

Auctions in Markets: Common Outside Options and the Continuation Value Effect*

Stephan Lauer[†] Gábor Virág[‡]

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Abstract

We study auctions with outside options determined through bargaining in the market. We show that auctions with less information revelation to bidders may yield higher revenues. It is never optimal for the auctioneer to reveal information after the auction, while it may be optimal to do so before. These results differ from the case where the value of the outside option arises exogenously in the post-auction market. Non-transparent auctions are preferred by the auctioneer if: (i) the value of the outside option depends less on market conditions per se than on whether the action chosen in the external market reflects those conditions adequately; (ii) bidders have imprecise signals about conditions prevailing in the post-auction market. Consequently, the auctioneer prefers to hide information about aggregate market conditions exactly when the value of information in the post-auction market is the highest for the bidders.

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[†]University of Michigan, Department of Economics, slauerma@umich.edu.

[‡]Corresponding author, University of Rochester, Department of Economics, gvirag@mail.rochester.edu.

1 Introduction

We study an auctioneer’s preferences over different auction formats, when his auction is embedded in a larger market in which losing bidders can purchase a substitute of the good sold in the auction. We investigate whether an individual auctioneer prefers to run transparent or opaque auctions. Our findings show that an auctioneer has the strongest incentives to run an opaque auction when (i) buyers have imprecise information about the outside option provided by the market and (ii) when buyers’ payoffs in the subsequent market are sensitive to the action taken there. These observations imply that the auctioneer has incentives to hide information about market conditions exactly when the value of information for buyers is highest.

The model consists of two stages. First, a fixed number of buyers participate in a second-price auction in which a single, indivisible good is for sale. After the auction any losing bidder is paired with another seller who offers an identical good as the original auctioneer. To model bargaining between the two sides, we assume that the buyer makes a take-it-or-leave-it price offer to the seller. The seller’s cost is unknown to the buyer, and it depends stochastically on the unknown state of the world, which is either *high* or *low*. In the high state sellers are more likely to have higher costs, so the buyers’ continuation payoffs are lower. Before submitting their bids in the auction, buyers receive noisy private signals about the state. The signals induce heterogeneity in beliefs among buyers¹ who are otherwise identical. Higher signals are indicative of the high state, and lower continuation values. We focus on the (unique) symmetric and monotone equilibrium of the second-price auction. In this equilibrium each bidder bids his valuation *minus* the continuation utility that arises from the post-auction bargaining problem. Then, we ask whether the auctioneer has an incentive to commit *ex ante* to reveal a perfectly informative signal about the state before the auction. We provide the following results: If the signal is revealed, then the revenue decreases if the bidders’ private signals are sufficiently uninformative (see Proposition 4), and increases if payoffs in the bargaining game depend more on the realized state than on how well the price offer fits that state (see Proposition 3). We also show that the auctioneer always prefers not to disclose the bids to the losing bidders after the auction (Proposition 6). Finally, the second-price auction always yields higher revenues than the first-price auction; the comparison between the English auction and the second-price auction depends on the informativeness of the bidders’ signals, and on the induced payoffs of the bargaining game (Proposition 7).

¹In many market situations agents have not only imprecise, but also asymmetric beliefs about the state of the economy. Such heterogeneity is also important for our results: if the bidders had access to the same (possibly imprecise) signal, then the situation would resemble a simple Bertrand competition with very different implications for auction design.

To provide intuition for our results, we decompose the revenue impact of revealing an informative signal about the state into two opposing effects. Interestingly, although our model is not a common value auction model, but the insights from the *linkage principle* are still active. The fact that common outside options introduce common value elements into bidding has been noted before (for example in Milgrom and Weber (1982)), so our setup is naturally related to such auctions. Milgrom and Weber (1982) observed that in such auctions the auctioneer prefers to reveal information to encourage bidder competition, if bidders observe affiliated signals about the object (or the outside options). This rule is known as the Linkage Principle. However, our analysis also identifies the *continuation value effect*, which favors opaque auctions. A transparent auction reveals information about outside market conditions, thereby improving the continuation payoffs of losing bidders. The improvement of the continuation payoffs leads to lower bids, decreasing the auctioneer’s revenue.²

The continuation value effect and the linkage effect respond differently to changes in the parameters of the continuation problem. The continuation value effect is strong when bidders enter the auction with imprecise signals, since in this case revealing the state has a larger impact on losing bidders’ information about market conditions. Consequently, the quality of the decisions made in the post-auction market improves substantially. The continuation effect is also strong when the optimal price offer is sensitive to the state, since in this case improving one’s information about the market conditions has a larger impact on one’s decision, and hence on one’s continuation utility. Since in both of these cases the continuation value effect is strong, revealing information about the state reduces expected revenues. When the two states provide intrinsically different continuation payoffs regardless of the action taken in the post-auction market, the linkage effect is strong. The intuition is immediate from the common value auctions literature: revealing information to all bidders makes the knowledge of all bidders more symmetric, which intensifies the competition between them. This reduces bidders’ rents and increases the auctioneer’s revenues.

The decomposition also illustrates that revealing any information after the auction (e.g. the winning bid) decreases revenues. When information is revealed after the auction, the linkage effect is absent because bidders cannot incorporate the revealed information into their bids. However, information revelation improves bidders’ outside options, and thus lowers the revenues of the auctioneer through the continuation value effect.

The insights from our analysis extend to other settings with dynamic market interaction.

²Technically, the continuation value effect is related to the convexity of the continuation payoffs in buyers’ posteriors. Revealing information leads to a mean-preserving spread of the posteriors. Therefore, when continuation payoffs are (strictly) convex, revealing information increases expected continuation payoffs.

Lauermann, Merzlyn and Virag (2010) consider a large dynamic matching and bargaining game in which buyers and sellers (auctioneers) are matched every period. Each buyer participates in a sequence of auctions, and information learned in one auction allows the buyer to refine his bids in future auctions. Individual sellers, who choose whether to reveal information, face a problem that is similar to the problem of the auctioneer in our model. Duffie and Manso (2007) study a model where traders learn about aggregate market conditions in a dynamic matching market, which is interpreted as a decentralized over-the-counter asset market. Agents meet in small groups and exchange information each period. They study how fast information spreads if agents never hide any information from those they meet.³ Interestingly, real world over-the-counter asset markets do not always favor information percolation. In fact, the opacity of such markets raised regulatory concerns, motivating recent regulatory interventions to increase their transparency.⁴ Our analysis provides a possible explanation why opaque trading institutions may develop in decentralized markets. Indeed, the auctioneer has incentives to hide information exactly when the value of information for buyers is highest, which implies that in a dynamic interaction information may spread very slowly exactly when the value of information for buyers is highest.

We discuss extensions and limitations of our model in Section 5. Our model might apply particularly well to those situations in which (i) there is no strategic interaction among the participants in the future, and (ii) the auctioneer cannot charge fees to buyers for information. The first point distinguishes our analysis from the literature on auctions with resale, see, for example, Garratt and Tröger (2006), Hafalir and Krishna (2008) and Haile (2001); and from the literature on multi-unit auctions, see, for example Mezzetti et. al. (2008). For an analysis of the case where information may also be sold see Gershkov (2009), Eso and Szentes (2007) and Hörner and Skrzypacz (2010). The most related papers to ours are Bergemann and Pesendorfer (2007), Board (2009), and Forand (2010) who assume that information is not sold.⁵ In a setup of multi-unit auctions, Perry and Reny (1999) show that the insights

³Since the same agents never meet again, it seems reasonable to assume that they do not hide information from each other. However, we show that if continuation values are affected by exchanging information, then hiding information may be privately beneficial. In the different framework of repeated random matching games (see Deb (2009) for example), agents also play against strangers each period. To sustain cooperation, information about a deviation must be transmitted in a timely manner to the other agents in the economy, so that the deviator can be punished effectively. Therefore, it must be checked whether it is in an agent's interest to reveal the state of the economy (whether any agent has cheated before).

⁴The over-the-counter market for corporate bond was traditionally opaque. In 2002, the market underwent a fundamental change when the Transaction Reporting and Compliance Engine (TRACE) was introduced; see Bessembinder and Maxwell (2008).

⁵Board (2009) assumes that the auctioneer may know a signal z that influences who should obtain the object. He shows that hiding information may be revenue enhancing, because it reduces information rents of the bidders. Bergemann and Pesendorfer (2007) study a related framework, where each bidder's valuation is affected by an idiosyncratic signal, and they characterize optimal information rules. Forand (2010) studies the

from the linkage principle are qualified when there are multiple items sold. In particular, revealing information may decrease the bids of all bidders. In our paper a single object is auctioned, so the fact that the linkage principle has to be qualified comes from a different source. Finally, Tadelis and Zettelmeyer (2009) provide an empirical assessment of the effects of information policy on sellers' revenue in used car auctions.

The remainder is organized as follows. Section 2 describes the main model, Section 3 provides the main results for the second-price auction format, and Section 4 contains revenue comparisons between the first-price, second-price and English auctions. Section 5 provides some robustness checks. Section 6 concludes. Most of the proofs are in the Appendix.

2 Model and preliminary analysis

2.1 Setup

The interaction unfolds in three stages. First, the auctioneer and the N bidders receive signals about the state of the world. Second, the auctioneer runs an auction for an indivisible object. Third, each losing bidder chooses a price offer in a bilateral bargaining problem.

Information. There are two states of the world, $w \in \{H, L\}$, and the realization is not observed by the bidders. The probability of the high state is ρ_0 . The state of the world is interpreted as the aggregate market condition. The bidders receive private signals that are correlated with the state, and these signals are denoted by s_1, s_2, \dots, s_N . In state w , the bidders' signals are distributed independently and identically according to G_w on support $[\underline{s}, \bar{s}]$. We assume that G_w admits a differentiable density function g_w . With a signal s , the Bayesian posterior probability of the high state is denoted by $\rho(s) = \rho_0 g_H(s) / ((1 - \rho_0) g_L(s) + \rho_0 g_H(s))$. We assume that the likelihood ratio of the signal g_H/g_L is strictly increasing, and thus the posterior is strictly increasing in s . To simplify exposition, we assume that $\rho(\bar{s}) = 1$ and $\rho(\underline{s}) = 0$, that is, there are some perfectly informative signals.⁶

Auction. All bidders participate in an auction where a single indivisible object is for sale. We analyze bidding in standard auction formats, including the first-price, the second-price, and the ascending (English) auction.

Preferences and payoffs. The winning bidder receives the object and pays a price p , while the losing bidders do not make payments in any of the auctions studied. The valuation for

case of competing auctioneers. In equilibrium, the auctioneers attract bidders by revealing their information (under commitment).

⁶Later, in a discussion we show that this assumption does not drive our results.

the object, v , is the same for all bidders and publicly known. The utility of the winner is equal to $v - p$, while that of the losers' is equal to their continuation payoffs defined below.

Outside Option After the auction each losing bidder proceeds to a “market” which is modeled as follows. After losing, the bidder is matched with another seller with probability μ_ω . If matched, the buyer can make a take-it-or-leave-it offer to the seller. To ease exposition we assume that when making the offer the losing bidder does not observe whether he is matched yet.⁷ The seller accepts the offer whenever it is below his costs. The buyer does not know the cost of the seller but believes that, conditional on state ω , the costs are distributed on some interval $[\underline{c}, \bar{c}]$ according to a distribution function F_ω which is atomless and has a differentiable density. The expected utility from an offer x is

$$u_w(x) = \mu_w(v - x) F_w(x).$$

Let $a(\rho) = \arg \max \rho u_H(x) + (1 - \rho) u_L(x)$ be the price offer given belief ρ and let $V(\rho) = \rho u_H(a(\rho)) + (1 - \rho) u_L(a(\rho))$ be the value function. Let $U_w(\rho)$ be the payoff in state w of a buyer who takes action $a(\rho)$.⁸ It is important to stress that both V and U_w are exogenous for the auction stage as they embody information about the optimal action *after* the auction given any belief.

When only the matching probabilities—but not the distribution of the seller’s costs—differ across states, the optimal action is independent of the state. In this case, $V(\rho)$ is simply a linear function of the probability of the high state, $V(\rho) = \rho V(1) + (1 - \rho) V(0)$. If the distribution does depend on the state, the optimal action depends on the belief ρ . In the latter case, it follows from standard arguments from the economics of information that the value function V is convex in beliefs. The curvature of the value function plays an important role in our analysis.

The following example illustrates the continuation problem we are considering:

Example 1: A continuation value problem. Let $\mu_H = \mu_L = 1$ and let $v \geq 1$, $k \geq 0$ with $F_L(x) = \frac{d-0.5kx^2+t}{v-x}$ and $F_H(x) = \frac{k(x-0.5x^2-0.5)+t}{v-x}$.⁹ The example is constructed so that the

⁷This assumption can be relaxed if one impose conditions on the relative values of μ_H and μ_L so that a monotone equilibrium exists. Alternatively, instead of matching probabilities, one can think of μ_H and μ_L as probabilities with which the offer gets communicated to the potential seller

⁸The above notation implicitly assumes that $a(\rho)$ is the unique optimal price when the belief is ρ . Whenever there are multiple actions that are optimal (set $\alpha(\rho)$) and that deliver different payoffs conditional on the realized state, let $U_H(\rho) = \max_{x \in \alpha(\rho)} u_H(x)$ and let $U_L(\rho) = \min_{x \in \alpha(\rho)} u_L(x)$. In the Appendix we show that for almost all ρ all optimal actions provide the same payoffs in the two states, that is, $U_w(\rho) = u_w(x)$ for all $x \in \alpha(\rho)$.

⁹To ensure that F_L, F_H are proper distribution functions, we assume that $k \leq 1$, $t \geq k/2$, and that $0.5 + \frac{d}{k} + \frac{t}{k} \geq v \geq d + t - 0.5k + 1$. This region is non-empty if $k \leq 1$. To ensure (strict) monotonicity of V

optimal price offer is identical to the belief, that is, $\alpha(\rho) = \rho$. The value function is

$$V(\rho) = t + (1 - \rho)(d - 0.5k) + 0.5k\rho^2.$$

The value function is strictly convex in ρ if $k > 0$, highlighting the value of information. If $k = 0$, the value function is linear and payoffs depend only on the state, not on the action.

3 Second-price Auction

To start our analysis of the effects of the information policy on revenues we consider a sealed-bid second-price auction. This auction format lends itself to a very tractable analysis, since bidding incentives are relatively simple. We characterize equilibrium bidding behavior and compare revenues for three different information policies. In Section 3.1 we consider the case where no information (other than who has won) is released, in Section 3.2 we consider the case where the auctioneer has perfect information about the state and reveals his information, and in Section 3.3 the case where the winning bid is revealed.

3.1 Equilibrium without information revelation

We study symmetric equilibrium where each bidder's bid is strictly monotone in his signal. Milgrom and Weber (1982) showed that in the standard common value setup each bidder bids his valuation assuming that he ties at the top spot. Equilibrium bids are similar in our setting. Let $\rho_{tie}(s)$ denote an agent's belief conditional on being tied at the top, and let $\rho_{lose}(s)$ denote the probability of the high state conditional on losing with a signal s .¹⁰ The relevant continuation value is assessed conditional on tying (ρ_{tie}) as in Milgrom and Weber (1982). However, in our setup the continuation value also depends on the action taken in the continuation problem, and this action is the optimal action for the belief upon losing (ρ_{lose}). The following Proposition provides existence and uniqueness results¹¹:

Proposition 1 *If V is strictly decreasing, then there exist monotone and symmetric equi-*

we assume that $d \geq k/2$.

¹⁰The posterior upon tying at the top is $\rho_{tie}(s) = \frac{\rho_0 g_H^2(s) G_H^{N-2}(s)}{\rho_0 g_H^2(s) G_H^{N-2}(s) + (1-\rho_0) g_L^2(s) G_L^{N-2}(s)}$. The posterior upon losing is $\rho_{lose}(s) = \frac{\rho_0 g_H(1-G_H^{N-1})}{\rho_0 g_H(1-G_H^{N-1}) + (1-\rho_0) g_L(1-G_L^{N-1})}$.

¹¹We have shown that if there are only two possible actions in the continuation problem, then any symmetric equilibrium is monotone when V is decreasing. We conjecture that this holds for any continuation value problem, but a proof is currently unavailable.

libria. In every such equilibrium for almost all s each bidder bids according to

$$b(s) = v - [\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))]. \quad (1)$$

If V is not strictly decreasing, then a monotone and symmetric equilibrium does not exist. A monotone and symmetric equilibrium exists if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$.

The proof in the Appendix establishes that (1) follows from necessary first order conditions for the bidders' problems, and that the global optimality conditions are satisfied when V is decreasing. For the rest of the paper, except when it is stated otherwise (see Example 3), we assume that V is decreasing, and concentrate on the monotone equilibrium in the game with no information revelation.

If V is linear, then information has no value and U_H, U_L are constant functions of ρ . Consider the following common value auction in the framework of Milgrom and Weber (1982). There are two states, the value of the object is $v - U_H$ in the high, and $v - U_L$ in the low state, and bidders receive signals that are i.i.d. conditional on the state. This common value auction is formally equivalent to our setup when V is linear, and in both models the equilibrium bid function is $b = v - V(\rho_{tie})$. This shows that one can model auctions with common outside options as common value auctions if the value of the outside option depends on the state, but not on an action taken in the continuation problem. On the other hand, if the value of the outside option depends on the action taken, then (as we show below) the auction design influences the amount of information learned by the bidders, and thus $\rho_{lose}(s)$ depends on the information policy adopted by the auctioneer. Since this belief does not enter the equilibrium bid function of standard common value auctions, therefore the case in which the value of the outside option depends on actions in the post-auction market cannot be captured by the common value framework of Milgrom and Weber (1982).

3.2 Revenue comparison when the state may be revealed

3.2.1 Main result

We now derive the ex-ante expected revenue when the auctioneer who knows the state precisely¹² reveals the state before the auction (ER^{Before}), and compare it to the ex-ante expected revenue when the state is not revealed (ER^{None}). In order to obtain insights into the main trade-offs involved we calculate the difference in revenues ($ER^{Before} - ER^{None}$) as the difference of two opposing effects.

¹²At the end of this Section we discuss the case where the auctioneer's information about the state is not fully precise, and show that our results still hold qualitatively.

To formally introduce those two effects, let ER^{After} denote the ex-ante expected revenue in the second-price auction when the state is revealed *after* the auction. We decompose the change in revenue from revealing the state before the auction:

$$ER^{Before} - ER^{None} = \underbrace{(ER^{Before} - ER^{After})}_{\text{Linkage Effect}} - \underbrace{(ER^{None} - ER^{After})}_{\text{Continuation Value Effect}}.$$

The first of the two opposing effects is the *linkage effect*. This effect measures the increase in revenues when the state is revealed as it makes bidders more aggressively by making their information more similar and intensifying competition.¹³ The second effect, the *continuation value effect* measures the decrease in revenue when the state is revealed as the result of improving outside options, and lower willingness to pay in the current auction. We show that the two revenue differences work in the opposite direction, that is, they are both positive.

The ex ante expected revenues of the auctioneer when the state is revealed before or after the auction are derived next. When the state is revealed *before* the auction takes place, all bidders bid $v - V(1)$ in the high state, and all bidders bid $v - V(0)$ in the low state. The ex-ante expected revenue is

$$ER^{Before} = v - (1 - \rho_0)V(0) - \rho_0V(1) = v - V(0) + (V(0) - V(1))\rho_0. \quad (2)$$

When the state is revealed *after* the auction, the full information optimal action is taken in both states, yielding utilities $V(1)$ and $V(0)$. In this case, our model can be reduced to Milgrom and Weber (1982) as discussed before. From their analysis, it follows that the equilibrium bid function is $v - (\rho_{tie}(s)V(1) + (1 - \rho_{tie}(s))V(0))$. Let $g^{(2)}(s)$ denote the density function of the second largest signal of the N signals from an ex-ante perspective.¹⁴ The bidder with the second highest signal determines the revenue in the second-price auction, and the expected revenue is

$$ER^{After} = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} \rho_{tie}(s)g^{(2)}(s)ds. \quad (3)$$

The following result collects our findings:

¹³This effect was introduced in Milgrom and Weber (1982) in the context of common value auctions, although we modified their intuition to obtain an intuition more robust to changes in auction formats and other considerations. See also Perry and Reny (1999) for a discussion.

¹⁴Formally, $g^{(2)}(s) = \rho_0 N g_H(s)(1 - G_H(s))G_H^{N-2}(s) + (1 - \rho_0) N g_L(s)(1 - G_L(s))G_L^{N-2}(s)$.

Proposition 2 *The linkage effect and the continuation value effect are given by:*

$$LP = ER^{Before} - ER^{After} = (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g^{(2)}(s) (\rho_0 - \rho_{tie}(s)) ds, \quad (4)$$

$$CV = ER^{None} - ER^{After} = \int_{\underline{s}}^{\bar{s}} g^{(2)} [\rho_{tie}(V(1) - U_H(\rho_{lose})) + (1 - \rho_{tie})(V(0) - U_L(\rho_{lose}))] ds. \quad (5)$$

The revenue of the auctioneer decreases when the state is revealed before the auction rather than not revealed at all if and only if $CV > LP$.

Proof. Using (1), the ex-ante expected revenue of the auctioneer from not revealing the state is

$$ER^{None} = \int_{\underline{s}}^{\bar{s}} g^{(2)}(s) b(s) ds = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g^{(2)} \rho_{tie} ds + \int_{\underline{s}}^{\bar{s}} g^{(2)} [\rho_{tie}(V(1) - U_H(\rho_{lose})) + (1 - \rho_{tie})(V(0) - U_L(\rho_{lose}))] ds.$$

Then the result follows directly from formulas (2) and (3). **Q. E. D.**

Suppose that information has no value, that is, the utility in the continuation problem depends only on the state, but not on the action. In this case for all beliefs $\rho \in [0, 1]$ it holds that $V(1) - U_H(\rho) = V(0) - U_L(\rho) = 0$. Inspection of the continuation value effect shows that it is zero. Since only the linkage effect remains active, revealing information is profitable as shown by Milgrom and Weber (1982). This is not surprising, since (as we argued after Proposition 1) in this case our model reduces to the Milgrom and Weber (1982) setup. Finally, the linkage effect measures the difference of equilibrium rents the bidders make under the two information policies. This difference is captured by the auctioneer just like in Milgrom and Weber (1982).¹⁵

3.2.2 Comparative statics of information revelation

We now discuss conditions under which revealing or hiding information may be profitable. We consider an example to show that revealing the state is profitable if the value of information is low, if the payoff differences between the states are high and if bidders have precise signals. General comparative statics results follow after the example.

Example 2. Assume that there are two bidders and the two states are equally likely ex-ante. The signal distribution function is $G_L(x) = 1 - (1 - x)^\beta$ in the low state, and $G_H(x) = x^\beta$ in the high state for some $\beta \geq 1$. Importantly, the beliefs ρ_{lose} and ρ_{tie} ¹⁶ depend

¹⁵See the first paragraph and the corresponding footnote in Section 4.

¹⁶Formally, $\rho_{tie} = \frac{(g_H)^2}{(g_H)^2 + (g_L)^2}$ and $\rho_{lose} = \frac{g_H(1 - G_H)}{g_H(1 - G_H) + g_L(1 - G_L)}$.

only on β , but not on the other parameters of the example. To measure the precision of signals, introduce $a = 1 - 1/\beta$ and note that when $a = 0$ the signals are uninformative as they have the same distribution in the two states. As a increases the signals become more precise, and in the limit when $a \rightarrow 1$ the signals in the low state converge to 0 in probability, and the signals in the high state converge to 1 in probability, that is, signals are perfectly informative in the limit. The value function is the same as in Example 1, that is $V = t + k\rho(\rho - 0.5\rho^2 - 0.5) + (1 - \rho)(d - 0.5k\rho^2)$. The utility from taking the action that is optimal with belief ρ is $U_H(\rho) = k(\rho - 0.5\rho^2 - 0.5) + t$ if the state is high, and $U_L(\rho) = d - 0.5k\rho^2 + t$ if the state is low. The difference of the expected utility in the two states is $U_H(\rho) - U_L(\rho) = d + k\rho - 0.5$, so d can be interpreted as a payoff difference between the two states. A higher value of k means that information is more valuable, because the utility losses from taking a suboptimal action in the low state ($U_L(0) - U_L(\rho) = \frac{k\rho^2}{2}$) and in the high state ($U_H(1) - U_H(\rho) = k(\frac{1}{2} - \rho + \frac{\rho^2}{2})$) are both increasing in k . Using that $V(0) = U_L(0)$ and $V(1) = U_H(1)$, straightforward algebra shows that

$$CV = k \int_{\underline{s}}^{\bar{s}} g^{(2)}(s)(0.5\rho_{lose}^2(s) + 0.5\rho_{tie}(s) - \rho_{tie}(s)\rho_{lose}(s))ds. \quad (6)$$

The fact that $d = V(0) - V(1)$ implies that the linkage effect can be written as

$$LP = d(0.5 - \int_{\underline{s}}^{\bar{s}} g^{(2)}(s)\rho_{tie}(s)ds). \quad (7)$$

The continuation value effect is increasing in k , since the larger the value of information is, the more it helps the losing bidders in the continuation problem. The linkage effect is increasing in d , since the greater the payoff differences between the states are, the more it helps to reveal the state in reducing bidders' rents by intensifying the competition between the bidders. Figure 1 captures these comparative statics results by depicting the continuation value effect and linkage effect as a function of k and d .

Let us now provide general counterparts to the comparative statics result depicted in Figure 1. Consider an auction where the continuation value function is indexed by parameters δ and κ , while the signal distributions are indexed by a parameter α . The interpretation of these variables is the same as those of d , k and a in Example 2: δ measures the payoff differences between states, κ measures the value of information and α measures the precision of the signals of the bidders. In what follows we formalize these three measures for a general environment.

First, to measure the value of information in the post-auction market, let us introduce functions $W_L^\kappa(\rho)$, $W_H^\kappa(\rho)$ that measure the baseline utility of an agent (in states L and H) who

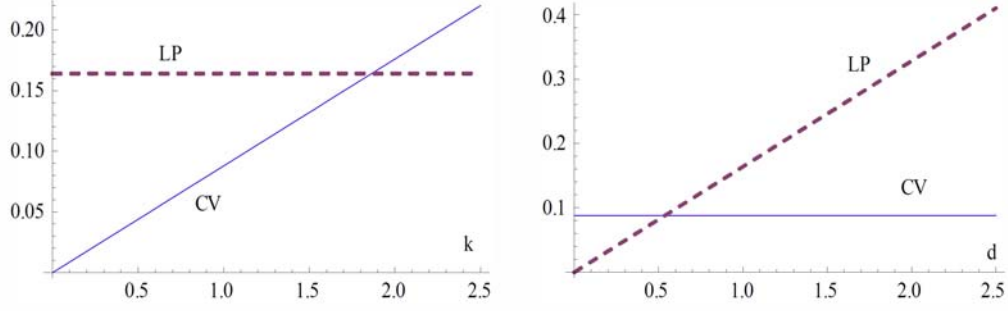


Figure 1: CV and LP as functions of the value of information k and the payoff difference between states d (with signal precision $\alpha = 0.4$, $d = 1$ and $\alpha = 0.4$, $k = 1$).

take the action optimal for belief ρ . Assume that $W_L^\kappa(\rho), W_H^\kappa(\rho)$ are decreasing in κ for all $\rho \in (0, 1)$ and $W_H^\kappa(1) = W_L^\kappa(0) = t$, that is, they are not affected by κ . Then the utility loss from not taking the optimal action in the high state, that is, $V(1) - W_H^\kappa(\rho) = W_H^\kappa(1) - W_H^\kappa(\rho)$, is increasing in κ for any $\rho < 1$, and a similar statement is true for the loss in the low state. Consequently, the value of information is increasing in κ . Second, to measure intrinsic payoff differences between states, let $U_L^\kappa(\rho) = \delta + W_L^\kappa(\rho)$, and $U_H^\kappa(\rho) = W_H^\kappa(\rho)$; that is, an increase in δ implies a boost of the utility in the low state that does not depend on the action taken, and leaves the utility in the high state unchanged. Finally, note that the value function induced by the above specification is

$$V_\delta^\kappa(\rho) = \rho U_H^\kappa(\rho) + (1 - \rho) U_L^\kappa(\rho) = \delta(1 - \rho) + \rho W_H^\kappa(\rho) + (1 - \rho) W_L^\kappa(\rho), \quad (8)$$

where functions $W_H^\kappa(\rho), W_L^\kappa(\rho)$ do not depend on δ .

We are ready to state our general comparative statics result as the value of information (κ) and the payoff differences between states (δ) change:

Proposition 3 (*Incentives for Information-Revelation and the Shape of V.*) *The continuation value effect is strictly increasing in the value of information κ , and is independent of the payoff difference between states δ . The linkage effect is strictly increasing in the payoff difference between states, and is independent of the value of information.*

Proof. First, (8) and the fact that $W_H^\kappa(1) = W_L^\kappa(0) = t$ imply that for any fixed κ it holds that $V_\delta^\kappa(0) - V_\delta^\kappa(1) = \delta + W_L^\kappa(0) - W_H^\kappa(1) = \delta$ and thus the linkage effect can be written as

$$LP = \delta \int_{\underline{s}}^{\bar{s}} g^{(2)}(s) (\rho_0 - \rho_{tie}(s)) ds, \quad (9)$$

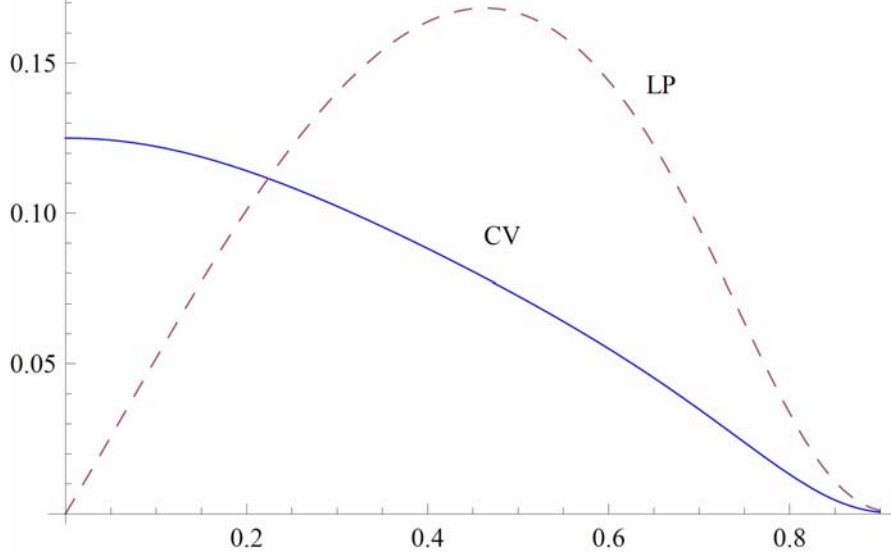


Figure 2: CV and LP as functions of α ($k = d = 1$).

which is clearly increasing in δ and is independent of κ . Second, by construction for all ρ , $V_\delta^\kappa(1) - U_H^\kappa(\rho_{lose}) = t - W_H^\kappa(\rho_{lose})$ and $V_\delta^\kappa(0) - U_L^\kappa(\rho_{lose}) = \delta + t - U_L^\kappa(\rho_{lose}) = t - W_L^\kappa(\rho)$ are increasing in κ and constant in δ , since functions W_H^κ and W_L^κ are constant in δ . Therefore, the continuation value effect

$$CV = \int_{\underline{s}}^{\bar{s}} g^{(2)}[\rho_{tie}(V_\delta^\kappa(1) - U_H^\kappa(\rho_{lose})) + (1 - \rho_{tie})(V_\delta^\kappa(0) - U_L^\kappa(\rho_{lose}))] ds \quad (10)$$

is increasing in κ and independent in δ . **Q. E. D.**

Let us return to Example 2 and study the effect of signal precision (a) on revenues. Taking any d and k such that a monotone equilibrium exists (that is, $d \geq k/2$), let us vary a ranging from full precision ($a = 1$) to being completely uninformative ($a = 0$). We depict the resulting graph in Figure 2.

If signals are uninformative, then the linkage principle effects is absent, because bidders know the value of the object, and thus their rents are competed away already. However, the continuation value effect is strong, because bidders value any extra information about the state a lot when they do not have precise information to begin with. When signals are perfectly informative both effects disappear, because the bidders know the state, so it does not make a difference whether the auctioneer reveals his information or not. More interestingly, when signals are *almost* perfectly informative the linkage principle dominates.¹⁷

¹⁷The reason is that when the losing bidders have very precise information about the state, any increased precision has only a second order effect on their continuation utilities.

The figure shows that for some a^*

$$LP \geq CV \iff a \geq a^* \in (0, 1),$$

that is, the linkage effect wins over if and only if signals are precise enough. Consequently, the auctioneer has incentives to reveal his information only if the bidders have precise information already when entering the auction.

Since the linkage effect tends to dominate when signals are precise, the auctioneer has more incentives to reveal the state when the bidders have fairly good signals on their own. On the flip side, when bidders are less well informed, the auctioneer prefers to hide information to avoid increasing the continuation utilities of the bidders. This outcome is unfavorable to bidders, since the auctioneer hides his information exactly when the bidders need it the most to make better decision on the market unfolding after the auction. The result has implications for the information percolation literature as well. Suppose that bidders are matched with sellers in a dynamic game, and ask whether the seller has incentives to reveal the maximum amount of information about market conditions. Our result shows that unless bidders arrive at the transaction place with precise information about the market already, they may not be able to learn the state quickly, since the auctioneer has no incentive to reveal it to them. In other words, information will "percolate" only if it was learned reasonably well already before bidders arrive on the market.

We now show that the above results hold more generally. Let the signal distribution functions G_H^α, G_L^α be parameterized by α and let $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s) + (1-\rho_0) g_L^\alpha(s)}$ denote the posterior upon receiving signal s . We adopt the convention that signals are uninformative when $\alpha = 0$ holds, and perfectly informative when $\alpha = 1$. In particular, we assume that for all s it holds that $G_H^\alpha(s) = G_L^\alpha(s)$ when $\alpha = 0$, so each signal realization s has the same meaning. To capture that signals become perfectly informative when α is close to 1 we assume that for any $y > 0$ it holds that

$$\lim_{\alpha \rightarrow 1} \Pr(\rho^\alpha(s) \geq y \mid \omega = L) = \lim_{\alpha \rightarrow 0} \Pr(\rho^\alpha(s) \leq 1 - y \mid \omega = H) = 0.$$

Our next result shows that it remains true in the general case that hiding the state is revenue enhancing when bidders have uninformative signals:

Proposition 4 (*Incentives for Information-Revelation when Signals are Uninformative.*) *The continuation value effect is stronger than the linkage effect if signals are uninformative ($\alpha = 0$), and information has value, that is, V is not linear.*

Proof. When $\alpha = 0$ for all s it holds that $g_H^\alpha(s) = g_L^\alpha(s)$ and thus $\rho_{tie}^\alpha(s) = \rho_0$ holds¹⁸. Therefore, $\int_{\underline{s}}^{\bar{s}} g^{(2)}(s) (\rho_0 - \rho_{tie}(s)) ds = 0$. This then implies that when signals are not informative then the linkage effect disappears, that is,

$$\alpha = 0 \Rightarrow LP = 0.$$

However, as long as V is not linear it holds for a set of beliefs $\rho \in (\underline{\rho}, \bar{\rho})$ that $V(1) - U_H(\rho)$, $V(0) - U_L(\rho) > 0$, and the continuation value effect is strictly positive (see formula (5)).

Q.E.D.

Finally, we would like to confirm that it continues to hold in the general framework that when signals become *almost* perfectly informative, then the linkage effect dominates the continuation value effect. To establish this result we need to make a further assumption on the exact way signals become perfectly informative:

Assumption CR: *There exists $T > 0$ and $\hat{\varepsilon} > 0$ such that for all $0 < \varepsilon \leq \hat{\varepsilon}$*

$$\frac{\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \geq 1 - \varepsilon} g^{(2)}(s) (1 - \rho(s))^2 ds}{\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \leq \varepsilon} g^{(2)}(s) \rho(s) ds} \leq T.$$

To interpret this assumption, imagine that for any fixed α the signals are distributed symmetrically in the sense that $\Pr(\rho(s) \leq t \mid L) = \Pr(\rho(s) \geq 1 - t \mid H)$ for any $t \in [0, 1]$. In this case the relevant ratio converges to 0, as it can be shown.¹⁹ Indeed, the only way to violate Assumption CR is to assume that signals are much more precise in the low state than in the high state in the limit. At the end of the online Appendix we consider such an example, and show that the conclusion of our Proposition below fails.

Proposition 5 (Incentives for Information-Revelation when Signals are very Informative.) *Under Assumption CR the continuation value effect is weaker than the linkage effect if the signals are informative, that is when $\alpha \in (\hat{\alpha}, 1)$ for some $\hat{\alpha} < 1$.*

Proof. See the online Appendix.

¹⁸To see this, note that in this case $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s) + (1 - \rho_0) g_L^\alpha(s)} = \rho_0$, and $\rho_{tie}^\alpha(s) = \frac{\rho^\alpha(s) g_H^\alpha(s) (1 - G_H^\alpha)^{N-2}}{\rho^\alpha(s) g_H^\alpha(s) (1 - G_H^\alpha)^{N-2} + (1 - \rho^\alpha(s)) g_L^\alpha(s) (1 - G_L^\alpha)^{N-2}} = \rho_0$ for all s , since $G_H^\alpha = G_L^\alpha$ also holds.

¹⁹The key idea is that the ratio $\frac{\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \geq 1 - \varepsilon} g^{(2)}(s) (1 - \rho(s)) ds}{\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \leq \varepsilon} g^{(2)}(s) \rho(s) ds}$ (that is, omitting the square sign from the numerator) is equal to 1 for any α , as the two states are completely symmetric. On the other hand, $\lim_{\alpha \rightarrow 1} \frac{(1 - \rho(s))^2}{1 - \rho(s)} = 0$ if it is known that $\rho(s) \geq 1 - \varepsilon > 0$, since in this case the high state is very likely and thus the posterior converges to 1 in probability.

Some assumption on the convergence rates is necessary for the result, since in the online Appendix we provide an example where Assumption CR is violated and the continuation value effect dominates the linkage effect for any precision level α .

As the signals become almost perfectly informative the (integrand of the) linkage effect (see formula (4)) dominates the (integrand of the) continuation value effect (see formula (5)) except for signals which indicate that the high state is very likely. Moreover, the continuation value effect is only relevant even for such signals if the precision of such signals are not too high in the limit, otherwise upon receiving a high signal, one is almost sure that it is the high state and thus the state revelation by the auctioneer would not change actions in the continuation problem by much. Therefore, if the signal precision converges not much slower in the high state than in the low state (an equal rate is more than sufficient), then the linkage effect is stronger than the continuation value effect.

To show that our results do not depend on our strong assumption about the auctioneer's signal, it is important to study the more realistic case where the auctioneer's information about the state is not fully precise. This also allows studying the case where the auctioneer can choose the precision of the signal he wishes to reveal. In the online Appendix we provide the general framework, analysis, and conduct some numerical calculations. Here, we report our findings for the special case where the auctioneer receives one of two possible signals s_H or s_L . Moreover, let $\Pr[s_H | H] = \Pr[s_L | L] = z \in [0.5, 1]$, that is, z measures the precision of the auctioneer's signal, which is common knowledge. While few general results are available about the behavior of the two effects (CV and LP) when signals are imperfectly precise, two observations still hold. First, when bidders have uninformative signals ($\alpha = 0$), the LP effect is still zero, while $CV > 0$ so revealing information (even not fully precise information) hurts the revenues of the auctioneer. Second, when the bidders are fully informed ($\alpha = 1$), by construction $LP = CV = 0$.

To obtain additional insight, we study Example 2 in the online Appendix with $k = d = 1$ and allow precision z to vary between 0.5 and 1. In Figure 3 we depict the two effects for a fixed level of precision z , and varying α between 0 and 1. The picture is qualitatively similar to the full precision case: the auctioneer reveals information if and only if the bidders have precise private signals. We obtain several further observations. First, for any fixed level of the signal precision of the bidders (α) if an auctioneer wants to reveal a signal with precision z , then he also wants to reveal any signal with higher precision. Consequently, the auctioneer has no incentive to garble his signal. Suppose that he observes the state fully ($z = 1$), but can commit to announce a signal with an arbitrary precision. We show the auctioneer would always choose to reveal his fully informative signal or no signal at all, but it is never revenue

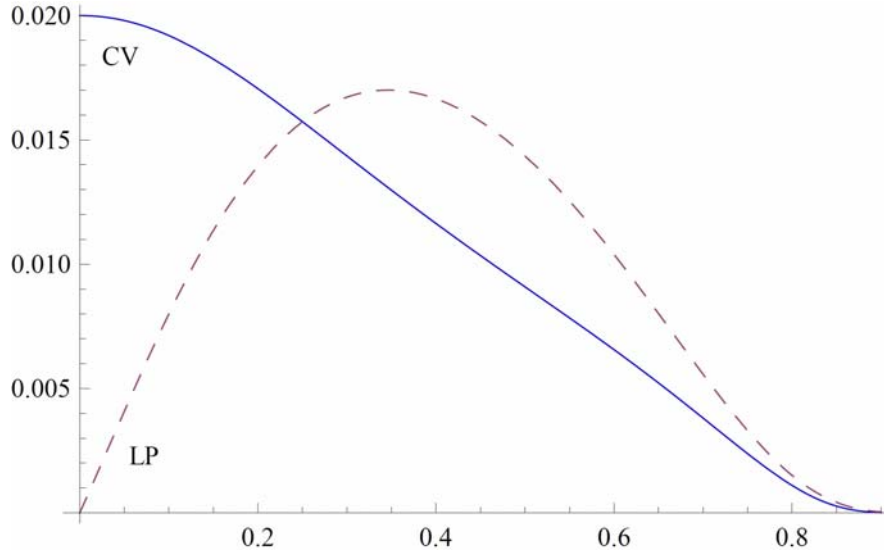


Figure 3: CV and LP as α varies and $z = 0.7$.

maximizing to choose a signal with intermediate precision.²⁰ Second, for some parameter values, revealing the fully informative signal is revenue enhancing compared to not revealing any signal, but revealing a partially informative signal hurts the auctioneer. Investigating the non-monotonicity of the revenue impact in the precision of the auctioneer’s signal is an interesting question left to future research.

3.3 Revenue when the winning bid is revealed

Another important question in auction design is whether any *bid* information should be revealed by the auctioneer. In settings in which the auctioneer does not possess information superior to the bidder’s information, one may ask whether the auctioneer prefers information exchange between bidders by revealing information about the bids. In the previous Section we showed that revealing the state after the auction decreases revenues. The argument there relied on the fact that the linkage effect is absent when information is revealed after the auction, while the continuation value effect is present. This suggests that revealing any information after the auction should hurt the revenues of the auctioneer. We show that this is indeed the case when the winning bid is revealed by the auctioneer. Let ER^{Bid} denote the expected revenue when the winning bid is revealed, assuming that V is decreasing.

²⁰In this paper we compare the information structures of complete revelation and no revelation at all. We also show that the seller should never garble his information. However, we do not consider asymmetric policies that reveal signals with different precision for different bidders. The usefulness of such signals was shown by Mares and Harstad (2003).

Proposition 6 *Revealing the winning bid after the auction decreases revenue relative to not revealing any information, $ER^{Bid} < ER^{None}$.*

Proof. By the same reasoning as in the case without information revelation, when the auctioneer reveals the winning bid the bid function is

$$b_b(s) = v - [\rho_{tie}(s)U_H(\rho_{tie}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{tie}(s))].$$

Comparing it with the case of no bid revelation yields

$$b_b(s) < v - [\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))] = b(s),$$

which follows from the fact that $\rho \in \arg \max_{q \in [0,1]} \rho U_H(q) + (1 - \rho) U_L(q)$. That is, the bid of a type s if the winning bid is revealed is lower than in the benchmark case of no information revelation. Therefore,

$$ER^{Bid} = \int_{\underline{s}}^{\bar{s}} g^{(2)}(s)b_b(s)ds < \int_{\underline{s}}^{\bar{s}} g^{(2)}(s)b(s)ds = ER^{None}.$$

So, revealing the bids after the auction decreases revenues. **Q.E.D.**

This result is similar to that of Perry and Reny (1999) who show that in a multi-unit (Vickrey) auction revealing information can decrease the bid of all bidders. We obtain the same result as them, although for different reasons.

4 First-price, second-price and English auctions

In the auction design literature, the linkage principle implies that the more an auction format links the payments to the types of the other agents, the higher the expected revenue is. In a first-price auction the expected payment conditional on winning does not depend on the types of the other bidders, while in a second-price auction and in an English auction it does. Therefore, the linkage principle implies that the second-price and English auctions yield higher expected revenues than the first-price auction. Similarly, an English auction links payments to others' bids (types) even more, since all the types except for the two highest are revealed by the bids at which those bidders are dropping out. Therefore, the English auction yields higher expected revenue than the second-price auction.²¹

²¹This argument relies on the effect of information revelation on equilibrium rents, as the equilibrium allocation in the standard setup is not affected by the information policy. This is not the case in Board (2009), where the information policy's effect on equilibrium allocation modifies the insights of the linkage principle.

In our model with endogenous outside options the linkage effect is counteracted by the continuation value effect, when we analyze whether the auctioneer should reveal his exogenous information about the state of the world. It is natural to ask whether the same is true when one compares the three standard auction formats fixing the information policy. Assume that the auctioneer does not reveal any information and runs a first-price, second-price or English auction. First, we observe that the second-price auction still revenue dominates the first-price auction.²² The key is that in both auctions the losers learn the same information; they only learn that there was a bidder with a higher signal than theirs. This implies that they take the same actions in the continuation decision problems, and therefore the presence of endogenous outside options does not change the comparison between the two formats.

The important novelty when comparing second-price and English auctions is that the English auction reveals more information, so it allows the losers to take better decisions in the aftermarket, and thus the continuation value effect favors the second-price auction over the English auction. Since the continuation value and the linkage effects work in opposite directions, one needs to assess whether the second-price or the English auction raises higher revenues. To present our result, we concentrate on a three bidder example, where a monotone equilibrium exists and the auctioneers' revenue is higher in the second-price auction than in the ascending auction.²³

Proposition 7 *The expected revenue of the English auction is sometimes less than the expected revenue of the second-price auction.*²⁴

In this specification the continuation value effect is stronger than the linkage effect, and revealing information via holding a more open auction decreases revenues. If one considers variations in the parameter values, then the revenue comparison has the same qualitative features as in the case of state revelation. For example, if the two states provide very different utility values (that is d is high), then the linkage effect dominates, and the auctioneer prefers the English auction. If α_H decreases or α_L increases, then information is more valuable, which favors the less transparent second-price auction.

²²This can be done by modifying the analysis of Krishna (2008), Section 7, pages 105-108. The formal argument is provided in our online Appendix.

²³The calculations are in the online Appendix.

²⁴As proven in the Appendix, this happens if $\rho_0 = 1/2$, $N = 3$, $g_L = 2(1 - s)$ and $g_H = 2s$, and

$$U_H(\rho) = \rho^{n-1} - \alpha_H \rho^n (n - 1),$$

and

$$U_L(\rho) = d - \alpha_L \rho^n (n - 1)$$

with $\alpha_H = \alpha_L = 1/n$, $n = 1.1$, $d = 1$.

5 Discussion

A monotone equilibrium does not always exist in our model. To discuss how our revenue comparison results change when a monotone equilibrium does not exist, let us revisit Example 2 and assume that the signals of the bidders are precise, that is, $a \geq \bar{a}$ for some appropriate value of \bar{a} . Then any monotone equilibrium is such that the linkage effect is stronger than the continuation value effect, regardless of the values of k and d ,²⁵ which suggests that the linkage effect tends to be stronger. However, as the next Example shows, this is an artifact of the assumptions needed for a monotone equilibrium to exist.

Example 3:

Consider an example where the intrinsic payoff differences between the two states are completely absent, but it is very important to *know* the state when making the price offer. Let $v = 2$, $\mu_H = \mu_L = 1$ and $F_H(x) = \frac{1-a(1-x)^2}{2-x}$ and $F_L(x) = \frac{1-ax^2}{2-x}$ with $a \in (0, 0.5]$. Sellers have mostly intermediate costs in the high state and more extreme costs in the low state; as a result, the revenue maximizing prices are very different in the two states.²⁶ Since the cost distributions F_H, F_L are not ranked by first order stochastic dominance, the induced value function may not be monotone in the belief. In fact, the two states are symmetric in that $V(\rho) = V(1 - \rho)$ for all $\rho \in [0, 1]$, and the value function is U-shaped. The resulting utilities in the two states are $U_H(\rho) = 1 - a(1 - \rho)^2$, and $U_L(\rho) = 1 - a\rho^2$. To fully specify the example, assume that $\rho_0 = 1/2$, $N = 2$, and $G_H(s) = s^2$, $G_L(s) = 2s - s^2$. Since the two states are symmetric, we concentrate on a "state-symmetric" equilibrium where $b(s) = b(1 - s)$ for all s . The posterior upon tying is $\tilde{\rho}_{tie}(s) = \Pr(H \mid s_1 = s, s_2 = s \text{ or } s_2 = 1 - s) = s$, and the posterior upon losing is $\tilde{\rho}_{lose}(s) = \Pr(H \mid s_1 = s, s_2 \in (s, 1 - s)) = s$. When the state is not revealed the equilibrium bid function is $b = v - [\tilde{\rho}_{tie}U_H(\tilde{\rho}_{lose}) + (1 - \tilde{\rho}_{tie})U_L(\tilde{\rho}_{lose})] = v - 1 + as(1 - s)$. The bid with state revelation is $b = v - V(0) = v - 1$ in the low state, and $b = v - V(1) = v - 1$ in the high state as well. Consequently, the revenue comparison favors not revealing the state. In general, if the two states are similar (that is $V(0) = V(1)$), then the linkage principle loses its bite, and although a monotone equilibrium does not exist, it follows that revealing the state decreases revenues.

The assumption of two states can be relaxed without changing the results as we discuss

²⁵To see this, note that a monotone equilibrium exists if $d \geq k/2$, so to be able to choose d, k such that a monotone equilibrium exists *and* $CV > LP$ it has to hold by (6) and (7) that $r = \frac{\int_0^1 g^{(2)}(0.5\rho_{lose}^2 + 0.5\rho_{tie} - \rho_{tie}\rho_{lose})ds}{0.5 - \int_0^1 g^{(2)}\rho_{tie}ds} > \frac{1}{2}$. However, numerical calculations show that $r \leq 1/2$ when bidders have precise signals (that is, $a \geq \bar{a}$).

²⁶The cost is 0 with probability $\frac{1}{2}$ ($\frac{1-a}{2}$) in the low (high) state, while the cost is less than 1 with probability 1 ($1 - a$) in the high (low) state. In the low state the cost is prohibitively high with probability a . In the high state a high price ($p = 1$) is optimal to capture all the sellers, but in the low state the most profitable offer is $p = 0$, since that already captures a large portion of the market.

next. We focus on comparing the revenues from the second-price auction with and without the revelation of the winning bid, the question addressed in Section 3.3 for the case of two states. Let $t \in [0, 1]$ denote the state of the world, and let $U_t(a)$ denote the continuation value when action a is taken in state t . Let g_t denote the conditional distribution of signals in state t , and let h the density function for the state of the world. Assuming that for all $U_t(a)$ is decreasing in t implies that there is an equilibrium with monotone bidding. Let $\rho_{tie}^t(s)$ be the density of state t if one ties at the top with signal s , and $\rho_{lose}^t(s)$ be the density of state t if one lost with signal s .²⁷ Then the optimal action after losing with signal s satisfies $a(s) = \arg \max_{x \in X} \int_{\underline{s}}^{\bar{s}} \rho_{lose}^t(s) U_t(x) dt$. Without bid revelation the equilibrium bid is $b_n(s) = v - \int_{\underline{s}}^{\bar{s}} \rho_{tie}^t(s) U_t(a(s)) dt$. With bid revelation the tying loser learns that he in fact tied with the winner and takes an action

$$a^*(s) = \arg \max_{x \in X} \int_{\underline{s}}^{\bar{s}} \rho_{tie}^t(s) U_t(x) dt. \quad (11)$$

Therefore, the equilibrium bid becomes $b_y(s) = v - \int_{\underline{s}}^{\bar{s}} \rho_{tie}^t(s) U_t(a^*(s)) dt$. By (11) it follows that $b_n(s) > b_y(s)$, so the revenue comparison is as in the two-state model.

The winning bidder may take an action in many situations of interest. Suppose that the winning bidder's continuation utility functions are U_H^w, U_L^w , which have similar properties to U_H, U_L , the continuation utility of the losers. We keep the assumption that the winner obtains a utility v from the object and that there are two states. Let us now concentrate on the question whether in the second-price format state revelation enhances or reduces revenues; the other questions can be studied similarly. Following similar argument as in the benchmark case, the equilibrium bid function without state revelation is

$$v + [\rho_{tie}(s)(U_H^w(\rho_{lose}(s)) - U_H(\rho_{lose}(s))) + (1 - \rho_{tie}(s))(U_L^w(\rho_{lose}(s)) - U_L(\rho_{lose}(s)))].$$

When the state is revealed, in the high state all bidders bid $v + V^w(1) - V(1)$, and in the low state all bid $v + V^w(0) - V(0)$. From these observations it is obvious that if the winner's continuation values are not very sensitive to the state of the world (that is $U_H^w - U_L^w$ is uniformly close to zero and thus V^w is close to being a constant function), then the revenue comparison is similar to the benchmark case where the winner's continuation problem was omitted. However, if the winner's continuation problem is important (compared to the losers'), then the continuation value effect *favours* information revelation²⁸, and thus

²⁷Formally, $\rho_{tie}^t(s) = \frac{h(t)g_t^2(s)G_t^{N-2}(s)}{\int_0^1 h(z)g_z^2(s)G_z^{N-2}(s)dz}$ and $\rho_{lose}^t(s) = \frac{h(t)g_t(s)(1-G_t^{N-1}(s))}{\int_0^1 h(z)g_z(s)(1-G_z^{N-1}(s))dz}$.

²⁸If more information is available after the auction, then the winner can make a better decision, which then makes bidders more aggressive since the winning prize has become more valuable.

transparent auctions are revenue enhancing.

Risk averse bidders in our framework lead to qualitatively similar results as our risk neutral benchmark. With risk averse bidders the revenue comparisons between the three formats (first-price -, second-price -, and English auctions) are ambiguous even in the standard interdependent value setup of Milgrom and Weber (1982). Therefore, we only compare revenues between the second-price auction with and without bid revelation.²⁹ Using similar considerations, as in the case of risk neutral bidders, one can calculate the equilibrium bid functions in a straightforward manner. Letting m denote the concave utility function, one can show that the equilibrium bid function without bid revelation b_n solves

$$\rho_{tie}U_H(\rho_{lose}) + (1 - \rho_{tie})U_L(\rho_{lose}) = m(v - b_n),$$

while if the winning bid is revealed, then the equilibrium b_b bid function becomes

$$\rho_{tie}U_H(\rho_{tie}) + (1 - \rho_{tie})U_L(\rho_{tie}) = m(v - b_b).$$

For the same reason as when bidders were risk neutral it still holds that $\rho_{tie}U_H(\rho_{lose}) + (1 - \rho_{tie})U_L(\rho_{lose}) < \rho_{tie}U_H(\rho_{tie}) + (1 - \rho_{tie})U_L(\rho_{tie})$, and thus $b_b < b_n$ follows, which implies that revealing the winning bid decreases revenues.

6 Conclusion

We study auctions with endogenous outside options determined through actions taken in the aftermarket. In contrast to the case of exogenous outside options, auctions with less information revelation may yield higher revenues. Opaque auctions decrease the information available to losing bidders, which leads to worse decisions in the aftermarket. This leads to worse outside options, and thus more aggressive bidding in the original auction. Effects that favor non-transparent auctions include a small payoff difference between the two states, a great value of information in the continuation problem, and imprecise signals of the bidders. The timing of information revelation is important: it is never optimal to reveal information after the auction, while it may be optimal to reveal information before the auction. We also show that a less transparent auction format, the second-price auction can yield higher revenues than an English auction, as it fosters less learning and provides lower continuation values for the bidders. The model is robust to introducing several states, and with respect to the winner having state dependent continuation functions.

²⁹If the seller considers revealing the *state*, then similar trade-offs apply as when the bidders were risk averse, but the calculations become more involved.

7 Appendix

In the Appendix we prove Proposition 1, and several useful results about the continuation problem. We start by establishing a monotonicity result:

Lemma 1 *For all $\rho' > \rho$ and $U_H(a') - U_L(a') > U_H(a) - U_L(a)$ it holds that if type ρ weakly prefers a' over a , then type ρ' strictly prefers a' over a . Therefore, for all $b' \in \alpha(\rho')$ and $b \in \alpha(\rho)$ it holds that $U_H(b') - U_L(b') \geq U_H(b) - U_L(b)$. If for some $\rho' > \rho$ it holds that $e \in \alpha(\rho')$ and $f \in \alpha(\rho)$, then $U_H(e) \geq U_H(f)$ and $U_L(e) \leq U_L(f)$; and for almost all $\rho \in [0, 1]$ if $c, d \in \alpha(\rho)$ then $U_H(c) = U_H(d)$ and $U_L(c) = U_L(d)$. Function U_H is monotone increasing, while U_L is monotone decreasing.*

Proof of Lemma 1:

Proof. Suppose that $\rho' > \rho$ and $U_H(a') - U_L(a') > U_H(a) - U_L(a)$, and type ρ weakly prefers a' over a , that is

$$\rho U_H(a') + (1 - \rho)U_L(a') \geq \rho U_H(a) + (1 - \rho)U_L(a). \quad (12)$$

Then $U_H(a') - U_L(a') \geq U_H(a) - U_L(a)$ implies that

$$(\rho' - \rho)(U_H(a') - U_L(a')) > (\rho' - \rho)(U_H(a) - U_L(a)).$$

Adding the last inequality to (12) implies that

$$\rho' U_H(a') + (1 - \rho')U_L(a') > \rho' U_H(a) + (1 - \rho')U_L(a),$$

which establishes the first claim. To prove the second statement, suppose that $U_H(b) - U_L(b) > U_H(b') - U_L(b')$. Then the first statement implies that type ρ' strictly prefers b over b' , which contradicts with the assumption that $b' \in \alpha(\rho')$. The second statement implies that $U_H(e) - U_L(e) \geq U_H(f) - U_L(f)$. Then $U_H(e) < U_H(f) \implies U_L(e) < U_L(f)$, which implies that e is worse than f for any beliefs, and thus $e \in \alpha(\rho')$ could not hold. This contradiction establishes the third claim. To prove the last claim let $\tau(\rho) = \max_{x \in \alpha(\rho)} U_H(x) - U_L(x)$. The second statement implies that τ is weakly increasing, and thus it is almost everywhere continuous. Moreover, at every continuity point ρ of τ it holds that for all $c, d \in \alpha(\rho)$, $U_H(c) - U_L(c) = U_H(d) - U_L(d)$.³⁰ Then suppose that $U_H(c) > U_H(d)$. In this case, it would

³⁰Suppose that $c, d \in \alpha(\rho)$, and $U_H(c) - U_L(c) > U_H(d) - U_L(d)$. Then for all $\rho' > \rho$ it holds by the second claim that for any $c' \in \alpha(\rho')$, $U_H(c') - U_L(c') \geq U_H(c) - U_L(c)$, and thus $\tau(\rho') \geq U_H(c) - U_L(c)$. Similarly, for any ρ'' and $d' \in \alpha(\rho'')$, $U_H(d') - U_L(d') \leq U_H(d) - U_L(d)$, and thus $\tau(\rho'') \leq U_H(d) - U_L(d)$. Therefore, the function τ must have a jump at such a ρ .

follow that $U_H(c) > U_H(d)$, implying that c dominates d and contradicting $d \in \alpha(\rho)$. This contradiction establishes the last result about functions U_H, U_L . The monotonicity claim about U_H, U_L then follows by construction. **Q.E.D.**

Next we prove that a useful envelope condition holds for almost all ρ . In particular they hold at every continuity point of U_H, U_L . Let us formally state our claim first:

Lemma 2 *For almost every ρ it holds that*

$$a \in \alpha(\rho) \implies V'(\rho) = U_H(a) - U_L(a) \quad (13)$$

and

$$V'(\rho) = U_H(\rho) - U_L(\rho). \quad (14)$$

Proof. Take any ρ and let $a \in \arg \max_{x \in \alpha(\rho)} U_H(x)$, $b \in \arg \min_{x \in \alpha(\rho)} U_H(x)$. Note, that by definition of $\alpha(\rho)$ it must hold that $a \in \arg \min_{x \in \alpha(\rho)} U_L(x)$, $b \in \arg \max_{x \in \alpha(\rho)} U_L(x)$ and thus for all $x \in \alpha(\rho)$

$$U_H(a) - U_L(a) \geq U_H(x) - U_L(x) \geq U_H(b) - U_L(b).$$

Next, note that for all $\rho' > \rho$ it holds that $V(\rho') \geq U(\rho', a)$. Therefore,

$$V(\rho') - V(\rho) \geq (\rho' - \rho)(U_H(a) - U_L(a)).$$

Also, the right hand derivative of a convex function exists everywhere, therefore the right hand derivative at ρ satisfies

$$V'_+(\rho) \geq U_H(a) - U_L(a).$$

A similar argument yields that the left hand derivative satisfies

$$V'_-(\rho) \leq U_H(b) - U_L(b).$$

Since $V'(\rho)$ exists almost everywhere, therefore for almost every ρ it must hold that $U_H(a) - U_L(a) = U_H(b) - U_L(b)$. Therefore, wherever a derivative exists (which is almost everywhere) it holds that $V'(\rho) = U_H(a) - U_L(a)$ for all $a \in \alpha(\rho)$, which establishes that (13) for almost all ρ . To establish that ((14)) holds for all continuity points of U_H, U_L (which is almost everywhere) note that the proof of Lemma 1 implies that for any such continuity point ρ it holds that if $c, d \in \alpha(\rho)$ then $U_H(c) = U_H(d)$ and $U_L(c) = U_L(d)$. Therefore, the argument establishing (13) applies to show that $V'(\rho) = U_H(c) - U_L(c) = U_H(\rho) - U_L(\rho)$, concluding the proof. **Q.E.D.**

We are ready to prove the existence of a monotone equilibrium stated in Proposition 1.

Proof of Proposition 1:

Proof: In the proof below, we use that U_H is weakly increasing, while U_L is weakly decreasing in ρ for any continuation value problem (see Lemma 1), that is in the high (low) state it is better to take actions that are optimal when the high (low) state is more likely. Then we prove that at all continuity points of U_H, U_L any optimal action provides the same utility in the continuation problem, thus there is a unique optimal action in terms of payoff consequences. Using this observation the first order condition (1) is shown to be necessary for optimal bidding for almost all s (that is for all s such that $\rho_{lose}(s)$ is a continuity point of U_H, U_L). To check that global sufficiency conditions and strict monotonicity of b also hold for the bidders' problem we use basic properties of affiliated random variables, extending the analysis of Milgrom and Weber (1982).³¹

From Lemma 1, we know that for almost all ρ all the optimal actions induce the same utilities in both states. We first concentrate on such values of ρ and then by construction the induced utilities by the optimal action(s) are equal to $U_H(\rho), U_L(\rho)$. We discuss what happens at other values of ρ at the end of the proof.

First, we show that the above defined bid function constitutes an equilibrium. Symmetry of b is immediate, while monotonicity follows from the facts that ρ_{tie}, ρ_{lose} are increasing, $\rho_{tie} < \rho_{lose}$ and that the monotonicity of V implies that $U_H(x) \leq U_L(x)$ for all $x \in A$. To see this, note first that the function $t(s) = \rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))$ may have a jump. However, $t = V(\rho_{lose}) + (\rho_{tie} - \rho_{lose})(U_H(\rho_{lose}) - U_L(\rho_{lose}))$, and since V and $\rho_{tie} - \rho_{lose}$ are continuous, $\rho_{tie} - \rho_{lose} < 0$ and any jump of $U_H - U_L$ is upward, therefore any jump of t must be downward. Second, the derivative of t exists almost everywhere (where V'' exists), and

$$\begin{aligned} & \frac{d}{ds} [\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))] = \\ & \rho'_{tie}(s)(U_H(\rho_{lose}(s)) - U_L(\rho_{lose}(s))) + \rho'_{lose}(s)V''(\rho_{lose})(\rho_{tie}(1 - \rho_{lose}) - (1 - \rho_{tie})\rho_{lose}) < 0 \end{aligned}$$

follows from the observations above. But this is equivalent to $t'(s) < 0$. Thus $b = v - t$ has either a jump upward, or is (almost everywhere) differentiable with a positive derivative, which implies that b is monotone increasing.

Next, we show that if it is known that $s_1 = s_2 = s$, then winning with $b(s)$ yields the same utility as losing and acting in the future as if the probability of the high state was $\rho_{lose}(s)$. Losing yields a continuation utility that is equal to $\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 -$

³¹To prove that b is a strictly increasing function if V is monotone we use two observations. First, the tying and the losing posterior are monotone in the signal. Second, the tying posterior is lower than the losing posterior, which holds if bids are monotone.

$\rho_{tie}(s))U_L(\rho_{lose}(s))$ by construction, while winning with bid $b(s)$ yields a utility $v - b(s)$, which is equal to the continuation utility upon losing.

It also has to be established that if $s_i = s$ then winning against a type $y > s$ with bid $b(y)$ is unprofitable, while if $y < s$ then winning against type y with bid $b(y)$ is profitable.³² Let us just inspect the $y > s$ case, the other one is similar. In this case winning, upon tying, yields a utility of

$$v - b(y) = \rho_{tie}(y)U_H(\rho_{lose}(y)) + (1 - \rho_{tie}(y))U_L(\rho_{lose}(y)).$$

To calculate the utility from losing, upon tying, let us introduce the relevant tying posterior when one bids $b(y)$ and has type s as $h(s, y) = \frac{\rho_0 g_H(s) g_H(y) G_H^{N-2}(y)}{\rho_0 g_H(s) g_H(y) G_H^{N-2}(y) + (1 - \rho_0) g_L(s) g_L(y) G_L^{N-2}(y)}$. By the fact that g_H and g_L satisfy the MLRP, it follows that $\rho_{tie}(y) > h(s, y)$. One can similarly define the losing posterior as $n(s, y) = \frac{\rho_0 g_H(s) (1 - G_H^{N-1}(y))}{\rho_0 g_H(s) (1 - G_H^{N-1}(y)) + (1 - \rho_0) g_L(s) (1 - G_L^{N-1}(y))}$. Again, the MLRP condition implies that $\rho_{lose}(y) > n(s, y)$. Then the utility upon losing (and tying) can be written as $h(s, y)U_H(n(s, y)) + (1 - h(s, y))U_L(n(s, y))$.

Next, we show that

$$\begin{aligned} & h(s, y)U_H(n(s, y)) + (1 - h(s, y))U_L(n(s, y)) \geq \\ & \geq h(s, y)U_H(\rho_{lose}(y)) + (1 - h(s, y))U_L(\rho_{lose}(y)). \end{aligned} \quad (15)$$

To see this, note that by construction

$$\begin{aligned} & n(s, y)U_H(n(s, y)) + (1 - n(s, y))U_L(n(s, y)) \geq \\ & \geq n(s, y)U_H(\rho_{lose}(y)) + (1 - n(s, y))U_L(\rho_{lose}(y)). \end{aligned} \quad (16)$$

Also, $\rho_{lose}(y) > n(s, y) > h(s, y)$ and the monotonicity of V implies that $U_H(n(s, y)) - U_L(n(s, y)) \leq U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))$. Thus it follows that

$$(h - n)(U_H(n(s, y)) - U_L(n(s, y))) \geq (h - n)(U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))). \quad (17)$$

Adding up (16) and (17) implies (15). Then using (15), the utility difference between losing and winning satisfies

$$\Delta = hU_H(n) + (1 - h)U_L(n) - (\rho_{tie}(y)U_H(\rho_{lose}(y)) + (1 - \rho_{tie}(y))U_L(\rho_{lose}(y))) \geq$$

³²Again, we need to use the relevant tying belief $\Pr(H | s_1 = x, s_2 = y)$ and the relevant action inducing belief upon losing $\Pr(H | s_1 = x, s_2 \geq y)$.

$$\begin{aligned} &\geq hU_H(\rho_{lose}(y)) + (1-h)U_L(\rho_{lose}(y)) - (\rho_{tie}U_H(\rho_{lose}(y)) + (1-\rho_{tie})U_L(\rho_{lose}(y))) = \\ &= (h-\rho_{tie})(U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))) \geq 0, \end{aligned}$$

where the last inequality follows, because $\rho_{tie} > h$ and $U_H(\rho_{lose}(y)) \leq U_L(\rho_{lose}(y))$ by monotonicity of V . Therefore, it is indeed more profitable to lose against a type y than to win if one's type is $s < y$. This concludes the proof of global optimality for the bidders.

Uniqueness of b as in (1) follows from the above argument as well, since upon tying indifference has to hold in an ex-post equilibrium which yields exactly (1) after taking it into account that the equilibrium is symmetric and monotone. The only caveat is that the bid function is not determined at the (at most countably many) discontinuity points of U_H, U_L . At such a belief ρ , the optimal action in the continuation problem is not unique which introduces multiple optimal bids when the belief is ρ . However, there are at most countably many such jump points, so this multiplicity arises only for a small set of types, and for all other beliefs the equilibrium bid is pinned down by formula (1).³³ If V is not (strictly) monotone, then it is easy to show that the bid function (1) is not strictly increasing.

For the last result it is sufficient to prove that V is (strictly) monotone if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$. First, if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$, then by Lemma 2 $V' < 0$ for all $\rho < 1$ whenever the derivative exists, which implies that V is indeed monotone decreasing. Second, suppose that for some $\rho^* < 1$ it holds that $U_H(\rho^*) \geq U_L(\rho^*)$. Then Lemma 1 implies that U_H is increasing, while U_L is decreasing in ρ , and thus for all $\rho \geq \rho^*$ it holds that $U_H(\rho) \geq U_L(\rho)$. Therefore, using the envelope formula (14) that holds for almost all ρ implies that for all $\rho > \rho^*$ it holds that $V(\rho) \geq V(\rho^*)$ and thus V is not (strictly) monotone decreasing. **Q. E. D.**

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³³When the value function is smooth such discontinuity of W_H, W_L cannot occur and the equilibrium bid is unique for all x . Moreover, the function b is continuous in this case.

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