex. The seller’s types disagree on which set of buyers to sell the option to. Still we show that there exists a set of mechanisms that guarantees each type a higher profit than what each of them would obtain revealing her identity. Like in the symmetric case, these mechanisms entail the simultaneous sale of an opaque good and an option. However,

²We borrow the opaque good term from the marketing literature. Opaque are goods that are offered on sale supposedly without disclosing relevant information about their attributes.

different mechanisms imply different distributions of this extra surplus among the types.

Our work contributes to different strands of literature. It is related to the Informed Principal literature (Myerson (1983), Maskin & Tirole (1990), Maskin & Tirole (1992), Yilankaya (1999), Skreta (2007), Tröger & Mylovanov (2008), Balestrieri (2008)) Indeed, in the jargon of Maskin & Tirole (1992), we consider an informed seller in a common value environment: the seller's type enters directly into the utility function of the buyers. Most of these works, however, consider environments with competition between the buyers as in auction models and vertical differentiation among the buyers' types. We contribute to this literature by considering an horizontally differentiated market. We offer an alternative approach to Myerson (1983) that allows to determine the optimal solution in a variety of new settings. The opportunity for the seller of pooling together her types is evaluated not only with respect of the resulting buyers' expected valuations for the good, but also considering the resulting buyers' willingness to pay for information.

Our analysis is also closely related to the literature about the optimal mechanism for a multi-product monopolist (McAfee & McMillan (1988), Thanassoulis (2004), Pavlov (2006), Balestrieri & João Leão (2008)). The uncertainty about the two types of the seller embedded in our environment happens to provide a natural link between our work and the ones that solve the profit maximization problem of a two-good monopolist. Indeed, our Hotelling environment with different transportation cost function specifications is similar to the one that appears in Balestrieri & João Leão (2008). Both in Pavlov (2006) and in Balestrieri & João Leão (2008) the optimal mechanism is characterized in terms of a set of lotteries. The buyers are price discriminated on the base of their degree of indifference between the two goods. The ones that are more indifferent prefer to buy a lottery with equal probability of winning each good. In the optimal mechanism, the goods are offered beside the lotteries for a higher price. Such extra value may be interpreted as the value of an informed (as opposed of random) purchase. In our environment, there is only one good and the problem of the seller is how to extract revenue from the buyers located further away.

More broadly, our work contributes to a vast literature that considers mechanism design problems in environments in which the seller controls and manipulates the buyers's access to information in order to maximize her profits. This literature covers a space across two fields: industrial organization and auction theory. Like in our case, the problem that these studies tackle is determining if a profit maximizing seller should facilitate or not the buyer's acquisition of information. In general, each buyer's valuation is modeled as a function of the buyer's type and some extra factor. This additional component may be the private information of the seller or an exogenous stochastic variable. In the first case, the problem is often the disclosure of quality related information (vertical differentiation) in the context of pre-specified mechanisms. In the second case, when a stochastic component is the source of the buyers' uncertainty, the seller controls the accuracy with which the buyers learn the value of the realized shock. In static models, the seller can add noise to the shock (without privately observing its realization); in dynamic models, where the shock realizes in the future, the seller can add different time-related options to her offers (advance selling options, refunds options).

Lewis & Sappington (1994) is a seminal contribution in the industrial organization side of this literature. They consider the trade-off faced by a seller who can control the accuracy with

which buyers learn their valuation of the good on sale. More precise private information for the buyers brings new price discrimination opportunities to the seller, but it also leaves higher informational rents to the buyers. They characterize different settings in which extreme disclosure policies (i.e. maximum precision, noise) are optimal.

Such extreme results are also obtained by Johnson & Myatt (2006). In their work they show how the monopolist's profits are a U-shaped function of the dispersion of the buyer's valuation. Given that, the seller considers two strategies. She may reveal information in order to identify the high valuation buyers and charge them a high price (niche-market strategy). Otherwise she may hide information and charge low price to a large number of buyers (mass-market strategy). In other words, the information disclosure policy becomes a tool to transform (rotate) the buyers' demand and maximize profits through niche- or mass-market strategies. In a similar fashion, Anderson & Renault (2006) show that costless advertising may not be always profitable for the seller in an environment where advertisement has a twofold role: improve the precision of the buyer's expected valuation for the good and decrease the search costs incurred by the buyers. In our setting we can also identify a similar to these papers effect on the demand: by not revealing the information the seller lowers the buyer's expected willingness to pay and differently so for different types while gaining a new market.

In the auction theory literature, Milgrom & Weber (1982) raised the question of whether the seller should reveal her information and answered it positively for a general affiliated values setting. Bergemann & Pesendorfer (2007) and Eső & Szentes (2007) derive the optimal auction in environments in which the seller controls the precision with which the buyers learn about their valuation for the good. The degree of uncertainty left to the buyers is an endogenous variable, and one can interpret the optimal mechanism design as having the sale of information by the seller.³ On top of that, each buyer's expected valuation is a monotonically increasing function of the buyer's private information. The provision of information by the auctioneer affects the degree of competition between bidders.⁴ This additional factor is crucial in determining the optimal disclosure policy. For example, Ganuza & Penalva (2010) consider the incentives of an auctioneer to provide private information to the bidders comparing different definitions of signal's precision. Competition and precision appear to be complementary factors to maximize the auctioneer's profits. In all these works, and in contrast to our study, any disclosure of information by the seller happens before the buyers take any action and there is no price on information. The additional crucial distinction of our study from the auction models of the seller's control of information channels and of the information acquisition is that in our setting the seller is the one that possesses information and has different incentives to share it depending on what she knows. In contrast to Milgrom & Weber (1982) we show that quite generally the seller would not want to disclose her information or do so only partially. Moreover, we show that instead of the conventional

³Shi (2007) characterizes an optimal auction with information acquisition. The cost of the information is an exogenous function of the signal precision and the revenues from selling information are not accrued by the seller. Several works in the literature about auctions with costly entry fee can be interpreted in terms of costly information acquisition for the bidders. As noticed in Eső & Szentes (2007), such works usually assume ex-ante homogeneity across the bidders.

⁴Hoffmann & Inderst (2009) extends the work of Eső & Szentes (2007) to a setting in which the provision of information is costly for the seller, and buyers do not compete. The buyer's type are vertically differentiated.

disclosure channels – reveal her information to everyone or affect the information of every type of a specific buyer in the same way, the seller with the private information chooses to disclose her information selectively, only to some types of the buyer and for a fee.

Price discrimination across buyers in terms of their valuation for information (instead of ex-post valuation for the good) is analyzed in models of mechanisms with refunds or advance-purchase discounts.⁵ In these models, like in ours, buyers pay higher prices for more information. However, differently from our work, there is an exogenous dynamic process according to which the buyers learn their true valuation for the good over time.

In the case of refunds, a buyer decides to buy a good on the base of his expectations over the good's valuation. If the buyer actually buys the good, then he learns his valuation and decides if returning it in exchange for a refund. Courty & Li (2000) characterize the optimal selling mechanisms with refunds in settings where each buyer's type is associated with a conditional probability distribution over valuations and the ranking of the types corresponds to a ranking of the conditional distributions in terms of first order stochastic dominance or mean-preserving spread.⁶

In the case of advance-purchase discounts, each buyer's uncertainty about his valuation for the good is resolved over time, and such learning is not conditioned on the purchase of the good. This strand of literature refers mostly to capacity constrained sellers who face uncertain demand (Gale & Holmes (1992), Gale & Holmes (1993); Dana (1998), Dana (1999*b*), Dana (1999*a*), Dana (2001)). An exception is Nocke, Peitz & Rosar (2011), who determine necessary and sufficient conditions for an advance selling mechanism to be optimal.

Our environment differs from the ones in the refund or the advance-purchase literature, because, in our case, the uncertainty of the buyer is due to some information that belongs to the seller. Given that, even the selection of the mechanism becomes a vehicle of information transmission for the buyers (the informed principal problem). Moreover, the feasible information sets of the buyer are not two (no information before buying, full information after buying) but are a continuum and they are endogenously determined in equilibrium as a part of the design of the optimal mechanism.

In most of the aforementioned models buyers are vertically differentiated with respect of their types or, alternatively, ex-ante identical. Exceptions are the works of Board (2009), Koessler & Renault (2011), Sun (2011), Celik (2010), Li (2010). Board (2009) and Li (2010) consider auction models with competition between the bidders. Koessler & Renault (2011) model each buyer's valuation through a general matching function of the seller's and the buyer's type. Sun (2011), Celik (2010) and Li (2010) use Hotelling models where the types are locations along the Hotelling line and the costs of transportations are linear. In all these works, however, any disclosure of information by the seller happens before the buyers make any purchasing decision and there is no price on information.

The rest of the paper is organized as follows. The model is set up in Section 2. In Section 3 we describe the optimal mechanism for the benchmark setting of complete information. In Section 4 we solve for the optimal mechanism under restricted disclosure policy, allowing the seller only full revelation or nor revelation. We also consider specific settings of linear,

⁵Cremer & Khalil (1992), Cremer & Khalil (1994), Cremer, Khalil & Rochet (1998) study costly information acquisition by uncertain buyers. Such costs are not revenues for the seller of the good.

⁶Zhang (2008) considers an optimal auction problem with refunds.

convex, and concave cost functions. In Section 5 we set up the Informed Principal problem allowing for arbitrary disclosure policies and characterize the agent's incentive constraints. Then, in Section 6 we solve a simplified problem, where the principal's choice is limited to committing to reveal or not reveal her private information and then setting the price. Finally, we solve for a general optimal selling scheme and provide examples of the optimal mechanism for the specific cost functions. Section 7 concludes.

2 The model

There are two players, the principal (or the seller) and the agent (the buyer). The principal sells a good or service which the agent wants to purchase. There are two sources of incomplete information. First, the exact characteristics of the good are known to the principal but not fully known to the agent. Second, the agent's consumption utility (conditional on specifics of the good) is not fully known to the principal. In addition, the nature of the principal's information is such that no possible realization of such information (the principal's type) is a priori better than some other realization for all possible types of the agent. That is, the types of the principal are horizontally differentiated (e.g., by taste) rather than vertically (e.g., by quality). Such an environment can be conveniently modeled by a Hotelling-like model. For convenience, we would refer to the principal as she and to the agent as he.

The information of both the principal and the agent can be represented as a location on a line, with s denoting the location of the principal (the seller), and x – the one of the agent. Both the principal and the agent are risk neutral. The utility of the agent located at x from purchasing the good from principal s at price P is

$$U_A(x, s) = V - c(|x - s|) - P,$$

where V is the base value of the transaction, P is the price, and $c(\cdot)$ is the cost function specifying the loss of consumption value to the agent from the difference in the ideal and the actual characteristics of the good, $c(0) = 0$, and c is strictly increasing. The cost of producing the good to the principal is without loss of any generality is taken to be 0, and so the principal's utility from the transaction equals P . The outside participation values for both the principal and the agent are assumed to be 0.

For the main model we are going to assume that the principal's type can take one of the two values $s \in \{0, 1\}$ with equal probabilities, while the agent's type can take a continuum of values $x \in [0, 1]$ and is drawn from a uniform distribution. Accordingly, one can interpret this setup as the principal facing a continuum of agents of a unit mass, uniformly located over the segment. Thus, each of the players knows her own type, while types are independently drawn and the distributions are commonly known.

The equilibrium notion we use throughout the paper is Bayesian-Nash equilibrium.

3 Optimal selling mechanism under complete information

In this section we derive the optimal selling scheme from the perspective of the principal under the assumption that she reveals her private information truthfully or, equivalently, if her information is known to the agent. This is the classic bilateral trade setting with one-sided information problem, and the optimal mechanism is a posted price. For the sake of completeness and to introduce the notation and the logical steps of the approach we derive the optimal mechanism explicitly. Our problem is a special case of the optimal auction problem (Myerson (1981)). Our exposition follows the treatment of the optimal auction in Krishna (2002).

Without loss of any generality we can assume that the principal is located at $s = 0$. By the Revelation Principle, for any mechanism and an equilibrium of it offered by the principal there exist an outcome-equivalent direct mechanism in which all types of the agents report their valuations truthfully. Thus, in order to find the maximal possible expected revenue to the principal and the corresponding allocation and payment rules, it suffices to limit the search over the direct mechanisms.

Any direct mechanism can be characterized by a pair of functions (q, p) , where $q(z)$ is the probability of the sale (the allocation function) and $p(z)$ is the expected payment of the agent reporting z . Let $U(z|x)$ denote the expected utility the agent of type x obtains in the mechanism when he reports z . For the truth-telling to be an equilibrium, the pair (q, p) has to satisfy the following incentive compatibility (IC) and individual rationality (IR) constraints:

IC constraints: $\forall x, z \in [0, 1]$,

$$U(x) \triangleq U(x|x) = q(x)(V - c(x)) - p(x) \geq q(z)(V - c(x)) - p(z) = U(z|x). \quad (1)$$

IR constraints: $\forall x \in [0, 1]$,

$$U(x) = q(x)(V - c(x)) - p(x) \geq 0. \quad (2)$$

For any pair x, z , the combination of IC constraints $U(x) \geq U(z|x)$ and $U(z) \geq U(x|z)$ gives

$$[U(x) - U(x|z)] - [U(z|x) - U(z)] \geq 0 \implies (q(x) - q(z))(c(z) - c(x)) \geq 0. \quad (3)$$

Therefore, the first implication of incentive compatibility is that $q(x)$ must be non-increasing since $c(x)$ is strictly increasing.

Since

$$\begin{aligned} q(x)(c(z) - c(x)) &= U(x|z) - U(x) \leq U(z) - U(x) \\ &\leq U(z) - U(z|x) = q(z)(c(z) - c(x)), \end{aligned}$$

we obtain that the derivative of U exists almost everywhere and that U can be expressed as an integral of its derivative:

$$U'(x) = -q(x)c'(x), \quad (4)$$

$$U(x) = U(0) - \int_0^x q(t)c'(t)dt. \quad (5)$$

Accordingly, from equations (1) and (5), the payment $p(x)$ of consumer located at x is expressed as follows (this is essentially the Revenue Equivalence Theorem, as in Myerson (1981))

$$p(x) = q(x)(V - c(x)) - U(0) + \int_0^x q(t)c'(t)dt. \quad (6)$$

The expected revenue to the principal from any incentive compatible mechanism with implied probabilities of sale $q(x)$ is

$$ER = \int_0^1 p(x)dx = -U(0) + \int_0^1 q(x)(V - c(x))dx + \int_0^1 \int_0^x q(t)c'(t)dt dx. \quad (7)$$

By changing the order of integration

$$\int_0^1 \int_0^x q(t)c'(t)dt dx = \int_0^1 \left(\int_t^1 q(t)c'(t)dx \right) dt = \int_0^1 q(t)c'(t)(1-t)dt, \quad (8)$$

$$ER = -U(0) + \int_0^1 q(x) [V - c(x) + c'(x)(1-x)] dx. \quad (9)$$

Thus, the problem of choosing the optimal selling mechanism reduces to the problem of choosing $q(x)$ that maximizes (9) subject to IR constraints (2). From equation (5) and $q(x) \geq 0$, if IR holds for $x = 1$, then it holds for all x . From equation (5),

$$U(0) = U(1) + \int_0^1 q(t)c'(t)dt, \quad (10)$$

and so equation (9) becomes

$$ER = -U(1) + \int_0^1 q(x) [V - c(x) - c'(x)x] dx. \quad (11)$$

Now, ignoring necessity of $q(x)$ to be non-increasing for a moment, ER is maximized by setting $U(1) = 0$ (as low as possible) and setting $q(x) = 1$ for all x with $\Phi(x) = V - c(x) - c'(x)x \geq 0$, and $q(x) = 0$ for all other x . The virtual valuation function is actually a marginal revenue function, indeed $\Phi(x) = [x(V - c(x))]'$. Certainly, the function $c(x)$ can be such that the virtual valuation function $\Phi(x)$ is not monotone, can cross 0 several times, and so $q(x)$ defined above may not be non-increasing. But in this case (which corresponds to the case where SOC condition for maximization of $x(V - c(x))$ does not hold globally), the familiar ironing technique should be used (or the appropriate global maximum should be chosen), see Myerson (1981).

Note also that the expression (9) for the expected revenue cannot be simply optimized as $U(0)$ also depends on the marginal type of the buyer to whom the good is sold. If buyer x is the marginal type whose IR constraint binds, the price of the good is $P = V - c(x)$. By changing the marginal type, there is the marginal effect on price equal to $-c'(x)$. This is why the function under the integral differs from the virtual value $\Phi(x)$ by exactly $c'(x)$ to account for this extra marginal effect. When solving for the optimal mechanism under incomplete information such extra effects are going to appear and will have to be carefully considered.

3.1 More general settings

One way to generalize the above model is to allow for an arbitrary distribution of the agent's types. We are going to assume that $x \sim F[0, 1]$ with symmetric around 1/2 density f , that is $f(x) = f(1-x)$ for all $x \in [0, 1]$, and that distribution of x is independent of the distribution of the principal's type.

Clearly, the incentive compatibility and individual rationality constraints remain the same, and so the implications of the incentive compatibility are not affected. What is affected is the computation of the expected revenue, equation (7),

$$ER = \int_0^1 p(x)f(x)dx = -U(0) + \int_0^1 q(x)(V - c(x))f(x)dx + \int_0^1 \int_0^x q(t)c'(t)f(x)dt dx.$$

Respectively,

$$\begin{aligned} \int_0^1 \int_0^x q(t)c'(t)f(x)dt dx &= \int_0^1 \left(\int_t^1 q(t)c'(t)f(x)dx \right) dt = \int_0^1 q(t)c'(t)(1 - F(t))dt \\ ER &= -U(0) + \int_0^1 q(x)\Phi(x)f(x)dx, \end{aligned} \quad (13)$$

where

$$\Phi(x) = V - c(x) + c'(x) \frac{1 - F(x)}{f(x)}.$$

Given that the lowest-value type of the agent is $x = 1$ and using (10), we can rewrite (13) as

$$ER = -U(1) + \int_0^1 q(x)\Phi(x)f(x)dx - \int_0^1 q(x)c'(x)dx = -U(1) + \int_0^1 q(x)\Psi(x)f(x)dx, \quad (14)$$

where

$$\Psi(x) = V - c(x) - c'(x) \frac{F(x)}{f(x)}. \quad (15)$$

This is the virtual valuation function appropriately derived for the considered model. If it is not monotone, ironing (and so deriving a monotone quasi-virtual valuation function) may be needed.

The second possible generalization is to allow for the possibility of having random base consumption values, $V \sim H[0, \bar{V}]$ with density h , and independent of the other distributions. Despite the now two-dimensional agent types, (V, x) , the analysis remains the same, as only $V - c(x)$ matters in determining the consumption value to the agent. Indeed, for any pair (V, x) a consumption-equivalent type (\bar{V}, x') can be associated, requiring $\bar{V} - c(x') = V - c(x)$. Clearly, by extending the range of x beyond 1, and by extending $c(x)$ for the values $x > 1$, such an equivalence can be well defined. But this is only so for the case of complete information, as with incomplete information, consumption value equivalence also depends on the perceived probabilities of buying from different types of the principal. (We will deal with this later, once we analyzed the main model for the incomplete information case.)

4 To reveal or not to reveal

Suppose now that the choice of the selling scheme to the principal is limited by either revealing her information and then running the optimal selling scheme of Section 3 or not revealing her info and then running the same selling scheme for all the types of the principal. We are considering this restricted problem in order to gain track on the effects of informational disclosure under various cost structures. Besides, in many settings, the seller may be limited to no or full disclosure policies.

Let us derive the best selling scheme conditional on the information not being revealed and then compare whether it is better to reveal the information or not to reveal. By the Revelation principle, for any mechanism and its equilibrium that is offered by the seller there exists a direct mechanism in which the agent finds it optimal to report his type truthfully. Such a direct mechanism can be represented by a pair of functions (q, p) , where $q(z)$ and $p(z)$ denote, respectively, the probability of getting the good and the price paid given the report $z \in [0, 1]$.

The implication of the IC constraints, inequality (19), becomes: $\forall x, z \in [0, 1]$,

$$(q(x) - q(z)) ([c(z) - c(x)] + [c(1 - z) - c(1 - x)]) \geq 0. \quad (16)$$

For the general cost functions, the simplest way to solve for the optimal selling scheme is to reorder the agent's types according to their expected distance costs. For each type x of the agent we can assign type $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$. Letting $y_0 = \min_{x \in [0, 1]} \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$ and $y_1 = \max_{x \in [0, 1]} \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$, we have that the new types of the agent belong to the segment $[y_0, y_1]$, the utility from purchasing the good to the agent of type y net of the price is $V - y$, and the distribution of the types y , F_y is given by $F_y(z) = \Pr(y < z) = \Pr(\frac{1}{2}c(x) + \frac{1}{2}c(1 - x) < z)$. In turn, inequality (16) becomes

$$\forall y, z \in [y_0, y_1], \quad (q(y) - q(z)) (z - y) \geq 0.$$

Thus, in any incentive compatible scheme, the lower is the expected distance costs of the agent the higher must be the probability of him receiving the good.

By following the same steps as in Sections 3 and 3.1, we obtain

$$ER = -U(y_1) + \int_{y_0}^{y_1} q(z) \left[V - z - \frac{F_y(z)}{f_y(z)} \right] f_y(z) dz. \quad (17)$$

As long as the virtual value $\Psi^{nr}(z) = V - z - \frac{F_y(z)}{f_y(z)}$ is monotone, the optimal selling mechanism is obtained by setting $q(z) = 1$ whenever $\Psi^{nr}(z) \geq 0$ (note that $\Psi^{nr}(y_0) > 0$), which means selling the good at the price $P = V - y^*$, where y^* solves $\Psi^{nr}(y^*) = 0$ or $y^* = y_1$ if $\Psi^{nr}(y_1) \geq 0$. If $\Psi(z)$ is not monotone, the ironing procedure needs to be applied to compute the monotone quasi virtual valuation function $\hat{\Psi}^{nr}(z)$, and then the optimal mechanism is derived in a similar way. In any case, the optimal selling scheme is the posted price.

In order to get a better feeling of how the optimal solution looks in this case and to compare with the full revelation of the information option, let us consider three subcases of the cost function: linear, convex, and concave.

4.0.1 Linear costs

For the case of linear costs, $c(x) = cx$, the resulting optimal scheme is trivial. Indeed, $\frac{1}{2}c(x) + \frac{1}{2}c(1-x) = \frac{c}{2}$, that is all the agents have the same expected utility from the good. Thus, as long as $V - \frac{c}{2} > 0$, it is optimal for the principal to set $P = V - \frac{c}{2}$ and serve the whole market. Thus, the profit to each type of the principal is $\pi = V - \frac{c}{2}$. If the principal reveals her private information, then the optimal cut-off type computed for the principal's type $s = 0$ is determined from the equation $V - c(x) - c'(x)x = 0$, that is $x^* = \frac{V}{2c}$ or $x^* = 1$ if $V > 2c$. The price and the profit are $P = V - cx^* = \frac{V}{2}$ and $\pi = \frac{V^2}{4c}$ if $x^* < 1$ or $P = V - c$ and $\pi = V - c$ if $V > 2c$.

By comparing $\frac{V^2}{4c}$ and $V - \frac{c}{2}$, we obtain that for a given c , if $V > 2c - \sqrt{4c^2 - 2}$, then it is better not to reveal the information about the principal's type. The basic trade-off in choosing whether to reveal the information or not is whether to serve only the "local" market—the types of the agents that are "nearby" and charge them a higher price or expand and serve the whole market but at a possibly lower price. If V is large and the whole market is served even when the information is revealed, then it is strictly better not to reveal the information, as then the higher price can be charged. For a specific value of $c = 1$, the cutoff value is $\hat{V} = 2 - \sqrt{2} \approx 0.6$ in which case around 30% of the market is served when the type of the principal is revealed. By not revealing the information, each type of the principal will be able to serve the whole market, and this explains why the cut-off value of V for this to happen is relatively small.

4.0.2 Convex costs

For the case of convex costs, i.e. strictly increasing $c'(x)$, we have $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1-x)$ is decreasing on $x \in [0, \frac{1}{2}]$ as $y'(x) = \frac{1}{2}(c'(x) - c'(1-x)) < 0$. Thus, $y_0 = c(\frac{1}{2})$ and $y_1 = \frac{1}{2}c(1)$. It is clear that density $f_y(z)$ is bounded from away from 0. Indeed, $F_y(z) = 2\Pr(z > y(x) \geq y_0 | x \leq \frac{1}{2})$, and as $|y'(x)|$ is bounded from above $f_y(z)$ is bounded from below, $f_y(z) \geq \frac{2}{\max|y'(x)|}$. Thus, $\frac{F_y(z)}{f_y(z)}$ and $z + \frac{F_y(z)}{f_y(z)}$ are bounded.

Consider the virtual value $\Psi^{nr}(z) = V - z - \frac{F_y(z)}{f_y(z)}$ from equation (17). If $V > y_0$, then $\Psi^{nr}(y_0) > 0$, and so a positive revenue can be earned if the principal reveals no information. In the optimal scheme, the principal will set up a price $P^* > V - y_0$, in which case agent's types in the middle of the segment (with $y(x)$ close to y_0) will purchase the good, while those who are at the edges may be left out. Note that if V is sufficiently high, then $\Psi^{nr}(z) > 0$ for all $z \in [y_0, y_1]$, in which case in the optimal scheme all the agents buy the good, and the optimal price $P^{nr} = V - y_1 = V - \frac{1}{2}c(1)$.

Compared to the case when all the information is revealed, it is clear that for $V \leq y_0$ and by continuity for $V < y_0 + \delta$ for some $\delta > 0$, it is better to reveal all the information. However, if V is sufficiently high, for instance when $\Psi^{nr}(z) > 0$ for all $z \in [y_0, y_1]$ and $\Psi(x) > 0$ for all x (from equation (15)), the principal sells to all the agents at the price of $P^{nr} = V - \frac{1}{2}c(1)$ when no information is revealed and at the price $P^r = V - c(1)$ if all the information is revealed. Clearly, it is better not to reveal any information.

The expected valuations of the agent, and possible optimal schemes are shown on Figure 1. Here the solid curve is the expected value of the agent under no revelation disclosure

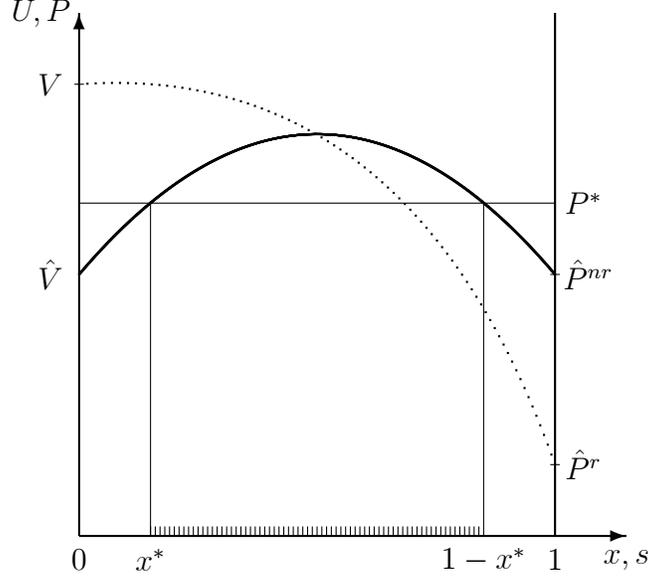


Figure 1: No revelation under convex costs.

policy. The dotted curve is the expected value of the agent if the type of the seller were $s = 0$ and known. If the base value V is in the intermediate range, the typical optimal solution will be to set price P^* , $\hat{V} = V - \frac{1}{2}c(1) < P < V - c\left(\frac{1}{2}\right)$, in which case the agents to the left of x^* and to the right of $1 - x^*$ will buy the good. Prices P^{nr} and P^r show what the optimal prices would be under no revelation and under revelation disclosure policies if V is sufficiently high.

4.0.3 Concave costs

For the case of concave costs, i.e. strictly decreasing $c'(x)$, we have $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1-x)$ is increasing on $x \in [0, \frac{1}{2}]$ as $y'(x) = \frac{1}{2}(c'(x) - c'(1-x)) < 0$. Thus, $y_0 = \frac{1}{2}c(1)$ and $y_1 = c\left(\frac{1}{2}\right)$. The only qualitative difference with the case of convex costs is that now it is agent's types at the edges that have the lowest expected costs, and if the optimal price is in between y_0 and y_1 those at the edges buy the good, while those in the middle are not. Similarly, if $V < y_0 + \delta$ for some $\delta > 0$, it is better to reveal all the information, while if V is sufficiently high, the price at which all the agent's types are served is higher when no information is revealed.

We can state a more general result considering only two possibilities for the principal: to reveal the information about her location or not to reveal it.

Lemma 1. *If the base consumption value V is sufficiently low, then it is better for the principal to reveal her information. If the base consumption value is sufficiently high, then it is better not to reveal anything.*

Proof. Clearly, $y_0 > 0$, and if $V \leq y_0$ then by not revealing any information no revenue can be collected. Revealing is better and will remain better by continuity for V slightly above y_0 . If V is sufficiently high then both $\Psi^{nr}(y(x))$ and $\Psi(x)$ are strictly positive for all x , thus every type of the agent will be served in each informational treatment. Clearly,

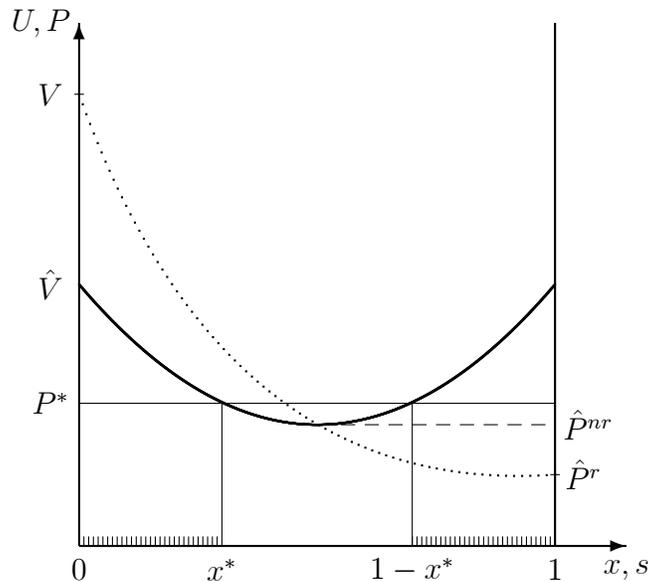


Figure 2: No revelation under concave costs.

$$y_1 = \max_{x \in [0,1]} \frac{1}{2}c(x) + \frac{1}{2}c(1-x) < c(1) \text{ as } c(x) \text{ is increasing, and so } \pi^{nr} = P^{nr} = V - y_1 > V - c(1) = P = \pi. \quad \square$$

If V is small, then when no information is revealed the expected value of the good for each type of the agent is very low. Thus, it is better to reveal all information and extract higher revenue from the nearby types. When V is intermediate, when she does not reveal her location the principal gains the access to a larger market though selling at a low price. When V gets larger the “new” market effect starts to dominate higher price effect. When V is large then the whole market is being served, but on top of it, the principal is able to charge a higher price when no information is revealed.

The expected valuations of the agent, and possible optimal schemes are shown on Figure 2. The difference with the convex costs is that for the intermediate values of V , in the optimal mechanism under no informational disclosure only the agent’s types in the middle, from x^* to $1 - x^*$ buy the good.

5 Incomplete information

5.1 Incentive compatibility

The characterization of the incentive constraints is no longer a straightforward exercise, as we have multiple types of the principal. The principal can suggest different mechanisms and equilibria depending on her information, and the agent can make inferences about the information of the principal from the mechanism she is being offered, which may affect his incentives.

By the Inscrutability Principle (see Myerson (1983)) we can always represent the menu of the mechanisms offered by the principal (depending on her type) as a single inscrutable

mechanism, in which agents infer nothing about the type of the principal when the mechanism (and its equilibrium) is offered. By the Revelation Principle, for any such inscrutable mechanism and its equilibrium there exists a direct inscrutable mechanism with truth-telling as an equilibrium.

Accordingly, we can limit our search of the optimal incentive scheme for all types of the principal to the set of inscrutable direct mechanisms, which can be represented by a collection of functions $(Q_0, P_0; Q_1, P_1)$, where for all $s \in \{0, 1\}$, $Q_s(z)$ is the probability of the sale and $P_s(z)$ is the expected payment of the agent reporting z when the realized type of the principal is s . From the perspective of the agent, who due to the inscrutability of the mechanism cannot distinguish principal's types, such a direct mechanism can be represented as a triple of functions (q_0, q_1, p) , which are, respectively, the probabilities of receiving the good if the principal is of type 0 and of type 1, and the expected payment, all functions of the agent's report. Thus, $q_s(z) = Q_s(z)$, for all $s \in \{0, 1\}$, and $p(z) = \frac{1}{2}P_0(z) + \frac{1}{2}P_1(z)$.

IC constraints: $\forall x, z \in [0, 1]$,

$$\begin{aligned} U(x) &\triangleq U(x|x) = \frac{1}{2}q_0(x)(V - c(x)) + \frac{1}{2}q_1(x)(V - c(1-x)) - p(x) \\ &\geq \frac{1}{2}q_0(z)(V - c(x)) + \frac{1}{2}q_1(z)(V - c(1-x)) - p(z) = U(z|x). \end{aligned} \quad (18)$$

Similarly to (3), we obtain

$$(q_0(x) - q_0(z))(c(z) - c(x)) + (q_1(x) - q_1(z))(c(1-z) - c(1-x)) \geq 0. \quad (19)$$

Unlike the complete information case, one cannot establish monotonicity of $q(x)$, since for $z > x$, $c(z) > c(x)$ and $c(1-z) < c(1-x)$. However, if $q_0(x)$ is constant at around some x , then $q_1(x)$ is increasing; and if $q_1(x)$ is constant at around x , then $q_0(x)$ is increasing. Intuitively, in any incentive compatible mechanism there is a pressure to sell the good from principal $s = 0$ (or $s = 1$) more often to agents closer to 0 (or 1). Similarly to (4) and (5) we have

$$U'(x) = -\frac{1}{2}q_0(x)c'(x) + \frac{1}{2}q_1(x)c'(1-x), \quad (20)$$

$$U(x) = U(0) - \frac{1}{2} \int_0^x q_0(t)c'(t)dt + \frac{1}{2} \int_0^x q_1(t)c'(1-t)dt. \quad (21)$$

Accordingly, the expected payment from the agent of type x is

$$\begin{aligned} p(x) &= \frac{1}{2}q_0(x)(V - c(x)) + \frac{1}{2}q_1(x)(V - c(1-x)) \\ &\quad - U(0) + \frac{1}{2} \int_0^x q_0(t)c'(t)dt - \frac{1}{2} \int_0^x q_1(t)c'(1-t)dt. \end{aligned} \quad (22)$$

5.2 The Informed Principal Problem

Each type of the principal wants to maximize her own revenue. Given that the agent does not know the principal's type, by offering a specific mechanism the principal may try to

influence beliefs of the agent about her type in the way that is more profitable for her. On the other hand, given the mechanism offered, the agent may reason about which type of the principal have offered the mechanism, and adjust his behavior accordingly. The principal can still commit to the rules of the mechanism she is offering but cannot commit or force the agent to believe that she would have offered a specific mechanism if she were the other type.

How to deal with these mechanism selection issue and the simultaneous objectives of all of the principal's types is the heart of the Informed Principal Problem. For our problem, we argue, the correct way to deal with it is to maximize the *ex ante* surplus, that is, to find the incentive compatible mechanism that maximizes the total revenue to all types of the principal. This is possible because of risk-neutrality of the agents and quasi-linearity of preferences, and can be argued as follows.

Suppose we have an equilibrium in which the tuple $(Q_0, P_0; Q_1, P_1)$ describes the direct incentive compatible inscrutable mechanism which is being offered by the principal, which, from the agent's perspective is represented by a triple of functions (q_0, q_1, p) . Suppose also that there exists a different incentive compatible triple (q'_0, q'_1, p') in which $Ep' = \int_0^1 p'(x)dx > \int_0^1 p(x)dx = Ep$, that is, the agent in expectation pays more under (q'_0, q'_1, p') than under (q_0, q_1, p) . Given (q'_0, q'_1, p') , one can set $Q'_0 = q_0$, $Q'_1 = q_1$ and find P'_0, P'_1 , such that $p'(z) = \frac{1}{2}P'_0(z) + \frac{1}{2}P'_1(z)$, while $ER'_i = \int_0^1 P'_i(x)dx > \int_0^1 P_i(x)dx = ER_i$ for $i = 0, 1$. For instance, let $P'_i(z) = 2\frac{ER_i}{Ep}p'(z)$, which corresponds to the case when extra profits $Ep' - Ep$ are divided between different types of the principal in proportion of their profits in the original mechanism. Then, each type of the principal instead of offering $(Q_0, P_0; Q_1, P_1)$ can offer instead $(Q'_0, P'_0; Q'_1, P'_1)$ and gain strictly more in expectation. Which means that $(Q_0, P_0; Q_1, P_1)$ cannot have been a solution to the informed principal problem and cannot have been offered in equilibrium by each type of the principal. Therefore, any solution to the informed principal problem should maximize the *ex ante* expected revenue to the principal.

Remark 1. *The same logic would apply for any support of the principal's types, whether a finite set or continuum. The key to the argument is that if there is any "money left on the table" to the agent, then there is a direct incentive compatible mechanism in which the agent pays more. As the agent is risk-neutral, his payment can be represented as a lottery on the principal's type. In turn, this allows to transfer the surplus among different types of the principal.*

Remark 2. *The possibility to make monetary transfers among different types of the principal allows, in principle, for the whole family of solutions to our problem, in which the total surplus over the principal's types is the same, but different types of the principal may earn different sums. Still, there must be bounds on how much the surplus can be transferred. This bounds can stem from the possibility of each principal's type to offer alternative mechanisms. That is, one can think that each principal has a best "alternative" mechanism, and so in the solution, she should not earn less than in the alternative. One extra necessary constraint on these alternative mechanisms is the condition that can be called "no fooling the agent" condition: that all the alternative mechanisms for all the principal's types must be consistent with each other according to the beliefs they induce from the agent and in terms of expected revenue. More precisely, the expectation of induced beliefs over all alternative mechanisms*

must coincide with the prior.⁷ In addition, the expected payment from the agent in these alternative mechanisms should not exceed the maximal expected payment among all the direct incentive compatible triples (q_0, q_1, p) . Indeed, one can consider the combined inscrutable mechanism composed of all the alternative mechanisms. These conditions must hold for this combined mechanism.

6 Optimal mechanism (symmetric)

The expected revenue collected from the agents in the incentive compatible mechanism $q_0(x)$, $q_1(x)$, $p(x)$ is

$$\begin{aligned} ER &= \int_0^1 p(x)dx = -U(0) + \frac{1}{2} \int_0^1 q_0(x)(V - c(x))dx + \frac{1}{2} \int_0^1 q_1(x)(V - c(1-x))dx \\ &\quad + \frac{1}{2} \int_0^1 \int_0^x q_0(t)c'(t)dt dx - \frac{1}{2} \int_0^1 \int_0^x q_1(t)c'(1-t)dt dx. \end{aligned} \quad (23)$$

By changing the order of integration as in equations (7-9), we can express

$$\begin{aligned} ER &= -U(0) + \frac{1}{2} \int_0^1 q_0(x) [V - c(x) + c'(x)(1-x)] dx \\ &\quad + \frac{1}{2} \int_0^1 q_1(x) [V - c(1-x) - c'(1-x)(1-x)] dx \end{aligned} \quad (24)$$

Finding the optimal mechanism boils down to maximizing (24) subject to IR constraints, IC constraints (19), and $q_0(x) \in [0, 1]$, $q_1(x) \in [0, 1]$ for all $x \in [0, 1]$. The maximization is non-trivial in two respects. First, similarly to the complete information case there exists a type x^* for which IR constraint is binding strictly, $U(x^*) = 0$, which from (21) would allow to express

$$U(0) = \frac{1}{2} \int_0^{x^*} q_0(t)c'(t)dt - \frac{1}{2} \int_0^{x^*} q_1(t)c'(1-t)dt.$$

Therefore,

$$\begin{aligned} ER &= \frac{1}{2} \int_0^{x^*} q_0(x) [V - c(x) - c'(x)x] dx + \frac{1}{2} \int_{x^*}^1 q_0(x) [V - c(x) + c'(x)(1-x)] dx \\ &\quad + \frac{1}{2} \int_0^{x^*} q_1(x) [V - c(1-x) + c'(1-x)x] dx \\ &\quad + \frac{1}{2} \int_{x^*}^1 q_1(x) [V - c(1-x) - c'(1-x)(1-x)] dx \end{aligned} \quad (25)$$

⁷We do not impose the constraint that in the mechanism offered by a specific type s of the principal, the agent must believe that it has been offered by type s .

Thus, the expected revenue consists of four components

$$ER = \frac{1}{2} \int_0^{x^*} q_0(x)A(x)dx + \frac{1}{2} \int_{x^*}^1 q_0(x)B(x)dx + \frac{1}{2} \int_0^{x^*} q_1(x)C(x)dx + \frac{1}{2} \int_{x^*}^1 q_1(x)D(x)dx, \quad (26)$$

which can be interpreted as follows. Function $A(x)$ is the complete information ($s = 0$) virtual valuation or marginal revenue from selling the good to the agent of type x assuming all the agents closer than him also purchase the good. Indeed, for $P = V - c(x)$, $R = (V - c(x))x$ and $MR = V - c(x) - c'(x)x$. Function $C(x)$ is the complete information (from the perspective of $s = 1$) virtual valuation or marginal revenue from selling the good to the agent of type x assuming all the agents further away than him also purchase the good. Indeed, in this case $P = V - c(1 - x)$, $R = (V - c(1 - x))x$ and $MR = V - c(1 - x) + c'(1 - x)x$. Note that $B(x) = A(x) + c'(x)$; and $D(x) = C(x) - c'(1 - x)$. Function $B(x)$ is the marginal revenue corrected by the extra marginal effect on the expected revenue if x is chosen as the marginal type to whom the good is sold (as if $s = 0$ is known). Indeed, in this case due to IR constraint the price charged is $V - c(x)$ with the marginal effect of $-c'(x)$. Thus, $B(x) - c'(x)$ is the total effect and it is equal to the marginal revenue $A(x)$. Similarly, from the perspective of $s = 1$, the extra marginal effect is positive and equal to $c'(1 - x)$.

Second, ignoring IC and IR constraints one can easily find the mechanism maximizing (26): set $q_0(x) = 1$, whenever $A(x)$ or $B(x)$ is positive, and 0, otherwise; set $q_1(x) = 1$, whenever $C(x)$ or $D(x)$ is positive, and 0, otherwise. But as IC and IR constraints must hold, one has to look at the relationships between $A(x)$ and $C(x)$ and between $B(x)$ and $D(x)$ more carefully. For instance, even under complete information of $s = 0$, letting the optimal cut-off x^* solve $A(x^*) = 0$, we have $B(x^*) > 0$ and so $B(x) > 0$ for some $x > x^*$. But setting $q(x) = 1$ would actually lower the revenue as $B(x)$ overstates the true marginal revenue by $c'(x)$.

One can show that in the our symmetric setting the optimal mechanism is necessarily symmetric [proof to be added], but for now we are going to assume that the solution is symmetric. That is,

$$\forall x \in [0, 1], \quad q_0(x) = q_1(1 - x). \quad (27)$$

As a result, by rewriting expression (26) substituting (27) for $x \in [1/2, 1]$, we obtain

$$ER = \int_0^{x^*} q_0(x)A(x)dx + \int_0^{x^*} q_1(x)C(x)dx + \int_{x^*}^{\frac{1}{2}} q_0(x)B(x)dx + \int_{x^*}^{\frac{1}{2}} q_1(x)D(x)dx \quad (28)$$

where

$$A(x) = V - c(x) - c'(x)x, \quad (29)$$

$$B(x) = V - c(x) + c'(x) \left(\frac{1}{2} - x \right), \quad (30)$$

$$C(x) = V - c(1 - x) + c'(1 - x)x, \quad (31)$$

$$D(x) = V - c(1 - x) - c'(1 - x) \left(\frac{1}{2} - x \right).$$

The expressions for $B(x)$ and $D(x)$ change as everything is doubled due to symmetry but the marginal effect of changing the type whose IR constraint binds.

An implication of the incentive constraints: the objective (28) together with (19) implies that for almost all $x \in [0, 1/2]$, $q_0(x)$ and $q_1(x)$ are at extremes (0 or 1). Thus, there are no lotteries involved in optimal selling contract (except, perhaps, for a set of types of measure 0). [Proof to be added]

6.1 Linear costs

Here we derive the optimal mechanism imposing no constraints on the disclosure policy. In the Section 4.0.1 we saw that the linear costs case becomes degenerate under no revelation, as the expected value for all the buyer's types is the same. Accordingly, one may expect that one cannot do better as there is no surplus left to extract. It turns that one can do better.

Linear costs, $c(x) = cx$. We have $A(x) = V - 2cx$, $C(x) = V + 2cx - c$, $B(x) = V - 2cx + 1/2c$, $D(x) = V - 3/2c + 2cx$. Note that: $A(x)$ is decreasing and $A(x) = 0$ at $x_0^* = \frac{V}{2c}$, while $C(x)$ is increasing and for $V < c$, $C(x) = 0$ at $x_1^* = \frac{c-V}{2c}$. Note also, that $B(x) > A(x)$, while $D(x) < C(x)$.

For a moment, assume that $x^* = 1/2$, that is, that IR constraint holds with equality for $x^* = 1/2$ and so $B(x)$ and $D(x)$ are irrelevant.

Then, for $V \geq c$ (for which $x_0^* \geq \frac{1}{2}$), the optimal selling mechanism is $q_0(x) = q_1(x) = 1$ for all $x \in [0, 1/2]$ — the two types of the principal “pool,” selling the good at the price $p = V - \frac{1}{2}c$.

For $V < c$, the unconstrained optimum is

$$q_0(x) = \begin{cases} 1, & \text{for } x \in [0, x_0^*] \\ 0, & \text{for } x \in (x_0^*, \frac{1}{2}] \end{cases} ; \quad q_1(x) = \begin{cases} 0, & \text{for } x \in [0, x_1^*] \\ 1, & \text{for } x \in (x_1^*, \frac{1}{2}] \end{cases} . \quad (32)$$

Since $q_0(x) < q_1(x)$ for $x > \max\{x_0^*, x_1^*\}$, IC constraints are contradicted by this solution. To restore IC, one has to require $q_0(x) = q_1(x)$ whenever unconstrained maximization brings $q_0(x) < q_1(x)$. Thus, there are two possible corrections to (32) to consider: (i) lower q_1 to 0, or (ii) increase q_0 to 1 on the interval of interest. Note, however, that once for some x' , $q_0(x') = q_1(x') = 0$ or $q_0(x') = q_1(x') = 1$, IC implies that $q_0(x) = q_1(x)$ for all $x \in [x', 1/2]$, and that all these types receive the same expected utility $U(x)$. Thus, the shape of the optimal mechanism depends on the comparison between x_0^* and x_1^* .

If $x_0^* \leq x_1^*$, which occurs when $V \leq \frac{c}{2}$ (which implies $x_0^* < \frac{1}{4}$), case (i) applies, and the optimal selling mechanism is described by

$$q_0(x) = \begin{cases} 1, & \text{for } x \in [0, \frac{V}{2c}] \\ 0, & \text{for } x \in (\frac{V}{2c}, \frac{1}{2}] \end{cases} ; \quad q_1(x) = 0, \text{ for } x \in [0, \frac{1}{2}] ; \quad (33)$$

$$p(x) = \begin{cases} \frac{V}{4}, & \text{for } x \in (0, \frac{V}{2c}] \\ 0, & \text{for } x \in (\frac{V}{2c}, \frac{1}{2}] \end{cases} .$$

In it, the type of the principal is revealed, only those customers that are close to her are served, and the price for the good $p = 2p(x) = \frac{V}{2}$.

If $x_0^* \leq x_1^*$, which occurs when $V \in [\frac{c}{2}, c)$, case (ii) applies, and the optimal selling mechanism is described by

$$\begin{aligned}
q_0(x) = 1, \text{ for } x \in \left[0, \frac{1}{2}\right]; \quad q_1(x) = \begin{cases} 0, & \text{for } x \in \left[0, \frac{c-V}{2c}\right] \\ 1, & \text{for } x \in \left[\frac{c-V}{2c}, \frac{1}{2}\right] \end{cases}; \\
p(x) = \begin{cases} \frac{3V}{4} - \frac{c}{4}, & \text{for } x \in \left[0, \frac{c-V}{2c}\right] \\ V - \frac{c}{2}, & \text{for } x \in \left[\frac{c-V}{2c}, \frac{1}{2}\right] \end{cases}.
\end{aligned} \tag{34}$$

This selling mechanism can be interpreted as follows. The principal offers a menu: buy the “cat in a sack” good (principal’s type is not revealed) for the price $p = V - \frac{c}{2}$; or, buy the information about the location of the principal for the price p_I and then, if the principal is close to you, the good at the price p_G . The price of information is equal to the (expected) loss a customer of type $x_1^* = \frac{c-V}{2c} = \frac{1}{2} - \frac{V}{2c}$ obtains if he finds the principal is “far” from him. Accordingly, p_G can be found from

$$\frac{3V}{4} - \frac{c}{4} = p(x_1^*) = \frac{1}{2}p_G + p_I.$$

Thus,

$$p_I = -\frac{1}{2} \left(V - c \left(1 - \frac{c-V}{2c} \right) \right) = \frac{c-V}{4}. \tag{35}$$

$$p_G = V - c \frac{c-V}{2c} - 2p_I = 2V - c. \tag{36}$$

Now, we have done these derivations assuming that the agent with the lowest utility (hitting IR constraint with equality) is agent $x^* = \frac{1}{2}$. It turns out that this is without loss of any generality since optimization on $x \in [0, x^*]$ boils down to the exactly same solution as above. This implies $q_0(x^*) = q_1(x^*)$ or $q_0(x^*) = 1 > q_1(x^*) = 0$. In the former case, IC implies $U(x^*) = U(x)$ for all $x \in [x^*, 1/2]$, which means that one could have set $x^* = \frac{1}{2}$ from the start. In the latter case, since $C(x) > A(x)$, the optimization on $x \in [x^*, 1/2]$ necessitates $q_0(x) > q_1(x)$ for x close and to the right of x^* , which in turns implies $U(x) < U(x^*)$ for such x , contradicting strict *IR* for x^* .

6.2 Linear costs: Example.

Let us compute a numerical example for the optimal selling mechanism for the case of linear costs, $c(x) = x$. If $V \leq \frac{1}{2}$, the optimal mechanism is to reveal the type of the principal, $p = \frac{V}{2}$, and the profits $\pi = \frac{V}{2} \times \frac{V}{2} = \frac{V^2}{4}$. In the case $V > \frac{1}{2}$, $x_1^* = \frac{1}{2} - \frac{V}{2}$, $p\left(\frac{1}{2}\right) = V - \frac{1}{2}$; $p_I = \frac{1-V}{4}$; and $p_G = 2V - 1$. Altogether, the profits are

$$\begin{aligned}
\pi &= \frac{1-V}{2}p_G + 2\frac{1-V}{2}p_I + V \left(V - \frac{1}{2} \right) \\
&= \frac{1-V}{2}(2V-1) + 2\frac{1-V}{2}\frac{1-V}{4} + V^2 - \frac{V}{2} = \frac{V^2}{4} + \frac{V}{2} - \frac{1}{4}.
\end{aligned}$$

For instance, if $V = 0.8$, we have $x_1^* = 0.1$, $p_I = 0.05$, $p_G = 0.6$, the price of the good without information revelation is 0.3. Altogether the profits are 0.31, which is higher than 0.16 coming from full revelation, and 0.30 from no information revelation. If $V = 0.6$, then $x_1^* = 0.2$, $p_I = 0.1$, $p_G = 0.2$, no revelation price is 0.1. Total profits are 0.14 compared to 0.09 from full revelation and 0.1 from no revelation.

6.3 Concave and convex costs

[to be added]

7 Conclusion

[to be added]

References

- Anderson, S. P. & Renault, R. (2006), ‘Advertising content’, *American Economic Review* **96**(1), 93–113.
- Balestrieri, F. (2008), ‘A modified english auction for an informed buyer’, MIT, mimeo.
- Balestrieri, F. & João Leão (2008), ‘The market for surprises: Selling substitute goods through lotteries’, MIT, mimeo.
- Bergemann, D. & Pesendorfer, M. (2007), ‘Information structures in optimal auctions’, *Journal of Economic Theory* **137**(1), 580–609.
- Board, S. (2009), ‘Revealing information in auctions: the allocation effect’, *Economic Theory* **38**(1), 125–135.
- Celik, L. (2010), ‘Information unraveling revisited: Disclosure of horizontal attributes’, CERGE-EI, mimeo.
- Courty, P. & Li, H. (2000), ‘Sequential screening’, *Review of Economic Studies* **67**(4), 697–717.
- Cremer, J. & Khalil, F. (1992), ‘Gathering information before signing a contract’, *American Economic Review* **82**(3), 566–78.
- Cremer, J. & Khalil, F. (1994), ‘Gathering information before the contract is offered: The case with two states of nature’, *European Economic Review* **38**(3-4), 675–682.
- Cremer, J., Khalil, F. & Rochet, J.-C. (1998), ‘Strategic information gathering before a contract is offered’, *Journal of Economic Theory* **81**(1), 163–200.
- Dana, J. D. (1998), ‘Advance-purchase discounts and price discrimination in competitive markets’, *Journal of Political Economy* **106**(2), 395–422.
- Dana, J. D. (1999a), ‘Equilibrium price dispersion under demand uncertainty: The roles of costly capacity and market structure’, *RAND Journal of Economics* **30**(4), 632–660.
- Dana, J. D. (1999b), ‘Using yield management to shift demand when the peak time is unknown’, *RAND Journal of Economics* **30**(3), 456–474.

- Dana, J. D. (2001), ‘Monopoly price dispersion under demand uncertainty’, *International Economic Review* **42**(3), 649–70.
- Esó, P. & Szentes, B. (2007), ‘Optimal information disclosure in auctions and the handicap auction’, *Review of Economic Studies* **74**(3), 705–731.
- Gale, I. L. & Holmes, T. J. (1992), ‘The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty’, *International Journal of Industrial Organization* **10**(3), 413–437.
- Gale, I. L. & Holmes, T. J. (1993), ‘Advance-purchase discounts and monopoly allocation of capacity’, *American Economic Review* **83**(1), 135–46.
- Ganuza, J. J. & Penalva, J. (2010), ‘Signal orderings based on dispersion and the supply of private information in auctions’, *Econometrica* **78**(3), 1007–1030.
- Hoffmann, F. & Inderst, R. (2009), ‘Price discrimination and the provision of information’, University of Frankfurt, mimeo.
- Hotelling, H. (1929), ‘Stability in competition’, *Economic Journal* **39**(153), 41–57.
- Johnson, J. P. & Myatt, D. P. (2006), ‘On the simple economics of advertising, marketing, and product design’, *American Economic Review* **96**(3), 756–784.
- Koessler, F. & Renault, R. (2011), ‘When does a firm disclose product information?’, Paris School of Economics, mimeo.
- Krishna, V. (2002), *Auction Theory*, Academic Press, San Diego, California.
- Lewis, T. R. & Sappington, D. (1994), ‘Supplying information to facilitate price discrimination’, *International Economic Review* **35**(2), 309–27.
- Li, D. Z. (2010), ‘Cheap-talk information disclosure in auctions’, University of Oxford, mimeo.
- Maskin, E. & Tirole, J. (1990), ‘The principal-agent relationship with an informed principal: The case of private values’, *Econometrica* **58**(2), 379–409.
- Maskin, E. & Tirole, J. (1992), ‘The principal-agent relationship with an informed principal, ii: Common values’, *Econometrica* **60**(1), 1–42.
- McAfee, R. P. & McMillan, J. (1988), ‘Multidimensional incentive compatibility and mechanism design’, *Journal of Economic Theory* **46**(2), 335–354.
- Milgrom, P. R. & Weber, R. J. (1982), ‘A theory of auctions and competitive bidding’, *Econometrica* **50**(5), 1089–1122.
- Myerson, R. B. (1981), ‘Optimal auction design’, *Mathematics of Operations Research* **6**(1), 58–73.

- Myerson, R. B. (1983), ‘Mechanism design by an informed principal’, *Econometrica* **51**(6), 1767–97.
- Nocke, V., Peitz, M. & Rosar, F. (2011), ‘Advance-purchase discounts as a price discrimination device’, *Journal of Economic Theory* **146**(1), 141–162.
- Pavlov, G. (2006), Optimal mechanism for selling substitutes, Boston University - Department of Economics - Working Papers Series WP2006-014, Boston University - Department of Economics.
- Shi, X. (2007), Optimal auctions with information acquisition, Working Papers tecipa-302, University of Toronto, Department of Economics.
- Skreta, V. (2007), On the informed seller problem: Optimal information disclosure, Levine’s Bibliography 84364400000000222, UCLA Department of Economics.
- Sun, M. J. (2011), ‘Disclosing multiple product attributes’, *Journal of Economics and Management Strategy* **20**(1).
- Thanassoulis, J. (2004), ‘Haggling over substitutes’, *Journal of Economic Theory* **117**(2), 217–245.
- Tröger, T. & Mylovanov, T. (2008), Optimal auction design and irrelevance of private information, Bonn Econ Discussion Papers bgse21_2008, University of Bonn, Germany.
- Yilankaya, O. (1999), ‘A note on the seller’s optimal mechanism in bilateral trade with two-sided incomplete information’, *Journal of Economic Theory* **87**(1), 267–271.
- Zhang, J. (2008), ‘Auctions with refund policies as optimal selling mechanisms’, Queen’s University, mimeo.